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A 544536

The Gift of

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QA
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A
SYSTEM
OF
ASTRONOMY.

CONTAINING THE
INVESTIGATION AND DEMONSTRATION
OF THE
ELEMENTS
OF THAT
SCIENCE.

Motibus aſtrorum nunc quæ ſit cauſa canamus.

Lucr. L. V.

RADCLIFFE
OBSERVATORY
OXFORD.

LONDON:
Printed for J. Nourse, in the Strand;
Bookſeller in Ordinary to his MAJESTY.
MDCCLXIX.

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Professor William H. Butts
10-14-1930

THE PREFACE.

9-27-37 HCM

I am very ſenſible how difficult a thing it is to write well upon the ſcience of Aſtronomy; *by reaſon the ſubject is ſo comprehenſive, and conſiſts of ſo many parts, and is connected with ſo many other ſciences, which it requires the perfect knowledge of; and beſides, is a work of ſo much time, that a man had need have the life of a* Mathuſalem, *to go thro' the whole of it. The periods even of ſome of the planets are ſo long, that they perform but a few of them in a man's lifetime. And much more are thoſe of the comets, ſeveral of them being many hundred years in performing one revolution. Whence, it comes that moſt of their periodical times, and the poſition of their orbits, are unknown to us. And even the moon ſo near hand us, is affected with ſuch variable motions, that they cannot be exactly accounted for to this day.*

That Aſtronomy was one of the firſt ſciences that men ſtudied, is plain from the nature of the thing. For as it is neceſſary for people to know how time paſſes away, it would be natural for them to enquire into the lengths of months and years. The firſt thing that offers itſelf to our ſenſes, is the ſun's diurnal motion, by which he ſeems to move from eaſt to weſt, returning to the eaſt again in 24 hours. The next thing is his annual motion, by which he advances northerly and ſoutherly, making the days longer and ſhorter, and thereby determining the ſeaſons of the year. Theſe are *fundamental meaſures of time, and therefore it was natural or rather unavoidable to take notice of them, and to make proper obſervations thereof, for ſetting out the length of the year, and to divide it into its proper ſeaſons; which were neceſſary to be known, as men had different ſorts of work to perform at different ſeaſons of the year.*

Nothing could be more eaſy for them, than to obſerve the increaſe and decreaſe of the ſhadows upon the ſurface of the earth, according to the different altitudes of the ſun, at different times of the year. Whence by marking the extremity thereof, it would be very eaſy, within a few days, to ſee when the ſhadow came again to the ſame point. The only difficulty would be, to keep a true account of the days. But this they would ſoon learn to do, by a few trials. So I do not doubt, but the length of the year would ſoon be determined, which a few years experience would teach them to do, near the truth. By the ſame means the moon's motion, or the length of the month would be found; which would aſſiſt them in finding the length of the year more exactly. And thus one thing after another would be found out, ſo far as they could be any way uſeful in theſe times. It is true, the ſcience of Aſtronomy could be carried to no great length for want of letters, and the knowledge of other arts.

In all ages, as far as hiſtory reaches, the moſt learned and eminent men in all countries have been encouragers and promoters of Aſtronomy; and by general conſent, it has always been accounted the nobleſt of the ſciences; as it has no leſs than the whole world for its object. It has been ſtudied by kings and princes, and a great deal of time has been ſpent in cultivating this ſcience; in which a few of the laſt ages have done more, and brought it further to perfection, than all the ages of the world that went before.

It is reported, that the Babylonians *had 700 years obſervations engraven upon bricks; and that when* Alexander *took* Babylon, *he cauſed almoſt two thouſand years aſtronomical obſervations to be carried away; which if true, is ſuch a period of time, and ſuch a fund of obſervations, as we have not the like to ſhew at this time. But theſe coming into the hands of idle, careleſs, and tyrannical people, were moſt of them loſt, and aſtronomers had to begin again, ſo arts and ſciences, as well as the ſtars, have their periods, their ſtations and retrogradations.*

Joſephus *writes, that* Seth, *or his nephews, firſt obſerved the order of the heavens, and the courſes of the ſtars. And it is ſaid* Neptune *was the firſt ſailor, and guided his ſhip in the night by the ſtars; and for his great ſkill was called the God of the Sea. His ſon* Belus *was an aſtronomer, and inſtituted colleges of aſtronomers.* Job *was acquainted with the aſtronomy of theſe times; for he more than once mentions the conſtellations of* Arcturus, Orion, *and the* Pleiades.

The Egyptians *learned from the* Edomites *aſtronomy and navigation. For the Edomites uſed to make long voyages upon the Red Sea, for the ſake of merchandiſing. And when they were conquered by* David, *they fled into* Egypt, *and to the coaſts of the Mediterranean, and to the* Perſian Gulf. *They afterwards began the like voyages upon the Mediterranean. Their year was the luniſolar year derived from* Noah, *and at that time conſiſted of* 360 *days. By the beliacal riſings and ſettings of the ſtars, they found the year to conſiſt of* 365 *days, therefore they added* 5 *days to the old year. This was in the reign of* Ammon. *Afterwards they formed the ſtars into conſtellations; and alſo made obſervations of the planets, which they called after their gods.*

When the Ethiopians *invaded* Egypt, *the Egyptians that fled to* Babylon, *carried with them the Egyptian year of* 365 *days, and the art of Aſtronomy, and founded the æra of* Nabonnaſſar. *The practice of obſerving the ſtars, was carried into other countries;*

the Greeks *had all their aſtronomical learning from* Egypt. *Then* Atlas *formed the ſphere of the* Lybians, *and* Chiron *that of the* Greeks, *and the* Chaldeans *another ſphere for their own uſe.*

After the art of Aſtronomy was carried to Greece; Thales, Pythagoras, *and* Eudoxus, *went thither to be inſtructed therein.* Thales, *who lived* 580 *years before Chriſt, began to obſerve the ſolſtices and equinoctial points; and firſt of any body predicted eclipſes. He predicted a total eclipſe of the ſun* 585 *years before Chriſt, which happened May* 28. *And another eclipſe of the moon.* Pythagoras *had the true ſyſtem of the world from the Egyptian prieſts, which was afterwards laid aſide for another ſyſtem invented by* Ptolemy, 70 *years after Chriſt, and from thence called the* Ptolemaic Syſtem. *This continued in vogue till* Copernicus *revived the Pythagorean ſyſtem in* 1515, *or thereabouts.* Eudoxus, *in deſcribing the ſphere, placed the ſolſtices and equinoxes in the middles of the conſtellations,* Aries, Cancer, Libra, *and* Capricorn.

In the following ages Aſtronomy began to improve apace. Tycho Brahe, *a Daniſh Nobleman and a great aſtronomer, furniſhed an obſervatory with proper inſtruments, made obſervations of the ſtars, and publiſhed a catalogue of the places of* 770 *ſtars.* Keplar, *a German aſtronomer, diſcovered the orbits of the planets to be ellipſes, with the ſun in the focus; and found out the true laws of their motions.* Galileus, *an Italian philoſopher, by the help of teleſcopes, diſcovered the ſatellites of* Jupiter *and* Saturn, *the ring of Saturn, &c.* Hevelius *publiſhed a larger catalogue of ſtars than* Tycho's. *And Mr.* Flamſteed *publiſhed a catalogue containing three thouſand ſtars. And ſeveral others made valuable additions to this ſcience. But the true theory of the planetary motions was ſtill wanting, till Sir* J. Newton *publiſhed his* Principia; *wherein he has demonſtrated, from the* *laws of motion, not only the true figures of the orbits of the planets, but that the irregular motions of the ſatellites, are conformable to the laws of gravity. Agreeable to theſe principles, Dr.* Halley *compoſed a ſet of tables, for all the planets and ſatellites, printed but lately; which is the only entire ſet of tables that we have.*

The ancients had ſome notion of the celeſtial bodies gravitating towards one another. Thus Lucretius *L. V. thinks the world to be infinite; for otherwiſe, ſays he, the bodies that are on the outſide, would by their gravity, be drawn towards thoſe on the inſide; and would long ſince have all met together in a heap in the middle. Yet they knew nothing of the law by which they were moved. And very likely they might think that gravity was the ſame at all diſtances; and it was very natural for them to think ſo, as this is the ſimpleſt ſuppoſition that can be made.*

No ſcience is converſant with objects more numerous, greater, or more glorious, or at ſuch great diſtances; being for any thing we know diſperſed thro' infinite ſpace. Their motions being the principal thing to be conſidered, aſtronomers can by their art, determine how great their motions are, how far they go off, and how near they approach in their return; the times they take up in going and returning, the laws of their motions, and the forces of gravity in the ſeveral parts of their orbits. The grand law which obtains in the whole ſyſtem is this, that every part of matter attracts and is attracted, directly as the attracting matter, and reciprocally as the ſquare of the diſtance. This law is the bond of nature, it keeps the planets and comets within their proper limits, and hinders them from running into other ſyſtems. And hence it comes, that the revolutions of the planets are performed with ſuch exactneſs and regularity round the ſun, which is their common center of motion.

By this art aſtronomers can tell, for many years to come, the eclipſes of the ſun and moon, their quantities and durations, alſo the moon's full and change, the aſpects of all the planets, their latitudes and declinations at any time propoſed.

By the help of Aſtronomy, our navigation is improved, and by this means trade and commerce is carried on. For by having the moon's true place, the longitude of a ſhip at ſea, or of any place at land may be found; which is a material point in geography. All chronology is founded upon it, without which we could have no certain account of hiſtory.

By the wonderful diſcoveries made in this ſcience, we can comprehend the vaſt extent of the univerſe, and the amazing beauty of its ſtructure; which gives us a grand and magnificent notion of the world and all its parts; at the ſame time that we ſee the earth which we inhabit, is but a poor, ſmall, trifling part of the whole.

By improving the theory, the art itſelf is brought to greater perfection, beyond what obſervations alone could do. For by this means we can make a better uſe of obſervations, than if we had nothing elſe to direct us.

If Aſtronomy is ſo much eſteemed for the great advantages we receive by it; it is no leſs entertaining to the curious, by making us underſtand the ſyſtem of the world, and the proportion, ſymetry and harmony of the ſeveral parts. For what is more delightful than to ſee ſo many great and glorious bodies, at immenſe diſtances, performing their ſeveral revolutions, in their due and ſtated times.

This likewiſe, of all things, gives us the juſteſt notions of the wiſdom and power of the Deity; that can form ſuch a vaſt ſyſtem, and continue it in motion for ſo many ages. For tho' the mere laws of gravity are ſufficient to keep it going, when once put into motion; yet mere mechanical laws could never *give it thoſe particular motions at firſt; there wants a firſt mover to do this. All theſe motions being ſo nicely adjuſted to one another, ſhew that they were all originally contrived and executed by ſome powerful being, acting with council and deſign, which demonſtrates the profound ſkill of the great architect.*

A great deal of time and pains have been ſpent in finding out the true ſyſtem of the world, or ſuch an hypotheſis as could account for all the phænomena. And various ſyſtems have appeared at different times, each of which could account for ſome particulars. But it is found at laſt that none of them could account for all of them, but the Copernican hypotheſis. The Copernican ſyſtem then is the true ſyſtem. From hence the motions of the heavenly bodies, and their cauſes, are perfectly underſtood; and by which all their appearances may be demonſtrated. And the ſituations of all the ſtars are better known, than the ſituations of a great many places on the earth are. For to find their latitude and longitude exactly, we muſt be beholden to aſtronomical obſervations.

Altho' great advances in Aſtronomy have been made from time to time; yet in ſuch a vaſt ſubject, it is no wonder if ſome things be ſtill wanting. The moon's motions are ſo irregular, and ſo infinitely diverſified, that no theory of her motions has yet been found, but it will vary in a few years, and differ from the obſervations. It is a work of great labour to reduce all her motions to certain rules, and compoſe tables thereof. Which, when they are made, muſt be proved, by a great number of obſervations before they can paſs, without which they are not to be depended on.

It would be much for the eaſe of calculation, if the ſexagenary account was laid aſide, and the decimal one ſubſtituted in its room. For there are ſo many reductions in the one, that make it exceeding tedious, which are entirely avoided in the other. But that tyrant

rant Cuſtom *has already got poſſeſſion of the former, and is likely to keep it.*

As to this book, the deſign is the ſame as in the former volumes; and that is, to lay down the principles of Aſtronomy, in as ſhort a compaſs as I could, to make it intelligible; and to ſhew the manner of calculating ſome of the principal aſtronomical problems; omitting things of leſſer moment, to keep the book within due bounds, ſuitable to the other parts of the Curſus. This is a ſubject of ſo great ſcope and extent, that to take in all things belonging thereto, would fill a very large volume. But if the reader underſtand the more difficult matters, he cannot fail getting thro' the eaſier. To this end, I have deſcribed the ſyſtem of the world and all its parts; the motions of the planets and their periods. Then you have the principal aſtronomical problems, ſolved by ſpherical trigonometry and by the globes. Then I have given the elementary part of Aſtronomy, being what depends upon obſervations. Then the theory of the primary planets. The theory of the moon and ſatellites, and laſtly the calculation of eclipſes.

I have no room here for aſtronomical tables, I have made uſe of Dr. Halley*'s, which is ſufficient for ſhewing the learner the method of working. But I cannot be anſwerable for the truth of theſe or any other tables. It is ſufficient to ſhew the method of working. For if the reader knows how to work with one ſet of tables, it will be equally eaſy for him to work with any other ſet, which he ſhall think better and more exact, when better can be had.*

W. Emerſon.

THE

CONTENTS.

SECT. I.

OF the Syſtem of the World, and the explanation of the Phænomena, cauſed by the motions of the Sun, Moon and Stars. The diſtances and periods of the Planets. The nature of Eclipſes of the Sun and Moon. Pa. 2

SECT. II.

Aſtronomical Problems, depending on the circles of the Sphere, calculated by ſpherical Trigonometry. 45

SECT. III.

The uſe of the Globes in Aſtronomy, and the ſolution of the principal Problems of the Sphere thereby. 105

SECT. IV.

The determination of the magnitude and ſituation of the orbits of the Earth and Planets; their periodic times; the places of the Stars; and all ſuch aſtronomical matters as depend upon Obſervations. This contains the Elements of Aſtronomy. 128

SECT. V.

The Theory of the primary Planets, and Comets; the calculation of their places. The equation of time. The times when the Planets are ſtationary; when they tranſit the Sun's diſk; when they are in conjunction; their quantities of matter, diameters, denſities, figures, weight on their ſurfaces; and their actions upon one another. 194

SECT. VI.

The Theory of the Moon and Satellites; the Sun's diſturbing force upon them. The errors in the Moon's and Satellite's motions. Conſtruction of tables, and their uſe in finding the Moon's place, and the time of the Syziges. The Moon's libration, her quantity of matter, denſity, &c. To find her diſtance from a Star. The errors of the Satellites. pa. 272

SECT. VII.

The Eclipſes of the Sun and Moon, and of Jupiter's firſt Satellite. 326

ASTRONOMY.

DEF. I.

ASTRONOMY is the ſcience which treats of the motions of the heavenly bodies, and all the phœnomena ariſing therefrom.

DEF. II.

A Syſtem is a particular diſpoſition or ſituation of the ſtars and planets among one another, and the particular motions they are ſuppoſed to have, according to ſuch or ſuch an hypotheſis.

DEF. III.

Fixt Stars are thoſe that never change their places or ſituations among one another.

DEF. IV.

Planets are thoſe ſtars that change their places, and move round ſome center.

DEF. V.

Satellites are thoſe ſmall ſtars that move round the larger planets.

DEF. VI.

Comets are a ſort of ſtars that appear but ſometimes, and move off to a great diſtance, and become at laſt inviſible, and remain ſo a long time.

DEF. VII.

An Eclipſe is a real or apparent deprivation of light from the whole or part of a luminous body.

SECT. I.

Of the Syſtem of the World, and the Explanation of the Phœnomena cauſed by the Motions of the Sun, Moon and Stars. The Diſtances and Periods of the Planets; the Eclipſes of the Sun and Moon.

PROP. I. *Prob.*

Fig. *To deſcribe the planetary ſyſtem, and conſtruction of the world.*

MANY hypotheſes have been formed for explaining the motions and appearances of the heavenly bodies; and every hypotheſis that ever was framed, could account for ſome or other of them; but ſince at that time men were ignorant of the laws of motion, it could not be expected that they could hit upon the true ſyſtem, or ſuch a one as could explain all the phœnomena. And moreover ſince the uſe of teleſcopes, numerous other phœnomena have offered themſelves, which were quite unknown before; from whence it followed that almoſt all theſe hypotheſes fell ſhort in accounting for them; and therefore ſuch hypotheſes could not be true. While theſe things were thus diſputable, and one contended for one hypotheſis, and another for another, Sir Iſaac Newton laid down the laws of nature and motion, and comparing all the phœnomena in the heavens, he found out the true ſyſtem of the world, and demonſtrated by unanſwerable arguments, that it could not poſſibly be any otherwiſe without the utter ſubverſion of all the laws of nature. The true ſyſtem founded upon the laws of nature, and true mechanical principles, is that which follows.

Fig. 1.

1. In this glorious ſyſtem, the ſun is placed nearly immovable in the center of the orbits of all the planets, and in theſe orbits they move round the ſun, each in its periodic time. The ſun as has been ſaid keeps always in or near the ſame place; but has a rotation round its axis, from weſt to eaſt in about 25 days. The orbits of all the planets are nearly circular, having the ſun for their center; but in ſtrictneſs they are ellipſes, having the ſun in the focus of each of them. Theſe orbits are not all in one plane, and yet do not vary a great deal; they interſect one another in lines that paſs thro' the center of the ſun, the places of the orbits where they interſect are called the *Nodes*. All the planets move round the ſun the ſame way, which is from weſt to eaſt. And their diſtances from the ſun are nearly as repreſented in the figure. Let S be the Sun, M Mercury, V Venus, E the Earth, N Mars, J Jupiter, and R Saturn; and their diſtances from the ſun are as follow, the diſtance of Mercury from the ſun is 4, of Venus 7, of the earth 10, of Mars 15, of Jupiter 52, and of Saturn 95, of equal parts of a right line. Alſo the periodic time, or the time of a revolution of each planet in its orbit is as follows, Mercury revolves in 3 months, Venus in 7 ½ months, the Earth in a year, Mars in 2 years, Jupiter in 12 years, and Saturn in 30 years; ſo that in this ſyſtem the earth is one of the planets.

2. Beſides theſe which are called primary planets; three of them have others likewiſe revolving about them; all which likewiſe revolve from weſt to eaſt; theſe are called *Secondary Planets* or *Satellites*. The earth E has one *l* called the *Moon*, which revolves in 27 ⅓ days. Jupiter J has 4 of

theſe

Fig. 1. theſe moons or Satellites, and Saturn R has 5 of them. Nobody ever had the leaſt knowledge of theſe Satellites of Jupiter and Saturn, till the invention of teleſcopes. The diſtances of theſe Satellites from their reſpective primary planets are as follows; the moon from the earth is diſtant 60 of the earth's ſemidiameters. The four Satellites of Jupiter, are diſtant from Jupiter, 5, 8, 12, 23, of Jupiter's ſemidiameters; and their periodic times are $1\frac{3}{4}$, $3\frac{1}{2}$, 7, $16\frac{1}{2}$ days reſpectively. Likewiſe the diſtances of Saturn's 5 Satellites from Saturn, are 2, $2\frac{1}{2}$, $3\frac{1}{2}$, 8, and 24 ſemidiameters of Saturn's ring. And their periodic times are 2, $2\frac{3}{4}$, $4\frac{1}{2}$, 16, $79\frac{1}{4}$ days reſpectively. All theſe meaſures are only taken in round numbers.

3. Beſides the planets and their Satellites, there is another ſort of bodies revolving about the ſun, which only appear ſometimes, theſe are the *Comets*; and they move either in very long ellipſes, which run to an immenſe diſtance from the ſun; or in parabolic curves, and then they never return. Theſe move in all manner of directions round the ſun; ſome the ſame way as the planets, and ſome the contrary way, they cut the plane of the earth's orbit, in all ſorts of angles, ſome greater and ſome leſſer. And ſome of theſe bodies are ſeveral hundred years in making a revolution; and therefore the periodic times of few of them are known. Nor can their number be determined for want of a proper ſeries of obſervations. There has not been above forty of theſe bodies obſerved; and of theſe ſome may happen to be the ſame, coming about again. They are diſtinguiſhed from other ſtars, by a luminous ſtream of light which they emit (called the tail), when they come near the ſun.

4. The inclination of the orbits of the planets to the plane of the earth's orbit are as follows; the orbit of Mercury 7 degrees, of Venus $3\frac{1}{3}$, of

Mars 2, of Jupiter $1\frac{1}{3}$, and of Saturn $2\frac{1}{2}$ degrees. Fig. 1. Whence their inclinations are ſo ſmall, that they are almoſt coincident. The characters of the celeſtial bodies are, ☉ the Sun, ☽ the Moon, ☿ Mercury, ♀ Venus, ♂ Mars, ♃ Jupiter, ♄ Saturn; and ⊕ the Earth.

5. The fixed ſtars are at an immenſe diſtance; for by the niceſt obſervations, they have no annual parallax that is ſenſible; that is, they appear to the earth when on different ſides of its orbit to be exactly in the ſame places; and therefore the earth's orbit ſeen from a fixed ſtar, will appear only as a point. Conſequently, the fixed ſtars, all ſhine with their own native light. For it would be impoſſible for light, tranſmitted from the ſun to them, ever to be viſible, for it would be infinitely weak at that immenſe diſtance.

7. Tho' the diſtance of the ſun is almoſt nothing in reſpect of that of the fixed ſtars, yet it is immenſely great, in reſpect of the moon's diſtance. For the ſun's diurnal parallax is ſo ſmall, that is, the apparent ſemidiameter of the earth ſeen from the ſun, is ſo ſmall, that no inſtruments could be ſo exactly made as to find it. And hence the ſun's magnitude is vaſtly greater than the earth's. For ſuppoſing the ſun's parallax to amount to as much as a minute, then ſince the apparent diameter of the ſun is half a degree, this would make the ſun's diameter 15 times as big as the earth's; but the ſun's parallax is no ſuch thing, for it is found not to exceed 10 ſeconds, which will make the ſun's diameter 90 times as big as the earth's. That the ſun is a great body of fire is evident from the heat of it. And that it is of a globular form, is plain from the apparent motion of the ſpots upon its ſurface; for while the ſun moves uniformly about its axis, the ſpots in the middle of the diſk move very quick, but near the edges grow ſlower and

Fig. 1. ſlower, agreeable to the motion of a globe about its axis. By obſervation of theſe ſpots, the ſun is found to revolve about its axis in 25 or 26 days.

8. In our planetary ſyſtem, none of the celeſtial bodies ſhine with their own native light, but only the ſun. So that all the planets both primary and ſecondary are opake bodies, that have no other light but what they receive from the ſun, and reflect it back towards the earth and other places. This is evident from the moon, for only that ſide of her is obſerved to ſhine which is directly oppoſed to the ſun, but the other ſide which is from the ſun is quite dark, except ſo far as it is illuminated by reflexion from the earth; for the more of the illuminated ſide that is turned towards the earth, the more we ſee her enlightened; the reſt being dark. And the more of her dark ſide that is turned towards the earth, the more of her appears dark. Thus at the full ſhe appears all enlightened, and at the change all dark. The like phœnomena are obſerved in Mercury and Venus, which ſhew all the phaſes of the moon, according to their various ſituations; and ſometimes appear like a black ſpot upon the body of the ſun. Mars likewiſe appears gibbous when near the quadratures with the ſun. The Satellites of Jupiter are eclipſed when they are behind his body, being then immerſed in his ſhadow; they likewiſe caſt their ſhadows upon the body of Jupiter. And in Saturn the ſhadow of the ring upon his body proves his opacity. And the weakneſs of the light of thoſe that are far diſtance from the ſun, ſhows that it is not innate but borrowed.

9. All the planetary bodies are ſpherical, and all or moſt of them have a rotation round their axes. Thus the earth revolves in 24 hours; and we find by obſervation, that Venus revolves in 23 hours, Mars in $24\frac{2}{3}$ hours, Jupiter in near 10 hours,

and the moon in 27 days being her periodic time, Fig.
and always turns the ſame face towards the earth. 1.
Theſe motions are found by means of certain ſpots
upon their ſurfaces, which give us the time of their
rotation. In the other planets, for the want of ſuch
marks, the times of their rotation have not yet
been found. And that they are ſpherical bodies
appears from the ſlow motion of the ſpots near the
edges, and their ſwifter motion near the middle.
Likewiſe the line ſeparating the illuminated part
from the dark is always elliptical, which proves
their figure to be ſpherical.

10. Beſides the Satellites of Saturn, he is encom- 2.
paſſed with a thin ring AB, concentrical with him,
and whoſe diameter is more than double Saturn's
diameter CD; and the breadth of the ring is equal
to the hollow ſpace between the ring and his body.
This is a very odd phœnomenon, and the like is
not to be found in any other celeſtial body. Its
plane makes an angle of 31 degrees with the eclip-
tic, and coincides nearly with the orbits of the
Satellites; from whence it is probable that this ring
revolves in that plane, in the nature of a Satellite,
or rather of a vaſt number of Satellites, which
touch one another, and compoſe the ring. In
ſome places of his orbit his ring cannot be ſeen,
when its plane paſſes thro' the earth. In other po-
ſitions it is viſible more or leſs.

11. The body of Jupiter is alſo encompaſſed with ſeveral dark lines, called Jupiter's *Belts*; theſe run about his body parallel to his equinoctial. But theſe belts are obſerved not to be permanent, but to increaſe and decreaſe; and undergo ſeveral changes, and ſo do the ſpots upon his ſurface.

12. The ſurface of the moon is rough and uneven, and full of innumerable mountains, vallies and deep pits. This is plain from the line that divides the light from the dark part, for this is

Fig. 2. always jagged and uneven; the higher parts running more into the dark than the lower. The high mountainous parts caſt a ſhadow to a great diſtance, which gradually grows ſhorter, as the ſun's light comes more perpendicular; and by degrees the light ſhines into the hollows and vallies, and then the ſhadows of the edges gradually diſappear. There are ſeveral ſpots in the moon, ſome brighter, and ſome duller; but no changes have ever been obſerved in them, and they keep their places and magnitude and colour. There are ſuppoſed to be no clouds, rain or water in the moon; for ſhe always has the ſame appearance. There are no rivers, no ſeas; for ſuch places as are ſmooth and look moſt like ſeas, are full of ſmall hills and hollows, and ſuch irregularities; and therefore can contain no water. She always turns the ſame face nearly, towards the earth. The moon is far leſs than any of the planets, and yet appears vaſtly greater; even as big as the ſun. And the reaſon is, becauſe it is vaſtly nearer than any of the reſt.

SCHOLIUM.

The Syſtem of the World, as here deſcribed, is called the *Copernican Syſtem*; but other ſyſtems have been eſtabliſhed and received at different times, which are founded upon the earth being fixed in the center of the world. They are theſe that follow.

The *Ptolemaic Syſtem*, is that which ſuppoſes the earth at reſt in the center of the ſyſtem, about which the planets are ſuppoſed to move in circles; as firſt the Moon, the next above her, Mercury; and then Venus; and beyond her, the Sun; then Mars; then Jupiter; and above him, Saturn; and laſtly, the ſphere of the fixed ſtars. The planets and ſtars are all ſuppoſed to move round the earth from eaſt to weſt in 24 hours.

The *Tychonic Syſtem*, ſuppoſes the earth abſolutely at reſt, and that the moon and ſun revolve about the earth; the moon in a month, and the ſun in a year; and at the ſame time, the reſt of the planets, Mercury, Venus, Mars, Jupiter, and Saturn, revolve round the ſun; the three laſt encompaſſing the earth. And beſides, they are all ſuppoſed to have a diurnal motion round the earth, as likewiſe all the ſtars. Fig. 2.

The *Semi-tychonic Syſtem*, ſuppoſes the planets to revolve round the ſun, while the ſun and moon revolve about the earth as their center of motion; and it ſuppoſes the earth to move about its axis from weſt to eaſt in 24 hours. This differs from the tychonic only in this, that it ſuppoſes a diurnal motion in the earth; but like the tychonic, denies an annual one.

None of theſe ſyſtems can account for all the phœnomena; and therefore I ſhall ſay no more about them.

PROB. II.

To explain the riſing and ſetting of the ſun, moon, and ſtars; the reaſon of day and night, &c.

The phœnomena of the apparent motion of the ſun, is explained by the motion of the earth, thus. 3.

Let FGHI be the earth, O its center; and let it revolve about an axis perpendicular to the plane of the figure, in the order IFGH; that is, from weſt to eaſt. Let A be the ſun, draw AFOHC, and GOI perpendicular to it; let a ſpectator be at I; then ſince the tangent at I (which repreſents the horizon) will be parallel to AFH, and A at an immenſe diſtance; they will nearly meet in A, and the ſun at A will be riſing in the horizon at I. As the earth moves round, the ſpectator is carried towards F, and the ſun at A ſeems to riſe higher and

Fig. 3. and higher; and when the ſpectator is arrived at F, then the ſun is at the higheſt. As the earth ſtill turns round, and the ſpectator is carried from F towards G, the ſun appears to deſcend, as if it moved towards D; and when the ſpectator is arrived at G, then the ſun appears in the tangent at G; that is, in the horizon at G; and therefore the ſun is ſetting. Afterwards, all the time the ſpectator is moved thro' GHI, the ſun appears under the horizon, till it comes at I, where the ſun ſeems to riſe again. Now it is evident, that while the ſpectator is carried thro' the illuminated half of the earth IFG, it is day-light; and at the middle point F, is noon day. Alſo whilſt the ſpectator is carried thro' the dark hemiſphere GHI, it is night; and at H, it is midnight. And thus the viciſſitude of day and night appears, by the rotation of the earth about its axis.

What has been ſaid of the ſun, is equally true of the moon, or any ſtar placed at A. And therefore all the celeſtial bodies ſeem to riſe and ſet by turns, one after another, according to their various ſituations. For let A, B, C, D be four ſtars; when the ſpectator is at I, the ſtar A riſes; and when at G, it ſets. When the ſpectator is at F, B riſes; and when it is at H, it ſets. And when the ſpectator is at G, C riſes; and when it is at I, it ſets. Alſo when the ſpectator is at H, D riſes; and when at F, it ſets. And the like for all others, according to their particular poſitions.

Cor. 1. *Hence it is the very ſame thing, as to the diurnal motions, whether the earth moves uniformly about its axis, while the heavens ſtand ſtill; or whether the heavens move uniformly round, whilſt the earth ſtands ſtill. The phænomena being exactly the ſame either way.*

UNIV

For whether the ſpectator moves uniformly in the arch IF, from weſt to eaſt, whilſt A is fixed; or A moves uniformly in the arch AD, from eaſt to weſt, whilſt I is fixed; the ſame angle will be deſcribed, and therefore the altitude of A, above the horizon, will be the ſame either way. Fig. 3.

Cor. 2. *Hence, as to all the diurnal phænomena, 'tis the ſame thing in effect, whether we ſuppoſe the earth to move, or the heavens to move. And ſince the latter ſeems to be the caſe (or is ſo in appearance,) therefore all calculations are made upon this ſuppoſition, that the ſun, or the moon, or any ſtar, moves in its parallel from eaſt to weſt.*

That all computations of the places of the ſun, moon and ſtars, are made upon this ſuppoſition, will appear when we come to the calculation of ſuch aſtronomical problems, as depend on the circles of the ſphere. For all calculations are eaſieſt computed upon this ſuppoſition.

SCHOLIUM.

That the earth really turns round its axis once in 24 hours, and by that means cauſes all the phœnomena of the diurnal motion, is proved thus. It is plain, that either the earth muſt revolve round its axis in 24 hours, or elſe the heavens, with all their furniture, muſt revolve round the earth, in the ſame time. But it cannot be the heavens that revolve; for example, ſuppoſe the ſun revolves after this manner. Every body revolving round another body as its center, it is neceſſary that the central body muſt always be in the plane in which the revolving body moves; whatever curve it deſcribes; otherwiſe it cannot be ſaid to revolve about that body, but about ſome other point. Therefore if the ſun moves about the earth in a day, the earth muſt be in the plane deſcribed by the ſun in a day.

Fig. 3. But this never happens but accidentally, two days in the year, at the equinoxes. For at all other times the ſun deſcribes the ſeveral parallel circles, that have not the earth for their center. Therefore the ſun can no more be ſaid to revolve about the earth, than a body moving round the baſe of a cone, can be ſaid to revolve about the vertex.

This may be proved experimentally thus, erect a gnomon upon any plane facing the ſouth, and obſerve the line which the ſhadow of the top of it, deſcribes upon that plane, thro' any day; and for ſeveral days in the year; and you'll find it always a curve line, except when the ſun is in the equinoxes. And therefore the earth is never in the plane of the diurnal orbit, but then. For if the end of the gnomon kept all the while in the plane of the diurnal orbit, the ſhadow deſcribed would be a right line, ſince the interſection of two planes is always a right line. Therefore the earth not being in the plane of its diurnal orbit, the ſun revolves not round the earth, as a center, in a day.

It is to no purpoſe to object, that if the gnomon was at the earth's center its ſhadow would be a right line. For by reaſon of the great diſtance of the ſun, this makes no ſenſible difference. For notwithſtanding that, it deſcribes right lines at the equinoxes; and at the ſolſtices, two curves exactly alike, with their concavities the contrary way.

What has been ſaid of the ſun is alſo true of the ſtars, only the ſtars not changing their declination, they are always in their reſpective parallels. And therefore the earth is never in the center of any of their motion, but ſuch as happen to be in the plane of the equator. Therefore ſince the ſun and ſtars move not round the earth in a day, it follows that the earth revolves about its own axis in a day. Hence the apparent diurnal revolution of the ſun and ſtars proceeds from the real revolution of the earth round its axis. Fig. 3.

It muſt appear ſttange to ſuppoſe, that all the ſtars, at immenſe diſtances, and at different diſtances too, ſhould all revolve preciſely in the ſame time of 24 hours; thoſe about the equinoctial moving incredibly ſwift, whilſt thoſe near the pole move very ſlow in appearance; and yet all of them conſpire to perform their revolutions in the ſame ſmall ſpace of time; a thing not to be imagined, and utterly inconſiſtent with all the laws of motion.

PROB. III.

To explain the different ſeaſons of the year, and account for the lengthening and ſhortening of days, &c. 4.

The orbit of the earth extended to the heavens, is called the *Ecliptic*; and is divided by aſtronomers into 12 equal parts, called Signs; theſe ſigns are, Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagitary ♐, Capricorn ♑, Aquarius ♒, Piſces ♓. Therefore let ♈ ♋ ♎ ♑ be the earth's orbit, S the ſun. This orbit is ſo ſmall in reſpect to the diſtance of the fixed ſtars, that the ſame aſpect of the heavens will appear, whether a man be placed in the earth or in the ſun. Now if the earth be at ♈, a ſpectator will ſee the ſun in ♎; and when the earth comes to ♉, he will ſee the ſun in ♏; and the ſun will appear to have moved thro' ♎ ♏. And whilſt the earth is moving to ♊, the ſun will ſeem to paſs thro' ♏ ♐; and hence a perſon in the earth obſerves the ſun to go thro' the ſame ſpace in the heavens, that a ſpectator at the ſun would ſee the earth go through. And becauſe he is not ſenſible of the earth's motion, he aſcribes that motion to the ſun, which in reality is unmoved. And hence, becauſe the relative motion is the ſame, whether of the two

is

Fig. 4. is moved, and all effects are the ſame as to their places; therefore aſtronomers generally ſuppoſe the ſun to move along the ecliptic, deſcribing its orbit round the earth at reſt.

Let NEAQ be the earth, which is a ſphere; and let NA be its axis, about which it revolves in 24 hours. N the north pole, A the ſouth; and EQ the equinoctial, or a circle equidiſtant from the poles N, A. And let PR be a parallel of latitude paſſing thro' any place. Draw a plane G ♈ I perpendicular to ♑ S ♋, this divides the illuminated hemiſphere from the dark one. The axis NA is inclined to the plane of the ecliptic or earth's orbit, in an angle of $66\frac{1}{2}$ degrees. And during the earth's motion in its orbit, the axis always remains in a parallel poſition, or pointing to the ſame ſtar. Moreover the earth moves uniformly round this axis; and therefore deſcribes equal arches in equal times. Now let the earth be at ♎; in this poſition, the circle dividing the light and dark hemiſpheres paſſes thro' the poles N and A. And conſequently by the nature of the ſphere, divides all the parallels as PR into two equal parts; and therefore any point in that parallel, as the earth revolves round, will ſtay as long in the light hemiſphere as in the dark. That is, the days and nights are equal. Then as it moves to ♏, the pole N comes into the light hemiſphere, by reaſon of the oblique poſition of the axis NA. And as it proceeds to ♐ and ♑, the light hemiſphere reaches farther and farther beyond N, till coming to ♑, it is at the fartheſt, reaching to G, and making the arch NG $23\frac{1}{2}$ degrees, the complement of N ♑ S, or $66\frac{1}{2}$ degrees. And then the oppoſite pole A is as far involved in the dark hemiſphere. Whence in north latitudes, or in the hemiſphere ENQ the days have been increaſing from ♎ to ♑, where they are at their longeſt; for the greater part of the parallel PDQ as PD is in the illumi-

Fig. 4.

illuminated hemiſphere, and only the part DR in the dark. But in the oppoſite or ſouthern hemiſphere the days have been decreaſing, and are at their ſhorteſt at ♑. For all the parallels to EQ have their greater part in the dark hemiſphere. If thro' the point G, a parallel be deſcribed, this parallel is called the *Arctic Circle*; and all the ſpace contained therein is illuminated, and therefore there is no night, during the earth's ſtay at ♑. And for the ſame reaſon, the ſpace within the parallel drawn thro' I, will be all dark, and all is night there. Likewiſe if a parallel be drawn thro' B, where S♑ cuts the arch NE, that parallel is called the *Tropic of Cancer*; and then the ſun will ſhine perpendicular upon the inhabitants in that parallel. This is the ſummer ſeaſon for thoſe that are in the hemiſphere ENQ, and the winter for thoſe that live in EAQ. And ſince EQ is equally divided by the circle of light and darkneſs GI; therefore the days and nights are always equal under the equinoctial.

Again, whilſt the earth moves thro' ♒ and ♓ to ♈, the circle of light and darkneſs comes nearer and nearer to the pole N, the angle N♑G and conſequently B♑E grows leſs and leſs, till they vaniſh in ♈; then the circle of light and darkneſs paſſes again thro' the poles N and A, and conſequently biſects all the parallels as PR; and therefore the days and nights are again equal, all over the earth.

Whilſt the earth moves thro' ♈, ♉, ♊ to ♋, the ſun will ſeem to go thro' ♎, ♏, ♐ to ♑; then the circle of light and darkneſs will fall ſhort more and more of the north pole N; and will go further and further beyond the ſouth pole A; whence the parallels cut by that circle will have the greater part PD in the dark, in the north hemiſphere; but in the ſouth hemiſphere, the greater part will be in the light. Therefore 'tis winter to the northern

hemiſphere

Fig. 4. hemiſphere ENQ, the days being at the ſhorteſt; and ſummer to the ſouthern hemiſphere EAQ, their days being at their longeſt. And within the parallel drawn thro' G, there will be no day whilſt the earth ſtays at ♋; and in the parallel drawn thro' I there will be no night. And at the pole A it will be day for 6 months, and at the pole N it will be night for ſix months; juſt the contrary of what happens, when the earth is at ♑. In this poſition, if a parallel be drawn thro' B, the ſun will ſhine perpendicular to the earth in that parallel, and it is called the *Tropic of Capricorn*. And a parallel drawn thro' I is called the *Antarctic Circle*.

Let the earth move from ♋ thro' ♌ and ♍ to ♎ again; it is evident the circle of light and darkneſs draws nearer and nearer to the poles N and A, by which the light and dark parts of the parallels became nearer an equality, and ſo do the days and nights. Therefore in the north hemiſphere ENQ, the days are increaſing; and in the ſouth hemiſphere they are decreaſing. And the days and nights become equal in every place, when the earth arrives at ♎. And in this manner, are the ſeveral ſeaſons cauſed; being owing to the obliquity of the axis of rotation of the earth, to the plane of the earth's orbit. For if the axis was perpendicular to it, there could be no variety in the length of days, in whatever part of the orbit the earth was; and all ſeaſons would be alike. Therefore the earth, in the time of paſſing thro' ♈ ♋ ♎ ♑, or thro' the whole ecliptic, undergoes all the different changes of its ſituation, and ſhews all the variety of ſeaſons, and ſo compleats the year, and begins a new one. And after this manner the earth revolves year after year about the ſun at reſt.

Cor. 1. *Hence the obliquity of the earth's axis to the ecliptic, (or which is the ſame thing,) of the equinoctial*

tial to the ecliptic, is the cauſe of the different ſeaſons, as ſummer, winter, ſpring and autumn, during the year. And without this, there could be no difference of ſeaſons; and conſequently it could not be eaſy to know the length of the year, without obſervations of the ſtars. Fig. 4.

For the length of the year is had from finding the time by obſervation, when the ſun is in the equinoctial points; and there being no ſuch points to obſerve by; there could be no method, but to obſerve by the poſition of the ſtars; when the ſame ſtar was again in oppoſition to the ſun; which none but an aſtronomer could do; and perhaps no body would undertake it, as it could be of no uſe when found.

Cor. 2. *The ſun appears 47 degrees higher in the ſummer tropic, than it does in the winter tropic.*

For in ſummer it ſeems to have aſcended thro' the arch BE; and in winter, to have deſcended thro' the arch BQ equal to BE; and their ſum is 47 degrees.

SCHOLIUM.

These phænomena may be repreſented thus. Take a ſmall globe that has the equinoctial and parallels drawn on it. And placing a candle upon a table move the globe round the candle in a circle parallel to the table, ſo as that the axis of the equator may be oblique to that circle, and be kept always in a parallel poſition whilſt it moves about. Then the candle will illuminate the globe, as it is carried round; juſt as the ſun does the earth in its orbit. And the poles and the parallels will be the ſame way affected with light and darkneſs as this globe.

It is taken for granted here, that the earth moves round the ſun at reſt in a year; by which means all the phœnomena of the ſeaſons are moſt eaſily

Fig. 4. and naturally accounted for. But in reality this is more than a mere hypotheſis; for by the laws of centripetal force (Prop. XX.) if two bodies revolve about each other, they revolve about their common center of gravity. Now the ſun being vaſtly greater than the earth, the center of gravity of the two bodies falls within the body of the ſun, which point is at reſt; and therefore whilſt they revolve about one another, the center of the ſun is very little moved, being at a very ſmall diſtance from the center of gravity; whence the ſun does not ſtir out of its place. And conſequently it is fact that the earth by its annual motion, moves about the ſun at reſt. They that aſſert that the ſun moves round the earth, muſt firſt prove that the earth is vaſtly greater than the ſun, which they never can do; and therefore that ſuppoſition is impoſſible in nature.

Neither do the planets revolve round the earth, for then they would deſcribe areas proportional to the times (ib. Prop. XI.), which they do not: and ſome of them do not include the earth in their orbits. Likewiſe the ſquares of the periodic times would be as the cubes of their diſtances from the earth (ib. Prop. XVII.), which they are not. But all theſe things agree with the ſun, as the center of their motions; conſequently the ſun and planets do not revolve round the earth in their ſeveral periodic times, but the earth and planets revolve about the ſun; the earth in a year, and the planets in their reſpective periodical times.

All people agree that the moon revolves about the earth; and if the ſun revolved about it alſo, its velocity would be 25 times the velocity of the moon; by reaſon of its great diſtance, contrary to the laws of nature, and Cor. 6. Prop. III. centripetal forces.

It may be obſerved, that the ſame phœnomena appear, whether the bodies or the ſpectator be moved; ſo that it cannot be proved by the obſervations themſelves, whether the motion is in the ſun, or in the earth. Fig. 4.

PROB. IV.

To explain the increaſe and decreaſe of the moon, and its different phaſes.

Let E be the earth, ACBD the moon revolving about the earth in her orbit 1357. The moon is the moſt ſplendid planet in the heavens, owing to its apparent magnitude and nearneſs to us. She performs her revolution from change to change in a month; during which time ſhe is always changing her figure, and putting on different phaſes or appearances. Half of her time ſhe is increaſing from the change to the full, and in the other half ſhe is decreaſing from the full to the change, when ſhe becomes inviſible. But all this is only in appearance, for the moon is a ſpherical opake body like the earth, and we can ſee no part of her but ſuch a part of her illuminated hemiſphere as is turned towards us, the reſt remains inviſible. From theſe appearances, one would ſuppoſe that the moon is generated anew every change, increaſes to the full, and then waſtes away again to nothing in the change. To explain this, we muſt underſtand, that ſhe has no light but what ſhe receives from the ſun, which is the light ſhe ſhines with. For the ſun ſhining upon the moon, illuminates that hemiſphere of her which is turned towards him, the oppoſite hemiſphere being dark and inviſible. And by her ſituation in reſpect of the ſun, ſometimes a greater and ſometimes a leſſer part of her enlightened hemiſphere is turned towards us. 5.

Fig. 5. Let S be the ſun at an immenſe diſtance; and ſuppoſe rays of light to be drawn from the ſun to the center of the moon, and that BA is every where drawn perpendicular to theſe rays through the center of the moon. Then it is evident, that the circle BA is the circle terminating light and darkneſs, or dividing the light from the dark hemiſphere. Alſo from E the center of the earth, draw lines every where to the center of the moon in her orbit; perpendicular to which draw the plane CD, then the circle of viſion CD will divide the viſible hemiſphere of the moon from the inviſible one.

Suppoſe the moon at 1, directly between the earth and ſun; then the circle AB of light and darkneſs, and the circle of viſion CD, coincide; and her whole illuminated hemiſphere is turned from us, and her dark ſide towards us; and we can ſee nothing of her; and now ſhe is ſaid to be at the *Change*, or it is *new Moon*. Then whilſt ſhe moves along her orbit from 1 towards 2, part of her illuminated hemiſphere begins to appear, and at 2 the part CA becomes viſible, and the part DA remains dark. In this poſition ſhe appears ſmall and *horned*, with her back towards the ſun, her horns looking eaſtward. Then while ſhe advances in her orbit from 2 to 3, her light increaſes more and more, as more of her enlightened ſide appears to us. And at 3, where 3E is perpendicular to SE, there CD is perpendicular to AB; then half of her enlightened hemiſphere will be viſible, and ſhe is then biſected, and ſhe appears a *half moon*. Afterwards moving from 3 to 4, ſhe ſtill increaſes in light; and at 4 the part CA of her illuminated diſk appears, which is more than half, and the part DA only remaining dark; and in this poſition ſhe appears *gibbous* or crooked on both ſides. Still proceeding in her orbit from 4 towards 5, her light ſtill increaſes, and at 5 the circle AB terminating Fig. light and darkneſs coincides with CD the circle 5. of viſion, and there the whole illuminated face of the moon is turned towards the earth, and becomes viſible. In this ſituation, being directly oppoſite to the ſun, ſhe is ſaid to be at *the full.* And now ſhe riſes when the ſun ſets, and ſets when he riſes. In like manner as ſhe proceeds in her orbit, ſhe begins to loſe part of her light, and at 6 the part BC is darkened, and the part DB only viſible; in this ſituation ſhe is again gibbous. Then moving on towards 7, ſhe loſes more and more light, and at 7, where E7 is perpendicular to SE, ſhe appears biſected again, and becomes a half moon. The moon ſtill going forward towards 8, the illuminated part which is viſible, becomes leſs and leſs, and at 8 ſhe appears crooked and horned, the horns looking now weſtward from the full to the change; contrary to what they did from the change to the full. The moon ſtill proceeding to 1, the light by degrees vaniſhes, and at 1, her whole dark ſide is turned towards us, and ſhe is again at the change.

It has been obſerved, that the moon has no light but what ſhe receives from the ſun; but this is not to be taken abſolutely, for there is a very ſmall quantity of light reflected to her from the earth, eſpecially near the change; which makes the dark part viſible in a clear air. For at that time, the whole illuminated hemiſphere of the earth is turned towards the moon, and enlightens her dark ſide, and makes it viſible. For ſince the moon's light enlightens the earth; much more will the earth's reflected light illuminate the moon, as the earth's hemiſphere is 15 times as great as the moon's. And hence ariſes that dim light, which is obſerved in the old moon, by which her body becomes viſible, altho' but obſcurely.

Fig. 5. The moon does not move in the plane of the ecliptic, but in an orbit making about five degrees inclination with it. And hence it comes to paſs, that ſometimes the moon riſes higher than the ſun, when the ſun is at higheſt; and ſometimes lower than him, when he is at the loweſt meridian altitude.

Cor. *Hence the moon has five remarkable phaſes or appearances.* 1. *New moon.* 2. *Full moon.* 3. *The horned moon.* 4. *The half moon.* 5. *The gibbous moon.*

PROB. V.

To ſhew the nature of eclipſes, and the reaſon of the eclipſes of the ſun and moon.

Any opake body expoſed to the light of the ſun, or of any other luminous body, will caſt a ſhadow behind it. This ſhadow is nothing but a ſpace void of light, into which if any other body happens to come, it cannot be ſeen for want of light. This body coming within the ſhadow of the other is ſaid to be eclipſed. Hence there muſt be three bodies concerned in any eclipſe, the luminous body, the body caſting a ſhadow, and the body involved in the ſhadow. Now ſince light moves in right lines, if the bodies be ſpherical, the ſhadow will be a cone or the fruſtum of one, whoſe axis paſſes thro' the center of the luminous body, and the center of the body caſting the ſhadow.

6. Let AB be a luminous body as the ſun, CD an opake ſpherical body expoſed to its light. Draw AC, BD to interſect in P; then the cone CDP is the ſhadow of CD. Into this ſhadow no light can come from the luminous body AB. Draw BCE, ADF, and the ſpace ECPDF is the *penumbra* ariſing from the ſhadow of CD; in which ſpace ſome light comes from part of the body AB. Near to

Fig. 6.

CP and DP it is nearly dark, as there is no light comes there, but what comes from ſmall parts of the body AB near the edges. The further you go from CP and DP, the more light there is, as a greater portion of the luminous body can ſend light to theſe places. And near CE and DF the penumbra is exceeding ſmall, becauſe almoſt all the luminous body AB can ſend its light to theſe places. Therefore the penumbra is darkeſt at CP, DP; and vaniſhes at CE, DF. Now if any opake body as G happens to be in or near the ſhadow CDP, that part which is within the ſhadow will be totally dark; and the part that is in the penumbra, will have ſome faint light, which is greater, the further it reaches from DP. The body G then will be eclipſed either partially or totally. If the center of the body G happen to fall in the axis of the cone CDP, it will be centrally eclipſed; and if its diameter be leſs than the diameter of the cone of ſhadow in that place, it will be totally eclipſed.

Therefore if AB be the ſun, CD the earth moving in its orbit, and G the moon revolving about the earth; when the moon at G comes to the oppoſition of the ſun, or to be full moon, it will then come near, or fall into the ſhadow of the earth CDP, if it falls into the ſhadow it will be eclipſed. Now if the moon's orbit was in the plane of the ecliptic or the earth's orbit, there would be an eclipſe of the moon every full moon, and a central one. For at every full moon the centers of all the three bodies would be in a right line; and therefore the moon muſt needs go thro' the earth's ſhadow.

Again, when the moon, by revolving in its orbit, is come to the oppoſite ſide at H, where it is new moon, then its body will be between the ſun AB and earth CD, and conſequently will caſt a ſhadow upon the earth. And to thoſe that live

Fig. 6. upon that ſpot of earth, the ſun will ſeem to be eclipſed, or to loſe its light; tho' in reality it is that part of the earth that is eclipſed. And therefore if the moon's orbit be in the plane of the earth's orbit, there will be an eclipſe of the ſun every new moon; for their centers will then be all in a right line.

But the orbits of the earth and moon are not in one plane; and therefore there cannot be any eclipſe but when their oppoſition or conjunction happens at ſuch a time when they are at or near the interſection of their orbits, that is, at their *nodes*. For at the interſection, they will be all in a right line.

7. Let E be the earth, BLAC the moon's orbit; AGBD a circle drawn in the plane of the ecliptic, interſecting the moon's orbit at A and B, the *nodes*; AB the line of the nodes; the angle GAL or DBC the inclination of the moon's orbit to the ecliptic, 5 degrees. Now as the moon moves in the orbit BLAC, and the ſun ſeems to move in the orbit BGAD; they can never be in a right line with E, but when they are in the line of the nodes AB. Let the ſun be at F, and the ſhadow of the earth diametrically oppoſite, at H; and I the place of the moon when the full moon happens; then if the diſtance HI be leſs than the ſum of their ſemidiameters, there will be a partial eclipſe of the moon at H. But if they are at K and L, ſo far from the node B, that the diſtance LK exceeds the ſum of their ſemidiameters, there can be no eclipſe, for the moon will miſs the ſhadow. And if they happen in B, there will be a central eclipſe.

Again, let the ſun appear at F in the time of the conjunction, and the moon be at N, and if they be ſo near the node A, that the diſtance of their centers NF be leſs than the ſum of their apparent ſemidiameters; then there will be an eclipſe of the ſun at F, at the time of new moon. But if they are ſo far from the node A, that the diſtance PQ exceeds the ſum of the ſemidiameters, there will be no eclipſe at that new moon. For then the moon at Q can hide no part of the ſun at P from being ſeen; or the moon's ſhadow at Q miſſes the earth. Laſtly, if they happen to be at A in the time of new moon, there will be a central eclipſe of the ſun. Fig. 7.

Since the ſun is bigger than the earth, and the earth bigger than the moon; the ſhadow of the earth or moon will end in a point. And the diameter of the earth's ſhadow will be far bigger than the diameter of the moon. And even at the diſtance of the moon's orbit, the diameter of the ſhadow CD is thrice the diameter of the moon. Whence it comes to paſs that the moon is 4 hours in paſſing thro' the earth's ſhadow, or that a central eclipſe of the moon is 4 hours long. 8.

The moon CD being far leſs than the earth, its ſhadow CDF ends ſooner, or is ſhorter, than the earth's ſhadow. And ſometimes the cone of ſhadow ſcarce reaches to the earth, and very little further at any time. Therefore the ſection of the ſhadow, where it falls on the earth, covers but a very ſmall portion of the earth's ſurface AB. Thus the ſhadow F eclipſes the ſun only to thoſe that live upon that ſpot. And as the moon moves eaſtward over the ſun's diſk, the ſhadow F moves over the ſurface of the earth deſcribing the tract FG, going off at G. And all the inhabitants living under that tract, have the ſun ſucceſſively eclipſed, and totally. Thoſe that live at ſome diſtance from the tract FG, will have a partial eclipſe, as they fall within the circle of the penumbra HI. But beyond that, the inhabitants are not ſenſible of any eclipſe. The cone of ſhadow is ſo narrow where it falls upon the earth at F, that the duration of a total eclipſe continues but a few minutes in any one place. 9.

Fig. 9. place. And the area which the ſhadow covers does not exceed three degrees in diameter; tho' the penumbra reaches over a ſpace of 70 degrees, for which reaſon the whole length of a total eclipſe is about two hours.

Cor. 1. *All eclipſes of the ſun happen at the change of the moon; and all eclipſes of the moon are at the full moon.*

For it is only at the change that the moon comes between the earth and ſun. And 'tis only at the full, that the earth can come between the ſun and moon.

Cor. 2. *The nearer the ſun and moon are to the nodes, at the time of an eclipſe; the greater the eclipſe will be.*

Cor. 3. *In every eclipſe of the moon, the eclipſe firſt begins on her eaſtern ſide, and goes off on her weſt ſide.*

For her motion being ſwifter than the ſun's, or the ſhadow of the earth; ſhe overtakes the ſhadow and enters into it, on the weſt ſide, with her eaſtern limb, as ſhe moves eaſtward.

Cor. 4. *In eclipſes of the ſun, the weſtern ſide is firſt eclipſed, and ends on the eaſtern ſide.*

For the moon moving faſter, overtakes the ſun in his motion eaſtward.

Cor. 5. *As many eclipſes happen of the ſun in general, as of the moon.*

For the moon's ſhadow + the earth's diameter = earth's ſhadow + the moon's diameter. Whence the moon's ſhadow falls as oft on ſome part of the earth, as the earth's ſhadow upon the moon.

Cor. 6. *In any particular place, there are more eclipſes of the moon than of the ſun.*

For the ſhadow of the moon is leſs than the Fig.
earth's ſhadow, and therefore will not ſo often ob- 9.
ſcure a certain place of the earth, as the earth's
ſhadow will of the moon.

Cor. 7. *In ſome places of the earth, an eclipſe of the ſun may be total, in others partial, and in ſome none at all.*

Cor. 8. *Eclipſes of the ſun don't happen at the ſame time in different places of the earth.*

For an eclipſe is viſible firſt at its going upon the earth's diſk, then at F; then ſucceſſively at all the places along the tract FG, as the ſhadow goes along. And laſtly at its going off at G.

Cor. 9. *But an eclipſe of the moon happens at the ſame time to all the inhabitants of the earth.*

Thus all the inhabitants of the earth AB, to 8.
whom the eclipſe is viſible, will ſee the moon im-
merged into the ſhadow at D, at the ſame inſtant.
They will alſo at the ſame time ſee it paſs thro'
the ſhadow DC, and then emerge at C.

Cor. 10. *The moon, before ſhe begins to be eclipſed, or to enter into the ſhadow of the earth, grows very pale.*

The reaſon of this, is her being in the penumbra.

Cor. 11. *When it happens that the cone of the moon's ſhadow does not reach the earth; there will be no total darkneſs at the earth. And thoſe that live where the axis of the cone cuts the earth, will ſee a bright ring ſurrounding the moon. This is called an annular eclipſe.*

For nothing but the penumbra falls then on the earth; and the bright ring that appears, is the edge of the ſun appearing all round; the moon being then (apparently) leſs than the ſun.

Fig. 9. Cor. 12. *No eclipſe of the ſun happens to all places where the ſun is viſible; but only to ſome particular places on the earth.*

SCHOLIUM.

Sometimes in an eclipſe of the ſun, the moon appears to have a duſky red light; which is owing to the refraction of the rays of light paſſing thro' the earth's atmoſphere, eſpecially the red rays; and ſo falling upon the body of the moon.

PROB. VI.

To explain the ſtations and retrogradations of the planets, and their phaſes.

If an obſerver was ſituated in the ſun, he would ſee all the planets move regularly the ſame way about him from weſt to eaſt, ſome faſter and ſome ſlower, according to their reſpective diſtances or rather nearneſs. But to an obſerver upon the earth the motion of the planets will be very irregular; becauſe the earth being one of them, is in motion itſelf. Sometimes they ſeem to move *directly* from weſt to eaſt, according to their true motions. At other times they ſeem to loſe all their motion, and to be *ſtationary*. And ſometimes they ſeem to go contrary to their real motions, from eaſt to weſt, and ſo move backwards or become *retrograde*. Theſe motions have no relation to the diurnal motion of the celeſtial bodies; they are only conſidered as moving among the fixed ſtars; ſo that when a planet moves from the weſtermoſt ſtars towards others that lie eaſterly, it is ſaid to be *direct*; when it alters not its poſition among them, it is *ſtationary*. And when it leaves the eaſtermoſt, and moves towards the weſtermoſt ſtars, it is *retrograde*.

10. Since Venus and Mercury (which are called *inferior Planets*, becauſe they are nearer the ſun than

we are,) move ſwifter about the ſun than the earth does, and perform their revolutions in leſs time. Therefore only a part of the earth's orbit will be deſcribed, whilſt an inferior planet deſcribes its whole orbit; and therefore it will be frequently overtaking the earth in its motion. Fig. 10.

Let an inferior planet be at E and the earth at T, in the oppoſite point, moving in their orbits, round the ſun at S. Then whilſt the planet moves from E towards F, if the earth ſtood ſtill at T, it would ſee the planet move eaſtward or direct; and much more ſo, if itſelf moves towards V. When the planet comes to G, and moves from G towards H, it will appear to move as quick as the ſun, becauſe its motion tends towards the earth at V; and therefore its apparent motion is ſlower than before. From H to A, (by reaſon of its ſwifter motion than the earth) it will tend to a point beyond V as at W; and when the lines HV and AW are parallel (which they are tending to), the planet will ſeem ſtationary. Afterwards the planet moving faſter thro' AB, than the earth moves thro' WX, the planet will ſeem to move weſtward or be retrograde, and will continue ſo till coming to O, a little before it ſeems to move in a direct line from the earth at Y; and about O and D their places will be found again in two parallel lines, that is, OY, DZ will be parallel; and then it appears ſtationary again to the earth at the place YZ. Then the planet by its ſwifter motion thro' DE, will begin to turn about from its paralleliſm towards the eaſt, that is, it will be direct again. Beyond F it will increaſe its direct motion more and more, till it come again to the earth's oppoſition. And from thence every thing will follow as before. And this will be the phœnomenon of Venus and Mercury. And as Mercury moves ſwifter than Venus, it ſooner overtakes the earth, and therefore the ſtations

Fig. 10. tions and retrogradations will be more frequent in Mercury than in Venus.

11. Again let ABC be the orbit of the earth, *abc* the orbit of a ſuperior planet. 'Tis plain, that from the ſwifter motion of the earth, and the ſlower motion of the planet, that the planet will ſometimes be in conjunction with the ſun, and ſometimes in oppoſition to him. Let the earth be at A, and the planet at *a* in conjunction with the ſun, in the oppoſite point of its orbit. And while they both move in their reſpective orbits, let their contemporary poſitions be A*a*, B*b*, C*c*, D*d*, &c. Now 'tis evident, if the earth ſtood ſtill at A, the planet would ſeem to move eaſtward from *a* to *b*, &c; and much more will it appear to do that, when the earth moves the ſame way about the center from A to B, &c. and this cauſes the planet to move faſter than ordinary in appearance. In the poſitions B*b*, the motion of the earth at B is more oblique, and therefore the planet ſeems to ſlacken its motion. Between B and C, about G, the earth ſeems to move directly towards the planet, advancing to *c*; and therefore it ſeems here to move no faſter than the ſun does. About C*c*, the line connecting the earth and planet will continue for ſome ſmall time parallel to itſelf, as pointing always to the ſame ſtar, and then the planet will ſeem ſtationary. Advancing to D and *d*, the planet will come to the oppoſition of the ſun, and the earth moving with a ſwifter motion will ſeem to leave the planet behind; that is, the planet will now begin to be retrograde. Paſt D and *d*, the retrograde motion will decreaſe, by reaſon of the earth's motion being more oblique to that at *d*; and ſome where paſt D and *d*, the line connecting the earth and planet, will continue for ſome time parallel to itſelf; that is, the planet will again be ſtationary. Approaching towards E and *e*; the earth ſeems to move directly from

the planet, and then the planet ſeems to have the ſame motion eaſtward as the ſun has. Between E and F the earth begins to get to the oppoſite ſide, from the planet; which accelerates the planet's motion eaſtward. At F*f* they are approaching to the oppoſition; and ſomewhere between FG and *fg*, they are again in the oppoſition, or the planet is in conjunction with the ſun, as at firſt; where the apparent motion is greateſt. And thus the ſame phœnomena will be repeated over again. Theſe will be the phœnomena of the ſuperior planets, Mars, Jupiter and Saturn; with this difference, that the ſtations and retrogradations will be more frequent in Saturn than in Jupiter; and more frequent in Jupiter than in Mars. Becauſe the earth oftener overtakes Saturn than Jupiter, and Jupiter oftener than Mars; and paſſes between them and the ſun. Fig. 11.

10. Since the inferior planets move round the ſun at a leſs diſtance than the earth, therefore their orbits are included within the earth's orbit. And ſince their planes almoſt coincide within the ecliptic or plane of the earth's orbit; therefore theſe planets will ſeem to a ſpectator on the earth, ſometimes to go to the eaſt of the ſun, and ſometimes to the weſt of him; and ſo to move back and forward with a vibrating motion almoſt in a right line. Thus if the earth be at Z, the planet revolving in the orbit EGA, will ſeem to move in the lines DS, SB, and then back again in the line BS, SD, making equal vibrations from the ſun S, each way. And the angle DZS or BZS, that is, the apparent ſemidiameter of the orbit at Z, is called the *Elongation*. In Mercury it is 30 degrees, and in Venus 45 degrees. The ſuperior planets being further diſtant from the ſun than the earth, their orbits include the earth's orbit. And therefore they ſeem to move about the earth; ſometimes coming nearer

Fig. to it, and appearing bigger; at other times being
11. further off, and appearing leſſer. Thus when Mars is in oppoſition to the ſun, he is 5 times nearer than in his conjunction, and appears 5 times greater in diameter.

12. The inferior planets being opake ſpherical bodies, enlightened by the ſun, will have the ſame phaſes in regard to their enlightened parts as the moon has. But they cannot be perceived without a teleſcope. Thus if E be the earth; A, B, C, D ſeveral places of an inferior planet in its orbit, enlightened by the ſun S in the center of its orbit. 'Tis evident, that half of the planet which is turned towards the ſun is enlightened, whilſt the oppoſite ſide is in the dark. And ſince only that ſide which is turned towards the earth can be ſeen by an obſerver; therefore more or leſs of the illuminated ſide of the planet will be ſeen, according as the light ſide is more or leſs turned towards him. Thus the planet at A, has its dark ſide turned towards him, being upon the earth at E; and therefore it cannot be ſeen. Moving toward B part of the illuminated ſide will be towards him; and the planet will appear horned like the old moon. At B, half of it, will be towards him, and it will appear like a half moon. Between B and C, it will appear gibbous; and at C the whole illuminated face is turned towards him; and it would appear like the full moon. Between C and D, it would appear gibbous again. At D, like the half moon, between D and A horned, like the new moon; and at A inviſible. If the planet, when at A, happens to be in the plane of the earth's orbit, or in a node, it will appear like a ſpot in the ſun's face.

As to the ſuperior planets, ſince they move about the earth; their illuminated hemiſphere will always be turned towards the earth; except in Mars, who by reaſon of his nearneſs, has part of

his illuminated hemiſphere turned from us, in his Fig.
quadratures with the ſun; and then he appears 12.
gibbous.

Cor. 1. *The inferior planets are always direct when furtheſt from the earth, and appear then leaſt; and they are always retrograde when neareſt the earth, and appear then biggeſt.*

Cor. 2. *The ſhorter the periodic time of an inferior planet is, the more frequent are its ſtations and retrogradations; but the ſhorter time they continue, and are alſo leſs in quantity.*

Cor. 3. *The ſuperior planets are always direct, when in conjunction with the ſun; and retrograde when in oppoſition to him.*

Cor. 4. *The longer the periodic time of a ſuperior planet is, the more frequent are its ſtations and retrogradations; but they are leſs, and continue a longer time.*

Cor. 5. *The inferior planets never go further from the ſun, than the apparent ſemidiameters of their orbits ſeen from the earth; which in Mercury is 30 degrees, in Venus 45.*

Cor. 6. *When an inferior planet is direct, it appears full or gibbous; when it is retrograde, it is horned, or elſe diſappears; and when ſtationary, 'tis nearly biſected.*

For the former happens at its greateſt diſtance, and the latter at its leaſt, from the earth; in the middle time it is ſtationary.

Cor. 7. *In all the planets the direct motion is ſwifteſt, when the ſun is between the planet and the earth; and the retrograde motion is ſwifteſt, when they are both on one ſide the ſun, in a right line.*

Fig. For the former motion is compounded of the motion of the planet, and that of the earth; and the latter is the difference thereof.

PROB. VII.

To deſcribe the order and magnitude of the fixed ſtars.

If one looks up in a clear night, he will ſee the hemiſphere all over adorned with a multitude of ſtars, ſeemingly innumerable, which appear of different degrees of bigneſs and brightneſs. Some have ſuppoſed that they are all placed at the ſame diſtance from us, in a concave ſpherical ſurface, and that their appearing of different degrees of brightneſs is, becauſe they are in themſelves of different magnitudes, the greateſt appearing the brighteſt, and the leaſt appearing faint and dull, by reaſon of their ſmallneſs. But others with more reaſon, ſuppoſe them to be at different diſtances, and then thoſe that are neareſt will give the brighteſt light, while thoſe that are far off will but ſhine very faintly. And ſome may be ſo far diſtant as not be viſible. And indeed the uſe of the teleſcope diſcovers to us innumerable ſtars which cannot be ſeen at all with the naked eye; among which ſome are brighter, and ſome ſo dim as hardly to be perceived with the teleſcope. Beſides the greater the magnifying power of the teleſcope is, the more ſtars are viſible thro' it. Whence it is very probable, that there is an innumerable multitude of ſtars placed at certain diſtances thro' all ſpace; at leaſt, further than we can make any obſervations. And yet theſe ſtars may not be preciſely all of the ſame bigneſs, but may admit of ſome variety in their magnitudes.

But however this may be, Aſtronomers divide them into certain magnitudes according to their apparent bigneſs or brightneſs. The brighteſt, they call ſtars of the *firſt magnitude*; the next are ſtars of the *ſecond magnitude*; and ſo on to the *ſixth magnitude*, which is the laſt. And all ſtars that can only be ſeen thro' a teleſcope, they call *teleſcopic ſtars*, or ſtars of the ſeventh magnitude. But we are not to ſuppoſe that all the ſtars of any one magnitude are exactly of the ſame bigneſs, there being a great variety, ſome bigger and ſome leſſer in every ſort. And for that reaſon, there are ſome ſtars that are put into different claſſes, by different people.

The diſtance of the fixed ſtars is ſo immenſely great, that no method could ever be uſed to find the diſtance of the neareſt of them. For altho' in half a year's time we are tranſlated from one ſide to the other of the earth's orbit; yet this does not make the leaſt difference in the appearance of the ſtars or their magnitudes, or diſtances; no not with any inſtruments. So that if we were placed in any part of the ſolar ſyſtem, even in Saturn, we ſhould ſee the ſame face of the heavens, the ſame ſtars, and in the ſame places.

That the ſtars ſhine with their own innate light is evident, from this, that it is impoſſible the light of the ſun ſhould reach them at that diſtance, and be reflected back to us, ſo as to become viſible. And there can be no other light but the ſun to do it; for if there was, it muſt be viſible. And therefore it is extremely probable that the fixed ſtars, are ſo many ſuns, placed in different parts of ſpace, for the ſame uſe or a like office, that ours is placed here.

The largeſt catalogue of the ſtars that we have, is that of Mr. Flamſteed's, which contains 3000 ſtars, 13 of which are of the firſt magnitude; but then many of them are teleſcopic ſtars. And by this catalogue, the place of the leaſt ſtar, to be ſeen in

Fig. in the heavens, is better aſcertained than the places of moſt of the cities or towns in the world. And this is abſolutely neceſſary; for without knowing the exact places of the fixed ſtars, the orbits of the planets could never have been found. Upon the knowledge of this, all aſtronomy is founded.

That all the viſible ſtars in the firmament do not exceed that number, may ſeem ſtrange; but it may be demonſtrated thus. Take a tube whoſe length to the width is as 100 to 97, cover one end with a thin plate having only a ſmall hole to look thro'. And then in any ſtar-light night, if you look thro' it at any part of the heavens, and keep it ſteady; you'll ſcarce be able to reckon 100 ſtars. To facilitate the counting of them, you may fix three or four threads acroſs the mouth of the tube to divide it into ſmaller ſpaces. Now this tube takes in a proſpect of a tenth part of the hemiſphere, as may eaſily be demonſtrated by geometry.

It is the peculiar property of the fixed ſtars, that they never change their places, but keep always the ſame poſition and diſtances from one another without any variation; and have done the ſame from the earlieſt ages of antiquity, wherein any aſtronomical obſervations have been made. By this they become the ſtandard and rule for all motions that are made in the heavens, and of the ſituation of all bodies that come there. There are indeed ſome ſtars that diſappear for ſome time, and after ſome time make their appearance again; and others that never appear again; which is ſuppoſed to be owing to certain dark ſpots upon their ſurfaces, which cover either the whole or a part of their bodies; and in their rotation they ſometimes turn their dark ſide to us. But none of them was ever known to change their places.

Likewiſe there have ſometimes appeared new ſtars, in the heavens, or ſome that never were obſerved before; and after ſtaying a while, have diſappeared, and been no more ſeen. It is probable that their appearing and diſappearing is owing to the forementioned cauſe, *viz.* a dark ſide and a bright one, which they ſucceſſively turn towards us by a ſlow rotation round their axes. And that ſuch as never appear again have their whole bodies covered and overwhelmed with dark clouds and ſpots, impenetrable to the rays of light. Fig.

It is remarkable that thro' the beſt teleſcopes, a fixed ſtar only appears like a lucid point without any ſenſible magnitude. This ſhews at what an amazing diſtance the neareſt fixed ſtars are placed, ſince they cannot be magnified by the beſt teleſcopes. Their ſparkling is likewiſe a proof of their ſmallneſs; for a pencil of rays coming from a ſtar is ſo very ſmall, that any ſmall particle of the atmoſphere will ſtop it, or turn it out of the way; which is the cauſe of ſparkling.

There is a ſort of ſtars, called *nebulous ſtars*, from their appearing like a bright cloud. When theſe are looked at thro' a teleſcope, they are found to be a cluſter of very ſmall ſtars, not to be diſtinguiſhed ſingly, by the naked eye. And indeed to whatever part of the heavens the teleſcope is directed, numbers of ſmall ſtars will appear, which are inviſible to the naked eye.

As it is impoſſible to diſtinguiſh or know all the ſtars by particular names; therefore, for the more eaſy deſcribing or pointing out any one ſtar, aſtronomers have divided all the ſtarry heavens into certain figures, which they imagine any collection of the ſtars to repreſent; each figure conſiſting of a great number of ſtars. And theſe figures or images, they call the *Conſtellations*.

Fig.

PROB. VIII.

To deſcribe the number of conſtellations in the heavens.

For the more eaſy finding any ſtar, or its place in the heavens; aſtronomers have divided the whole ſurface of the celeſtial ſphere into parcels, which they call *Conſtellations*. A *Conſtellation* is a collection of ſtars, bounded by ſome figure in the ſhape of ſome animal or ſuch other thing, as they ſuppoſe the ſtars contained in it to repreſent. By help of this artifice they can find any particular ſtar, or direct one to it, by naming the conſtellation, and the part of it, in which it is to be found. Theſe conſtellations are painted on the celeſtial globe; and in each conſtellation the principal ſtars are marked ſeverally with the letters of the Greek alphabet; by which means, it is eaſy to find any one ſtar by that letter, which is a name for it, or has the office of one.

The celeſtial ſphere is likewiſe divided into three regions. The middle part is the *Zodiac*, which is a zone of 16 degrees breadth, running round the globe, parallel to the ecliptic, being 8 degrees broad on each ſide of it. This conſiſts of 12 conſtellations. The northern part reaches from the zodiac to the north pole of the ecliptic, and contains 28 conſtellations. The ſouthern part reaches from the zodiac to the ſouthern pole, containing 37 conſtellations. I ſhall ſet down the names of all theſe conſtellations, and the number of principal ſtars that each contains.

The 12 conſtellations in the zodiac, and their characters, are theſe,

Aries ♈,	20 ſtars.	Libra ♎,	20 ſtars.
Taurus ♉,	53	Scorpio ♏,	25
Gemini ♊,	33	Sagitary ♐,	22
Cancer ♋,	31	Capricorn ♑,	30
Leo ♌,	45	Aquarius ♒,	49
Virgo ♍,	50	Piſces ♓,	39

The ecliptic, which is the plane of the earth's orbit, continued to the fixed ſtars, runs thro' the middle of theſe, and is divided into 12 equal parts, called the 12 ſigns, whoſe names are theſe. But theſe diviſions do not now fall upon the diviſions of the conſtellations, as formerly. For by the preceſſion of the equinoxes, theſe diviſions are moved almoſt 30 degrees back, whence the conſtellations ſeem to have moved ſo far forward.

The northern part of the ſphere formerly conſiſted of 21 conſtellations. And thoſe ſtraggling ſtars, lying between, which were taken into none of the conſtellations, they called *unformed Stars*. But later aſtronomers, have made new conſtellations of them, to the number of 7. Whence the whole number of conſtellations in the northern hemiſphere are 28.

	Stars.		Stars.
1 Urſa minor	19	10 Lyra	17
2 Urſa major	39	11 Tygris	38
3 Draco	37	12 Cygnus	29
4 Cepheus	34	13 Sceptrum	17
5 Camelopardalus	28	14 Caſſiopeia	30
6 Jordanus	31	15 Perſeus	40
7 Bootes	38	16 Auriga	27
8 Corona borealis	9	17 Serpentarius	31
9 Hercules	34	18 Serpens	18

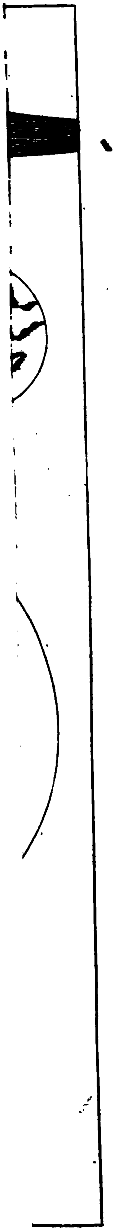
Fig.	Stars.		Stars.
19 Sagitta	8	24 Pegaſus	25
20 Aquila	24	25 Andromeda.	34
21 Antinous	12	26 Triangulum	5
22 Delphinus	11	27 Irinus	7
23 Equiculus	4	28 Coma Berenices	13

Among theſe conſtellations, there are ſeveral near the pole, that never ſet, but continue always above the horizon here in latitude 52.

Again, the ſouthern part of the ſphere contains 15 conſtellations which the ancients remarked; and 16 new ones that lie under the horizon to us, and are never ſeen; being about the ſouth pole. Theſe have been diſcovered by later aſtronomers; with ſeveral unformed ſtars, reduced lately into conſtellations. All which are theſe.

	Stars.		Stars.
1 Cetus	28	20 Phœnix	13
2 Orion	60	21 Indus	12
3 Eridanus	36	22 Pavo	14
4 Lepus	13	23 Apus	11
5 Canis major	19	24 Muſca	4
6 Columba	11	25 Cameleon	10
7 Canicula	11	26 Triangulum auſtrale	5
8 Monoceros	23	27 Piſcis volans	7
9 Navis	46	28 Dorado	4
10 Hydra	29	29 Nubes	3
11 Crater	11	30 Toucan	8
12 Corvus	9	31 Hydrus	10
13 Crux	4	32 Nubicula	3
14 Centaurus	35	33 Rhomboides	4
15 Lupus	21	34 Royal oak	10
16 Ara	7	35 Lynx	19
17 Corona auſtralis	13	36 Vertagus	23
18 Piſcis auſtralis	17	37 Unicorn	19
19 Grus	13		

And



pa. 40.

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And ſome more of leſſer moment are mentioned by ſome authors. Fig.

All the planets move in the Zodiac, and all their motions are contained within the bounds of it.

The milky way is a white broad circle going round the whole heavens; in ſome places it is double. When this is looked at with a teleſcope, it appears to be full of innumerable fixed ſtars, which are inviſible to the naked eye; and by the united force of their light, they illuminate that part of the heavens. Theſe we call Nebulous Stars, are of this kind. There are ſome of theſe towards the Antarctic Pole.

There are alſo in the heavens, ſeveral whitiſh ſpots, which viewed thro' a teleſcope, appear very bright; but no ſtars appear in them. So that if there be any in them, they muſt be ſo ſmall as not to be ſufficiently magnified by a teleſcope, to become viſible.

Some of the largeſt ſtars have particular names, as *Syrius*, *Arcturus*, *Pole Star*, *Aldebaran*, *Capella*, &c.

PROB. IX.

To give a juſt repreſentation of the ſolar ſyſtem. 13.

Suppoſe yourſelf placed on a wide moor or large extended plane, and in the middle of it let there be erected a globe 5 ½ feet diameter, which ſuppoſe to be the Sun, ☉.

At 80 yards diſtance from the ſun erect a globe ¼ inch diameter to repreſent Mercury, ☿.

At 140 yards from the ſun ſet up a globe near ¾ inch diameter for Venus, ♀.

At 200 yards from the ſun ſet up a globe ¾ inch diameter for the Earth, ⊕.

At

Fig. 13. At 300 yards from the ſun place a globe $\frac{3}{8}$ inch diameter for Mars, ♂.

At 1040 yards (or ſomething more than half a mile) place a globe near 6 $\frac{1}{2}$ inches diameter for Jupiter ♃.

At 1 $\frac{1}{12}$ mile from the ſun ſet up a globe 5 inches diameter for Saturn, ♄.

Again, at near 2 feet diſtance from the earth ſet up a globe $\frac{1}{5}$ inch diameter for the moon, ☽.

And at the reſpective diſtances 1 $\frac{1}{2}$, 2, 3 $\frac{1}{2}$, 6 feet from Jupiter, ſet up 4 little globes like the moon for the Satellites of Jupiter.

And at the diſtances 2, 2 $\frac{1}{2}$, 3 $\frac{1}{2}$, 8, 25 feet from Saturn reſpectively, ſet up 5 other little globes like the moon, for the Satellites of Saturn.

And thus you have very nearly the true proportion of all theſe great bodies to one another, and to their reſpective diſtances from each other, as they are in the heavens: which therefore is a true repreſentation of the ſolar ſyſtem. Where note an inch repreſents 10,400 miles, and a yard 370,000 miles, in round numbers.

Now ſuppoſe ſeveral large circles to be drawn thro' the places of the bodies ♄, ♃, ♂, ⊖, ♀, and ☿, whoſe common center is the ☉. And imagine theſe bodies to be carried about the ſun in theſe circles, by ſeveral men. And let each of theſe men move in his proper circle, with his left hand towards the ſun, and with ſuch a motion, as to walk ſo many miles in a minute, viz. ☿ 35, ♀ 24, ⊖ 21, ♂ 16 $\frac{1}{2}$, ♃ 9, ♄ 6 $\frac{3}{4}$. By this means the earth will move round the ſun in an hour, which here repreſents a year; and the others in proportional times, according to their ſeveral periods.

Whilſt the earth moves in his circle round the ſun, cauſe the moon to move the ſame way, and nearly in the ſame plane, about the earth, for its Fig. center; in a circle 2 foot diſtance from it; performing a ſynodic revolution in near 5 minutes. 13.

Alſo whilſt the earth moves in his orbit, cauſe it to revolve about its axis the ſame way as it moves about the ſun; ſo as to make one rotation in near 10 ſeconds, whilſt its axis always remains parallel to itſelf, and elevated 23½ degrees.

But we are not to ſuppoſe, that theſe bodies in the heavens are carried in this manner about the ſun in their proper orbits, by ſome intelligent beings that are always to attend them, as theſe bodies here deſcribed are carried by men. For there are two or three laws of nature which perform all this. Likewiſe we here ſuppoſe all theſe bodies to move in one plane; but in the heavens each of them moves in a ſeparate one, but then they are very little inclined to one another, and may therefore be looked upon to be nearly in the ſame plane. Likewiſe their ſeveral orbits are here ſuppoſed to be circles; but in the heavens, they are ellipſes very near to circles, and therefore may be ſuppoſed ſuch: this repreſentation being deſigned only to give a general idea of the planetary ſyſtem.

Laſtly, at the diſtance of about 300 miles, if you ſuppoſe ſome more globes ſet up, like the ſun; theſe will repreſent ſome of the neareſt fixed ſtars. For ſince the fixed ſtars have no ſenſible parallax, from the annual motion of the earth; the diſtance of the neareſt fixed ſtars, muſt be 2 or 3 thouſand times more than the diſtance of the ſun. And therefore cannot be leſs than 300 miles.

SCHOLIUM.

All theſe ſeveral diſtances being laid out in their true proportions, will give a more juſt idea of the ſolar ſyſtem, than any orrery or mechanical machine whatſo-

Fig. 13. whatſoever; tho' theſe ſorts of machines are contrived very well for ſhewing their reſpective motions; but by the ſmall compaſs they are contained in, can give no idea of their diſtances. And what a grand idea muſt this give us, of the immenſity of the univerſe; and of the regular diſpoſition of all the celeſtial bodies; and the wonderful harmony they obſerve in their motions? And nothing can be more ſublime than the ſpeculations and diſcoveries which aſtronomy affords us, in contemplating the ſyſtem of the world, and the beauty and analogy of all its parts.

SECT. II.

Aſtronomical Problems, depending on the Circles of the Sphere; ſolved by Calculation.

ALTHOUGH the celeſtial bodies are placed at different diſtances from the earth; ſome being vaſtly remote, while others are nearer hand. Yet an obſerver, living upon the earth, is not ſenſible of theſe different diſtances; and therefore he imagines all remote objects in the heavens to be placed in one concave ſurface, whereof the earth whereon he lives, is the center. Thus the ſun, moon and ſtars, ſeem to be placed in one and the ſame concave ſurface of the heavens, and therefore appear all at the ſame diſtance from the earth. Upon this account aſtronomers, for the eaſe of calculation, have ſuppoſed all the celeſtial bodies to be placed in the ſurface of this ſphere, while the earth is ſuppoſed to be no more than a point in the center of it. Fig.

Likewiſe, altho' in reality, the earth turning about its axis in 24 hours is the cauſe of day and night; and its moving thro' the ecliptic or its own orbit in a year cauſes all the ſeaſons of the year: yet becauſe all appearances and places of the celeſtial bodies will be exactly the ſame, whether the earth moves and the heavens ſtand ſtill; or the earth is at reſt, and the heavens move; therefore it is the ſame thing in effect whether of them we ſuppoſe to move. Therefore aſtronomers chuſe to ſuppoſe the earth to be at reſt, and aſcribe all theſe motions to the celeſtial bodies, and ſuppoſe them to have the very ſame motions which they ſeem to have; aſſigning no motion to the earth, as it ſeems

Fig. to have none. By this means all theſe relative motions are eaſier apprehended, as people are accuſtomed to think this way; and the calculations of all problems of the ſphere become more ſimple.

DEFINITIONS.

DEF. I.

14. The *Equinoctial or Equator* EQ is a great circle of the ſphere, which is 90 degrees diſtant from the poles of the world P, *p*; which are alſo the poles of the equinoctial. P the north pole, *p* the ſouth pole. And P*p* the axis of the world. This circle is made by producing the earth's equinoctial, till it cuts the ſtarry heaven, and its axis is made by producing the axis of the earth to the ſtars at P and *p*.

This circle divides the world into two hemiſpheres, the northern EPQ, and the ſouthern E*p*Q. It is called the equinoctial, becauſe when the ſun is in it, the days and nights are equal all over the world. And this happens twice in the year, in March and September.

This circle is divided into 360 degrees, beginning at the firſt point of Aries, where it interſects the ecliptic, and reckoned eaſtward. The whole heavens and all their furniture ſeem to turn round the earth, parallel to this circle, from eaſt to weſt, in 24 hours time; ſo that by the diurnal motion, a celeſtial body deſcribes 15 degrees in an hour, and 15 minutes of a degree in a minute's time; or a degree in 4 minutes.

And hence any body placed in the equinoctial will ſeem to deſcribe a great circle in the heavens; and thoſe at a diſtance will deſcribe leſſer circles, which will be the leſs, the nearer they are to the pole. And a ſtar which is in the pole, will be at reſt.

This circle belongs originally to the earth, and is transferred to the heavens only upon account of

the diurnal motion of the heavens, which is only Fig.
an apparent motion in them. 14.

The right aſcenſion of the ſun or a ſtar is counted upon this circle; and alſo the longitude of places upon the earth.

DEF. II.

The *Ecliptic* KL, is a great circle, cutting the equinoctial EQ at C, in an angle of 23 ½ degrees, or rather 23 degrees, 29 minutes. Theſe points of interſection are called the equinoctial points; that is, the *vernal equinox*, where the ſun is in the ſpring; and the *autumnal equinox*, where he is in autumn. The points K, L, which are 90 degrees from the equinoctial points C, are called the *Solſtitial Points*; K the *Summer Solſtice*; and L the *Winter Solſtice*. The ecliptic is divided into 12 equal parts, called the 12 *Signs*, and each ſign into 30 degrees.

This circle runs thro' the middle of the Zodiac; and in the ſame circle, the ſun ſeems to move round the earth in a year. For ſince the earth is carried round the ſun, in its proper orbit in the ſpace of a year; a ſpectator in the ſun would ſee the earth deſcribe this circle in the heavens; and a ſpectator in the earth would ſee the ſun deſcribe the very ſame circle, by its apparent motion. All the difference is, theſe two ſpectators would ſee the earth and ſun in oppoſite points of the orbit; or in conjunction with ſtars, which are diametrically oppoſite; but the apparent motion will be the ſame whether ever be ſuppoſed to move. The ecliptic is divided by the equator into two equal parts, in the equinoctial points Aries and Libra. And aſtronomers begin to reckon from the beginning of Aries or the vernal equinox, going eaſtward. The ſix ſigns which lie on the north ſide of the equinoctial, are called northern Signs, which be ♈, ♉, ♊, ♋, ♌, ♍; and thoſe ſix on the ſouth ſide the equinoctial, are called ſouthern Signs; which

Fig. 14. which are, ♎, ♏, ♐, ♑, ♒, ♓. As the beginning of ♈ and ♎, are the equinoctial points; ſo the beginning of ♋ and ♑, are the ſolſtitial points. Alſo the aſcending ſigns are ♑, ♒, ♓, ♈, ♉, ♊; and the deſcending ones, ♋, ♌, ♍, ♎, ♏, ♐. When the motion in the ecliptic is made according to the order of the ſigns, from ♈ to ♉, and from ♉ to ♊, &c. the ſame way that the ſun ſeems to move; it is ſaid to be forward or *in conſequentia*. But if the motion be the contrary way, from ♉ to ♈, from ♈ to ♓, &c. then it is ſaid to be backward or *in antecedentia*.

When two planets, &c. are referred to the ſame point of the ecliptic, they are ſaid to be in *conjunction* (whoſe mark is ☌); and thoſe that are referred to oppoſite points, are ſaid to be in *oppoſition* (the mark is ☍). If they are 3 ſigns diſtant, they are in a *quartile* aſpect (□). If 4 ſigns diſtant, a *trine* aſpect (△). If 2 ſigns off, a *ſextile* (✱). Each of the ſigns is divided into 30 parts or degrees; and each degree into 60 minutes, &c. like all circles of the ſphere. The ſun continues about a month in one of theſe ſigns, and goes through almoſt a degree thereof every day. Tho' the ſun always keeps in the ecliptic; yet the planets all deviate therefrom more or leſs, ſometimes being on the north ſide, and ſometimes on the ſouth. Yet they are all contained in the bounds of the Zodiac. Since the ecliptic cuts the equinoctial in an angle of 23° 29′; therefore the poles of the ecliptic are 23° 29′ diſtant from the poles of the world. The point of the ecliptic which riſes is called the *horoſcope*. The higheſt point of it is called the *nonageſimal degree*; the point in the meridian is the *culminating point*, or *medium cœli*. The oppoſite point, the *imum cœli*. The equinoctial and ſolſtitial points, are called the four *cardinal points*; and the four ſeaſons of the year begin with them. The ſpring with ♈, ſummer with ♋, autumn with ♎, and winter with ♑; the ſun being in them at theſe times. Fig. 14.

The longitude of the ſun or a ſtar is counted upon this circle.

DEF. III.

The *Horizon* HO, is a great circle, dividing the upper from the lower hemiſphere. This circle divides the heavens from the earth; and its poles are Z the *zenith*, or the point of the ſky over our heads; and N the *nadir*, being the point directly under our feet.

The *ſenſible horizon*, is the circle round us, where the ſky ſeems to touch the earth; and is made by a plane touching the earth at the point we ſtand on. The *rational horizon*, is that made by a plane parallel to the former, and paſſing thro' the center of the earth. But both theſe become the ſame when extended to the ſphere of the fixed ſtars.

The horizon ſhews the riſing and ſetting of the heavenly bodies; and divides the day from the night. For when the ſun or a ſtar is in the horizon, it is ſaid to be *riſing* if it comes above it; or to be *ſetting*, if it goes below it. And the time the ſun continues above the horizon, is called *Day*. And the time it is under the horizon is *Night*.

It is plain the horizon is a variable circle, for there is a different one for every different point of the globe. So that a traveller never keeps the ſame horizon.

This circle is divided into 4 quarters or quadrants; and the points of diviſion are the eaſt, weſt, north and ſouth, called the four *cardinal points*. The *eaſt* is where the prime vertical cuts the horizon towards ſun-riſe, and the *weſt* where the ſame circle cuts it towards ſun-ſet. The *ſouth* is where the meridian cuts the horizon towards the ſouth pole; and the north where the ſame circle cuts it

Fig. 14. towards the north pole; which to us is the elevated pole.

The azimuth or amplitude of the ſun or a ſtar is counted upon this circle. And upon this circle are drawn the mariners 32 points of the compaſs. It is owing to this circle that all the celeſtial motions become ſenſible; and it is a ſort of a gage or landmark, whereby to judge of the quantities of theſe motions.

D E F. IV.

Meridians are great circles as PA*p*, paſſing thro' the poles of the world, P, *p*; and therefore are perpendicular to the equator EQ. Theſe are alſo called *hour Circles*. And upon the earth theſe circles are circles of longitude. Upon theſe circles, the declination of a ſtar is counted. And the latitudes of places on the earth are counted on theſe meridians.

D E F. V.

Meridian of a place, is that meridian which paſſes thro' the zenith of that place, as PZ*p*N. This circle paſſes thro' the north and ſouth points of the horizon, H and O. When the ſun is in this circle above the horizon, it is *noon*; and under the horizon it is *midnight*. It divides the world into two hemiſpheres, the eaſtern and weſtern. Thoſe that live under the ſame meridian have noon and midnight, and all the hours, at the ſame time. A traveller going eaſtward or weſtward, continually changes his meridian. The celeſtial meridian is only the terreſtrial one extended to the heavens. On this circle the latitude of the place is reckoned as EZ, which is equal to the height of the pole OP.

D E F. VI.

Vertical Circles are great circles as ZBN, paſſing thro' the zenith and nadir; and therefore are perpendicupendicular to the horizon. The altitude of a ſtar is counted in one of theſe circles. Fig. 14.

DEF. VII.

Equinoctial Colure is a meridian PCp, paſſing thro' the equinoctial points at C. This is perpendicular both to the ſolſtitial colure, and to the equinoctial.

DEF. VIII.

Solſtitial Colure is a meridian PKPL, paſſing thro' the ſolſtitial points K, L; or the points where the ſun is higheſt and loweſt in the courſe of the year. This is perpendicular to the equinoctial colure and to the equinoctial.

DEF. IX.

Prime Vertical is that vertical circle ZCN, which paſſes thro' the zenith, and is perpendicular to the meridian, and to the horizon. This cuts the horizon in the *eaſt* and *weſt* points. The eaſt being toward ſun-riſe, and the weſt toward ſun-ſet.

DEF. X.

Circles of longitude in the heavens, are great circles paſſing thro' the poles of the ecliptic R and *m*; and therefore are perpendicular to the ecliptic. The latitude of the ſtars is counted on theſe circles.

DEF. XI.

The *Tropics* are two leſſer circles KD, FL, parallel to the equator, and paſſing thro' the ſolſtitial points K and L; therefore they are 23° 29′ diſtant from the equinoctial. The north one KD is the *tropic of Cancer*; the ſouth one FL, the *tropic of Capricorn*. When the ſun is at the tropics it ſeems to ſtand ſtill awhile, and then return back.

DEF. XII.

The *polar Circles* are two leſſer circles MR, *mr*, diſtant from the poles P, p, 23° 29′. The northern one

Fig. 14. one MR is the arctic circle; the ſouth one *mr*, the antarctic circle.

DEF. XIII.

Parallels of latitude, on the earth, are leſſer circles KD, parallel to the equinoctial. But in the heavens, they are leſſer circles, parallel to the ecliptic, as ST.

DEF. XIV.

Parallels of altitude, are leſſer circles as GI parallel to the horizon.

DEF. XV.

Parallels of declination, are leſſer circles parallel to the equinoctial as KD.

DEF. XVI.

Latitude of a place, is an arch of the meridian contained between the equinoctial and the place given, as ES is the latitude of S. This is equal to the height of the pole above the horizon.

DEF. XVII.

Latitude of a Star, is an arch of a circle of longitude contained between the ſtar and the ecliptic, as KS is the latitude of S.

DEF. XVIII.

Declination of a Star, is an arch of the meridian contained between the ſtar and the equinoctial. Thus ES is the declination of S.

DEF. XIX.

Altitude of a Star, &c. is an arch of a vertical circle contained between the ſtar and the horizon; as VB is the altitude of V.

DEF. XX.

Azimuth is an arch of the horizon contained between the ſtar's azimuth circle, and the north point. BO is the azimuth of V. Fig. 14.

D E F. XXI.

Amplitude is an arch of the horizon between that point where the ſun or a ſtar riſes, and the eaſt or weſt point. As CB is the amplitude of B.

D E F. XXII.

Right Aſcenſion, is an arch of the equinoctial reckoned from the vernal equinox to the ſun or ſtar's meridian. Thus CA is the right aſcenſion of V, if it is between C and K; or elſe CEA, if V is beyond the tropic K.

D E F. XXIII.

Aſcenſional difference, is an arch of the equinoctial, between the ſun's meridian, and the point of the equinoctial that riſes with him. Or it is the angle at the pole between the ſun's meridian, and the ſix o'clock meridian. Thus the aſcenſional difference of Y is the arch AC, or the angle YPC.

D E F. XXIV.

Oblique Aſcenſion or *deſcenſion* is the ſum or difference of the right aſcenſion, and the aſcenſional difference. Or it is the diſtance from the firſt point of Aries, to that point of the equator, that riſes or ſets with the ſun, or any ſtar.

D E F. XXV.

Longitude of the Sun, is an arch of the ecliptic contained between the firſt point of Aries, and the ſun; and *that of a Star*, contained between the firſt point of Aries, and the ſtar's circle of longitude. Thus CX is the longitude of the ſun at X.

D E F. XXVI.

Longitude of a Place, is an arch of the equator between the firſt meridian, and the meridian of the

Fig. place. And *difference of longitude*, is the angle at
14. the pole between the meridians of two places. As
APG = dif. longitude of A and G.

DEF. XXVII.

Hour of the day is an arch of the equinoctial between the meridian of the place, and the ſun's meridian. As if the ſun be at A, then AE is the hour at the places S, K, E, G, &c.

All altitudes, azimuths, amplitudes, &c. and in general all diſtances relating to the ſphere, are meaſured by degrees, and minutes, as explained in the trigonometry. In ſolving any Problems of the ſphere, there is no occaſion for drawing the whole ſphere; it will be ſufficient in moſt caſes to draw only a triangle or ſuch a part of it, as contains the conditions of the Problem. And then all the following Problems will be ſolved upon this hypotheſis, that the earth ſtands ſtill, and that all the celeſtial bodies are in motion. The ſolution of Aſtronomical Problems is only the application of Spherical Trigonometry to Aſtronomy.

PROB. I.

15. *Given the place of the ſun in the ecliptic; to find his right aſcenſion and declination.*

Let CGB be the ecliptic, CA the equinoctial, C the vernal equinox, G the place of the ſun, and GF a meridian paſſing thro' the ſun. Then CF is the right aſcenſion, and GF the declination. In all theſe Problems, we ſuppoſe the angle GCF, which the ecliptic makes with the equator, to be given, which is 23° 29′. Then in the right angled ſpherical triangle CGF, there is given CG the ſun's longitude, and angle GCF the obliquity of the eclipitc. Therefore by Caſe 1. of right angled ſpherical triangles, to find CF. Fig. 15.

Radius :
Coſ. obliq. ecliptic GCF : :
Tan. ſun's longitude CG :
Tan. right aſcenſion CF.

And by Caſe 2. to find GF.

Radius :
S. obliq. ecliptic GCF : :
S. ſun's longitude CG :
S. declination GF.

If the ſun's longitude be greater than a quadrant, his right aſcenſion will be ſo too; but his declination never exceeds 23° 29′. Alſo if the ſun be in the northern ſigns, his declination will be north; if he is in the ſouthern ſigns, his declination is ſouth.

Cor. *If either the right aſcenſion or declination be given, the ſun's longitude, or his place in the ecliptic, may be found, by changing the proportion, or working it backwards.*

PROB. II.

There is given the ſun's declination, to find his right aſcenſion.

In the right angled triangle CFG, there is given FG the declination, and angle C the obliquity of the ecliptic; to find CF the right aſcenſion. By Caſe 10. of right angled ſpherical triangles,

Radius :
Cotan. obliquity of the ecliptic GCF : :
Tan. declination FG :
S. right aſcenſion CF.

Here CF may be either leſs or greater than a quadrant.

Cor. *If the right aſcenſion is given, the declination may be found, by working the proportion backwards.*

PROB. III.

16. *Given the latitude of the place, and the ſun's declination; to find his aſcenſional difference; and his riſing and ſetting.*

Let CB be the horizon, CA the equinoctial, DB the ſun's parallel, B the ſun's place at riſing, CD the ſix o'clock meridian; AB the ſun's meridian at riſing. Then the angle DCB is the latitude, and ACB the complement of the latitude. And CD or AB is the declination. Then in the right angled ſpherical triangle CAB, there is given the angle C, and ſide AB, to find the ſide CA. Therefore by Caſe 10. of right angled ſpherical triangles,

Rad :
Tan. latitude DCB : :
Tan. declination AB :
S. aſ. difference CA.

Then the aſcenſional difference is to be turned into time, allowing 4 minutes of time to 1 degree; and that will be the time the ſun riſes before 6 if it be ſummer, or after 6 in winter.

Cor. 1. *And if the lat. and aſ. difference be given; the declination may be found, by working the proportion backwards.*

Cor. 2. *When ſun-riſing is found, double it and it gives the length of the night; and double the time of ſun-ſet, for the length of the day.*

PROB. IV.

Having the latitude of the place, and the ſun's declination to find his amplitude.

Let CB the horizon, CA the equinoctial, B the ſun, BA or CD his declination, angle ACB the complement of the latitude. Then in the right Fig. angled ſpherical triangle CAB, there is given the 16. angle C and ſide AB, to find the hypothenuſe CB. Therefore by Caſe 12. of right angled ſpherical triangles,

Coſ. latitude :
S. declination AB : :
Radius :
S. amplitude CB.

If the declination is north the amplitude is north; and if ſouth, it is ſouth.

Cor. 1. *If the latitude and amplitude be given; the declination may be found by the ſame proportion.*

Cor. 2. *If the declination and amplitude be given; the latitude may be found by working backwards.*

PROB. V.

Given the latitude of the place, and the ſun's declination; to find his altitude when eaſt. 17.

Let HC be the horizon, EC the equinoctial, S the ſun, HES the meridian paſſing thro' the ſun, CS the prime vertical. Then ES is the declination, and EC the aſcenſional difference, CS his altitude eaſt, and < ECS the latitude. Therefore in the right angled ſpherical triangle CES, there is given the ſide ES, and the angle ECS, to find the hypothenuſe CS. Therefore by Caſe 12. of right angled ſpherical triangles,

S. latitude ECS :
S. declination ES : :
Radius :
S. altitude when eaſt, or weſt CS.

Cor. 1. *If the latitude and altitude eaſt, be given; the declination may be found, by the ſame proportion.*

Cor.

Fig. 17. Cor. 2. *If the declination and altitude eaſt, be given; the latitude may be found, by working backwards.*

PROB. VI.

17. *Having given the latitude of the place, and the ſun's declination; to find the time when he is eaſt.*

Let HC be the horizon, EC the equinoctial, S the ſun's place, HES the ſun's hour circle, and CS the prime vertical. Then the angle ECS is the latitude, ES the declination, CS the altitude when eaſt, and EC the aſcenſional difference. Therefore in the right angled ſpherical triangle CES; there is given the ſide ES, and the angle ECS; to find the ſide CE. Whence by Caſe 10. of right angled ſpherical triangles,

Radius :
Cotan. latitude ECS : :
Tan. declination ES :
S. aſcenſional difference EC.

This converted into time, allowing 15 degrees to an hour, and 1 degree to 4 minutes, ſhews how long it is after 6 in the morning, or before 6 in the evening, when the ſun is eaſt or weſt.

Cor. 1. *If the latitude and time when the ſun is eaſt be given, the declination may be found, by working backwards.*

Cor. 2. *If the declination and time or aſcenſ. difference be given; the latitude may be found, by varying the proportion.*

PROB. VII.

18. *The latitude of the place, and ſun's declination being given; to find his altitude at 6.*

Let CO be the horizon, CS the 6 o'clock meridian, AC the prime vertical, S the place of the ſun, AS his parallel of altitude, OS his vertical Fig. circle. SCO the latitude of the place. Then in 18. the right angled ſpherical triangle COS, there is given the hypothenuſe CS and angle OCS, to find the ſide OS. Therefore by Caſe 2. of right angled ſpherical triangles,

Radius :
S. declination CS : :
S. latitude OCS :
S. altitude at 6, SO.

Cor. 1. *If the latitude and altitude at 6 be given, the declination may be found, by varying the proportion.*

Cor. 2. *If the declination and altitude at 6 be given; the latitude may be found, by working the proportion backwards.*

PROB. VIII.

The latitude of the place, and the ſun's declination being given; to find his azimuth at 6 o'clock. 18.

Let CO be the horizon, S the ſun's place, CS the 6 o'clock meridian, SO the ſun's vertical circle, angle SCO the latitude; then CO is his azimuth at 6. Therefore in the right angled ſpherical triangle COS, there is given the declination CS, and angle SCO, to find CO. Therefore by Caſe 1. of right angled ſpherical triangles,

Radius :
Coſ. latitude SCO : :
Tan. declination CS :
Tan. azimuth at 6, CO from the eaſt.

Then the azimuth from the north is readily found.

Cor. 1. *If the latitude, and ſun's azimuth at 6, be given; the declination may be found, by working backwards.*

Cor.

Fig. Cor. 2. *If the ſun's declination, and azimuth at 6*
18. *be given; the latitude may be found, by the ſame proportion.*

PROB. IX.

16. *The latitude of the place, and the ſun's amplitude being given; to find the aſcenſional difference.*

Let CB be the horizon, CA the equinoctial, B the place of the ſun, AB his hour circle, BCA the complement of the latitude. Then in the right angled ſpherical triangle CAB, there is given the angle ACB, the hypothenuſe CB the amplitude; to find the ſide CA or the aſcenſional difference. Therefore by Caſe 1. of right angled ſpherical triangles,

Radius :
S. latitude DCB : :
Tan. amplitude CB :
Tan. aſcen. difference CA.

Then CA turned into time allowing 15 degrees to an hour, and 1 degree to 4 minutes, gives the time the ſun riſes before 6 in ſummer, or after 6 in winter.

Cor. 1. *If the latitude of the place, and the aſcenſional difference be given, the amplitude may be found by working backwards.*

Cor. 2. *If the amplitude and aſc. difference be given, the latitude may be found by the ſame proportion.*

PROB. X.

16. *The ſun's amplitude and declination given; to find his aſcenſional difference.*

Let CB be the horizon, B the ſun's place, BA his hour circle, CA the equinoctial. Then CB is the amplitude, AB the declination, and AC the aſcenſional difference. Therefore in the right angled ſpherical triangle CAB, there is given the hypotheneuſe CB, and the declination AB; to find the aſcenſional difference CA. Therefore by Caſe 6. of right angled ſpherical triangles, Fig. 16.

Coſ. declination AB :
Coſ. amplitude CB : :
Radius :
Coſ. aſc. difference CA.

The aſcenſional difference converted into time, allowing 15 degrees for an hour; ſhews how long the ſun riſes before 6 in ſummer, or after 6 in winter.

Cor. 1. *If the ſun's declination, and the aſcenſional difference be given; his amplitude may be found, by the ſame proportion.*

Cor. 2. *If the amplitude, and the aſcenſional difference be given; the declination may be found, by working backwards.*

PROB. XI.

Having given the latitude of the place, and the ſun's azimuth at 6 o'clock; to find his altitude. 18.

Let CO be the horizon, CS the 6 o'clock hour circle, S the ſun's place, SO the ſun's vertical circle. Then OCS is the latitude, CO the ſun's azimuth from the eaſt at 6, and OS his altitude at that time. Therefore in the right angled ſpherical triangle COS, there is given the ſide CO, and the angle OCS; to find the ſide OS. Then by Caſe 7. of right angled ſpherical triangles,

Radius :
S. azimuth CO, from the eaſt : :
Tan. latitude SCO :
Tan. altitude at 6, OS.

Cor.

Fig. 18. Cor. 1. *If the latitude and altitude at 6 be given; the azimuth may be found, by the ſame proportion.*

Cor. 2. *If the altitude and azimuth at 6 be given; the latitude of the place may be found, by working backwards.*

PROB. XII.

17. *Given the latitude of the place, and ſun's altitude when eaſt; to find the aſcenſional difference, or the hour.*

Let HC be the horizon, SC the prime vertical, EC the equinoctial, S the place of the ſun, SE his hour circle. Then angle ECS is the latitude, SC the ſun's altitude when eaſt, and EC the aſcenſional difference. Therefore in the right angled ſpherical triangle ECS, there is given the hypothenuſe SC, and the angle SCE; to find the ſide EC. Then by Caſe 1. of right angled ſpherical triangles,

Radius :
Coſ. latitude ECS : :
Tan. altitude eaſt SC :
Tan. aſc. difference EC.

This turned into time, putting 4 minutes for every degree, gives the time after 6, when the ſun is eaſt; or before 6, when weſt.

Cor. 1. *If the latitude, and the hour, or aſcenſional difference be given; the altitude when eaſt or weſt may be found, by varying the proportion.*

Cor. 2. *If the altitude eaſt, and the aſc. difference be given; the latitude may be found, by the ſame proportion.*

PROB. XIII.

17. *Given the ſun's declination and his altitude when eaſt; to find the time, or the aſcenſional difference.*

Let HC be the horizon, EC the equinoctial, S the place of the ſun, SC the prime vertical, SE the ſun's meridian; then < SCE is the latitude, Fig. SE the declination, SC the altitude at 6, and EC 17. the aſcenſional difference. Then in the right angled ſpherical triangle SCE, there is given the hypothenuſe SC, and ſide SE; to find the ſide CE. Therefore by Caſe 6. of right angled ſpherical triangles,

Coſ. declination SE :
Coſ. altitude eaſt SC : :
Radius :
Coſ. aſc. difference CE.

Then CE turned into time, allowing 15 degrees to an hour, &c. gives the time after 6, when the ſun is eaſt; or before 6, when weſt.

Cor. 1. *If the ſun's declination, and the time he is eaſt, be given; his altitude may be found, by the ſame proportion.*

Cor. 2. *If the ſun's altitude when eaſt, and the time be given; his declination may be found, by working backwards.*

PROB. XIV.

Having the ſun's declination given, and altitude at 6; to find his azimuth. 18.

Let CO be the horizon, CS the 6 o'clock hour circle, S the place of the ſun, SO his vertical circle. Then angle SCO is the latitude, CS his declination, CO the complement of his azimuth, and SO his altitude at 6 o'clock. And in the right angled ſpherical triangle CSO, there is given the hypothenuſe CS, the ſide SO; to find the other ſide CO. Therefore by Caſe 6. of right angled ſpherical triangles,

Coſ. altitude at 6, SO :
Coſ. declination CS : :
Radius :
Coſ. azimuth at 6, from the eaſt, CO.

Fig. 18. Cor. 1. *If the declination and azimuth at 6 be given; the altitude at 6 may be found, by working the proportion backwards.*

Cor. 2. *If the altitude and azimuth at 6 be given; the declination may be found, by the ſame proportion.*

PROB. XV.

19. *Given the latitude, and time of the day, and the ſun in the equinoctial; to find his altitude.*

Let AC be the horizon, CS the equinoctial, S the ſun's place, SA the ſun's vertical circle. Then ACS is the complement of the latitude, AS the ſun's altitude, CS the arch of the equinoctial correſponding to the time from 6 o'clock; which is known by reducing the time from 6, into degrees, allowing 15 degrees for an hour. Then in the right angled ſpherical triangle ACS, there is given the hypothenuſe CS, and angle C; to find the ſide AS. Therefore by Caſe 2. of right angled ſpherical triangles,

Radius :
S. hour arch from 6, CS : :
Coſ. latitude, comp. C :
S. altitude AS.

Cor. 1. *If the latitude and altitude be given; the time may be found by the ſame proportion.*

Cor. 2. *If both the time, and ſun's altitude be given; the latitude may be found, by working the proportion backward.*

PROB. XVI.

19. *Given the latitude and time of the day, and the ſun in the equator; to find his azimuth.*

Let AC be the horizon, CS the equinoctial, S the ſun's place, and SA his vertical circle. Then ACS is the complement of the latitude, AC his azimuth from 6, CS the arch of the equinoctial correſponding to the time. Then in the right angled ſpherical triangle ACS, there is given the hypothenuſe CS, and angle C; to find the ſide CA. Therefore by Caſe 1. of right angled ſpherical triangles, Fig. 19.

Radius :
Sine. latitude (comp. C) : :
Tan. hour arch from 6, CS :
Tan. azimuth from 6, CA.

Cor. 1. *If the latitude and azimuth be given, the ſun being in the equator; the time may be found, by working backwards.*

Cor. 2. *If the azimuth and time be given; the latitude may be found, by the ſame proportion.*

PROB. XVII.

Given the latitude, and ſun's altitude, being in the equator; to find his azimuth. 19.

Let AC be the horizon, CS the equinoctial, S the ſun's place, SA his vertical circle. Then ACS is the complement of the latitude, AS his altitude, AC his azimuth from 6. Therefore in the right angled ſpherical triangle ACS, there is given the angle C, the ſide AS; to find the ſide AC. Whence by Caſe 10. of right angled ſpherical triangles,

Radius :
Tan. latitude (comp. C) : :
Tan. altitude AS :
S. azimuth from 6, AC.

Cor. 1. *If the latitude and azimuth be given; the altitude may be found, by working backwards.*

Cor. 2. *If the ſun's altitude and azimuth be given, being in the equinoctial; the latitude may be found, by the ſame proportion.*

PROB. XVIII.

14. *Given the latitude of the place, and ſun's meridian altitude; to find its declination.*

Let GH be the altitude of the ſun, HE the complement of the latitude, or the height of the equator. Then ſubtract the leſſer altitude from the greater, and the remainder GE is the declination. And the declination will be north, when the height of the ſun is greater than the complement of the latitude; but ſouth, if leſſer. As is evident by the figure.

Cor. *If the ſun's meridian altitude be given, and his declination; the latitude may be found, by the converſe of this.*

PROB. XIX.

To find at what hour any particular ſtar comes upon the meridian, on a given day.

This is performed by the tables of the ſun's right aſcenſion, and of the ſtars. Find the right aſcenſion of the ſun, and of the given ſtar, in the tables; and ſubtract the right aſcenſion of the ſun from the right aſcenſion of the ſtar, adding 24 hours or 360 degrees, when the ſtar is leſs. And the remainder is the difference of the right aſcenſions, or the time of the ſtar's ſouthing, from noon.

If this difference of the right aſcenſions is had in degrees; it is eaſily turned into time, allowing 15 degrees to an hour.

PROB. XX.

The day and hour being given; to find what ſtar is ſouth at that time.

In the table of the ſun's right aſcenſion, find his right aſcenſion for the given day. To this add the given hour reckoned from noon, if the right aſcenſion be for time. But if the right aſcenſion be for degrees, reduce that time to degrees, and add it to the ſun's right aſcenſion: the ſum will be the right aſcenſion of ſome ſtar. Therefore look for it, in the table of the ſtar's right aſcenſion; and againſt it, or the neareſt to it, you will find what ſtar it is. *Note*, you muſt throw away 24 hours or 360 degrees, when the ſaid ſum exceeds that. Fig.

SCHOLIUM.

In the foregoing problems we have only had regard to the ſun's motion; but in moſt of theſe problems, the ſolutions hold equally true for any ſtar or other object. Likewiſe what is ſaid about the eaſt points of the horizon, as the amplitude, altitude, &c. belongs equally to the weſt points. I ſhall now proceed to the ſolution of ſuch problems of the ſphere as depend on oblique ſpherical triangles.

PROB. XXI.

Having given the latitude of the place, the ſun's declination, and altitude; to find his azimuth, and time of obſervation, and angle of poſition ZSP. 20.

Let MZP be the meridian, Z the zenith, P the pole, S the place of the ſun, ZS his vertical circle, PS his meridian. Then ZP is the complement of latitude, SP the complement of declination, and SZ the complement of altitude; SZP the azimuth, and ZPS the hour angle from noon, and ZSP the angle of poſition. Therefore in the oblique ſpherical triangle ZPS, there is given the ſides ZP, SP and SZ; to find the angles SZP, SPZ, and ZSP. Therefore by Caſe 11. of oblique ſpherical triangles.

1. For

Fig. 20.

1. *For the azimuth* SZP.

$S.ZS \times S.ZP$:
Radius ſquare : :
$S.\frac{SP + ZP - ZS}{2} \times S.\frac{SP + ZS - ZP}{2}$:
Sine ſquare of $\frac{1}{2}$ SZP.

Then $\frac{1}{2}$ SZP will be known, which doubled gives SZP the azimuth.

2. *For the angle* ZPS.

$S.PZ \times S.PS$:
Radius ſquare : :
$S.\frac{ZS + ZP - PS}{2} \times S.\frac{ZS + PS - ZP}{2}$:
Sine ſquare of half ZPS.

Then $\frac{1}{2}$ ZPS being known, if it be doubled it gives the whole angle ZPS, which reduced into time, counting 15 degrees for an hour, gives the time from noon, that this happens.

3. *For the angle of poſition* ZSP.

$S.SZ \times S.SP$:
Radius ſquare : :
$S.\frac{ZP + ZS - SP}{2} \times S.\frac{ZP + SP - ZS}{2}$:
Sine ſquare of $\frac{1}{2}$ ZSP.

That angle doubled gives the angle of poſition ZSP.

PROB. XXII.

Given the latitude of the place, and the ſun's declination and azimuth; to find his altitude, and hour of obſervation.

Let MZP be the meridian, S the place of the ſun, ZS his vertical circle, PS his hour circle.

Then in the oblique ſpherical triangle ZPS, there is given the complement of latitude ZP, the complement of declination SP, and the azimuth SZP; to find the altitude or its complement ZS, and the angle ZPS. Therefore by Caſe 1. and 3, of oblique ſpherical triangles, Fig. 20.

1. *For the altitude or its complement ZS.*

by Caſe 1, S.SP :
S.SZP ::
S.ZP :
S.ZSP the angle of poſition.

Then by Caſe 3, for obliques,

$$\text{S.}\,\frac{ZPS - ZSP}{2} :$$

$$\text{S.}\,\frac{ZPS + ZSP}{2} ::$$

$$\text{Tan.}\,\frac{PS - ZP}{2} :$$

Tan. $\frac{1}{2}$ ZS, which doubled gives ZS the complement of the altitude.

2. *For the angle* ZPS.

Find the angle ZSP as before, then by Caſe 2, of obliques,

$$\text{S.}\,\frac{SP - ZP}{2} :$$

$$\text{S.}\,\frac{SP + ZP}{2} ::$$

$$\text{Tan.}\,\frac{SZP - ZSP}{2} :$$

Cot. $\frac{1}{2}$ ZPS, which doubled gives ZPS; this angle reduced to time, allowing 15 degrees to an hour, ſhews the hour from 12.

Fig.

PROB. XXIII.

20. *Given the latitude, ſun's declination, and hour of the day; to find the altitude and azimuth.*

Having the hour from noon, reduce it into degrees, allowing 15 degrees to an hour, then you have the hour angle.

Let MP be the meridian, S the ſun's place, SZ his vertical circle, and SP his hour circle.

Then in the oblique ſpherical triangle ZPS, there is given ZP the complement of latitude, SP the complement of the ſun's declination, and ZPS the hour angle. To find ZS and angle PZS. Let fall the perpendicular SM. Then by Caſe 7, of oblique triangles.

1. *For the azimuth.*

Radius :
Coſ. hour angle ZPS : :
Tan. PS the co-declination :
Tan. PM.

Then PM — PZ, or the difference of PM and PZ is = ZM. Then

S.PM :
S.ZM : :
Cotan. hour angle SPZ :
Cotan. PZS the azimuth.

Then if PM be leſs than PZ, SZP is of the ſame affection as SPZ, otherwiſe of different affection.

2. *For the altitude.*

Find the azimuth as before; then by Caſe 8, of oblique ſpherical triangles,

S. Azimuth PZS :
S.PS the co-declination : :
S. hour angle ZPS :
S. co-altitude CS.

PROB. XXIV.

Given the latitude of the place, the ſun's altitude and azimuth; to find the declination, and hour of the day. 20.

Let PM be the meridian, Z the zenith, P the pole, S the ſun, ZS his vertical circle, PS his hour circle.

Then in the ſpherical triangle ZPS, there is given ZP the co-latitude; ZS the co-altitude, and PZS the azimuth. To find the ſide PS and angle ZPS.

Let fall a perpendicular SM upon PZ, then by Caſe 8, of oblique ſpherical triangles,

1. *For the declination.*

Radius :
Coſ. Azimuth PZS : :
Tan. co-altitude ZS :
Tan. ZM. Then PZ + ZM = PM.
Then Coſ. ZM :
Coſ. co-altitude ZS : :
Coſ. PM :
Coſ. co-declination PS, or the ſine of the declination.

2. *For the hour.*

Find the baſe ZM as before; then by Caſe 7.

S.ZM :
S.PM : :
Cot. azimuth PZS :
Cot. hour angle ZPS.

Then the angle ZPS being turned into time, reckoning 15 degrees to an hour, ſhews the time from noon.

PROB. XXV.

Having given the latitude, the ſun's altitude, and the hour; to find his declination and azimuth. 20.

Let PZM be the meridian, Z the zenith, P the pole, ZP the complement of the latitude, ZS his

Fig. co-altitude, PS his co-declination, PZS his azimuth,
20. and ZPS the hour angle.

Then in the oblique ſpherical triangle ZPS, there is given PZ, ZS, and angle ZPS by turning the time into degrees; to find PS and angle PZS.

1. *For the declination.*

By Caſe 1, of oblique ſpherical triangles,
S. co-altitude SZ :
S. hour angle SPZ : :
S. comp. latitude ZP :
S. ZSP; then
by Caſe 3, S. $\frac{1}{2}$ diff. angles ZPS and ZSP :
S. $\frac{1}{2}$ ſum of ZPS and ZSP : :
Tan. $\frac{1}{2}$ diff. ſides ZP and ZS :
Tan. $\frac{1}{2}$ PS, which doubled is PS the complement of the declination.

2. *For the azimuth.*

Find the angle ZSP as before. Then by Caſe 2,
S. $\frac{1}{2}$ diff. of the ſides ZP and ZS :
S. $\frac{1}{2}$ ſum of the ſides ZP and ZS : :
Tan. $\frac{1}{2}$ diff. angles ZPS and ZSP :
Cotan. $\frac{1}{2}$ PZS, which doubled is PZS the azimuth.

PROB. XXVI.

20. *Given the latitude of the place, the ſun's azimuth and hour of the day; to find his declination and altitude.*

Let MP be the meridian, Z the zenith, P the pole, S the ſun; then ZS is the ſun's vertical, and PS his hour circle. Therefore,

In the oblique ſpherical triangle ZPS, there is given the ſide ZP, and the angles PZS and ZPS; to find PS and ZS; for the time from noon being turned into degrees gives the angle ZPS.

Then by Caſe 9, of oblique ſpherical triangles, Fig. 20.
Coſ. $\frac{1}{2}$ ſum angles PZS and ZPS :
Coſ. $\frac{1}{2}$ diff. angles PZS and ZPS : :
Tan. $\frac{1}{2}$ comp. latitude PZ :
Tan. $\frac{1}{2}$ ſum of the op. ſides PS and ZS.
Alſo S. $\frac{1}{2}$ ſum angles PZS and ZPS :
S. $\frac{1}{2}$ diff. angles PZS and ZPS : :
Tan. $\frac{1}{2}$ comp. latitude PZ :
Tan. $\frac{1}{2}$ diff. oppoſite ſides PS and ZS.

The ſum of that $\frac{1}{2}$ ſum and $\frac{1}{2}$ difference, gives the greater ſide, or that oppoſite to the greater angle; and their difference gives the leſſer ſide, or that oppoſite to the leſſer angle. And theſe are the complements of the declination and altitude.

PROB. XXVII.

The ſun's declination, altitude, and azimuth being given, to find the time of obſervation, and latitude of the place. 20.

Suppoſe MP the meridian, Z the zenith, P the pole, S the ſun, ZS the co-altitude, PS the co-declination. SZP the azimuth, ZPS the hour angle, ZP the co-latitude. Therefore,

In the oblique ſpherical triangle ZPS, there is given the ſides SP and SZ, and angle PZS; to find the angle SPZ and ſide ZP.

1. *For the time.*

By Caſe 1ſt, of oblique ſpherical triangles,
S. comp. declination SP :
S. azimuth SZP : :
S. comp. altitude SZ :
S. hour angle ZPS.

Then ZPS reduced to time, counting 15 degrees to an hour, gives the time from noon.

2. *For*

Fig. 20.

2. *For the latitude.*

The angle ZPS being found, then by Caſe 3d,
S. $\frac{1}{2}$ diff. ZPS and PZS :
S. $\frac{1}{2}$ ſum, ZPS and PZS : :
Tan. $\frac{1}{2}$ diff. ſides SP and SZ :
Tan. $\frac{1}{2}$ ZP; then this doubled gives ZP the complement of the latitude.

PROB. XXVIII.

20. *The ſun's declination, altitude and hour of the day being given, to find his azimuth, and the latitude of the place.*

Let MP be the meridian of the place, Z the zenith, P the pole, S the ſun, ZS his vertical circle, PS his hour circle.

Then in the oblique ſpherical triangle SPZ, there is given the ſides SP, SZ, and angle SPZ; to find the angle PZS and ſide ZP.

1. *For the azimuth.*

By Caſe 1ſt, of oblique ſpherical triangles,
S. complem. altitude ZS :
S. hour angle ZPS : :
S. comp. declination SP :
S. azimuth PZS, which may be greater than a right angle.

2. *For the latitude.*

Having found the angle PZS; then by Caſe 3d,
S. $\frac{1}{2}$ diff. angles SPZ and SZP :
S. $\frac{1}{2}$ ſum, angles SPZ and SZP : :
Tan. $\frac{1}{2}$ diff. ſides SP and SZ :
Tan. $\frac{1}{2}$ ZP, which doubled gives the complement of the latitude ZP.

PROB.

PROB. XXIX.

The ſun's declination, his azimuth, and hour of the day being given; to find his altitude, and the latitude of the place. 20.

Let MP be the meridian, Z the zenith, P the pole, S the ſun, ZS his vertical, PS his meridian. Then in the oblique ſpherical triangle ZPS, there is given SP (the co-declination), SZP (the azimuth), and ZPS (the hour angle); to find ZS, and ZP.

1. *For the altitude.*

By Caſe 4th, of oblique ſpherical triangles,

S. azimuth SZP :
S. co-declination SP : :
S. hour angle ZPS :
S. co-altitude ZS.

2. *For the latitude.*

Having found the co-altitude ZS as before; by Caſe 5th,

S. ½ diff. angles PZS and ZPS :
S. ½ ſum, angles PZS and ZPS : :
Tan. ½ diff. ſides PS and ZS :
Tan. ½ PZ, which doubled gives PZ the complement of the latitude.

PROB. XXX.

Having given the ſun's altitude, and his azimuth, and the hour of the day; to find his declination, and the latitude of the place. 20.

Let MP be the meridian, P the pole, Z the zenith, S the ſun's place, PS his hour circle, ZS his vertical circle. Then SZ is his comp. of altitude, SZP his azimuth, ZPS his hour angle, PS the comp. of declination, ZP the comp. latitude.

Then

Fig. 20. Then in the ſpherical triangle ZPS, there is given the ſide ZS, and the angles PZS and ZPS, by reducing the time to degrees; to find the ſides SP, and ZP.

1. *For the declination.*

We ſhall have, by Caſe 4th, of oblique ſpherical triangles,

S. hour angle ZPS :
S. co-altitude ZS : :
S. azimuth PZS :
S. co-declination SP.

2. *For the latitude.*

Having found PS as before, then by Caſe 5th,
S. ½ diff. angles PZS and ZPS :
S. ½ ſum, angles PZS and ZPS : :
Tan. ½ diff. ſides PS and ZS :
Tan. ½ ZP, and doubled gives ZP the complement of the latitude.

SCHOLIUM.

What has been laid down in the foregoing propoſitions, in regard to the ſun, holds equally true of a ſtar, or any other object that appears in the heavens.

PROB. XXXI.

21. *The latitude of the place, and declination of the ſun, being given; to find the time of day break.*

Let HZO be the meridian, HO the horizon, Z the zenith, P the pole, S the ſun 18 degrees below the horizon, PS his hour circle, and ZS his vertical circle. EQ the equinoctial, AB the ſun's parallel, DF the parallel of twilight below the horizon.

Then in the oblique ſpherical triangle ZPS, there is given ZP the complement of the latitude, PS the complement of the declination, and ZS the diſ-

tance of the ſun from the zenith 108 degrees. To Fig.
find the hour angle ZPS. Therefore all the ſides 21.
being given, it is ſolved by Caſe 11th, of oblique
ſpherical triangles, thus

S. ZP × S. SP :
Radius ſquare : :
$$S.\ \frac{ZS+ZP-SP}{2}\times S.\ \frac{ZS+SP-ZP}{2}:$$

Sine ſquare of $\frac{1}{2}$ ZPS; then $\frac{1}{2}$ ZPS being had, if it be doubled it gives the hour angle ZPS, which turned into time, allowing 15 degrees to an hour, gives the time from noon, that twilight begins, or ends.

PROB. XXXII.

The right aſcenſion and declination of a ſtar being 22.
given; to find its longitude and latitude.

Let LCQ be the ſolſtitial colure, EQ the equinoctial, P its pole, EC the ecliptic, L its pole, S a ſtar, PSA an hour circle or meridian, LSDB a circle of longitude. Then EA is the ſtar's right aſcenſion, and AQ its complement or the angle SPQ; and AS is its declination, and SP its complement. Alſo ED is the longitude of the ſtar, and DC or angle DLC the complement of it; and SD is its latitude, and SL the complement of it.

In the oblique ſpherical triangle LPS, we have given the ſide PS, and LP (23° 29′), and angle LPS; to find the angle PLS the comp. longitude, and the ſide LS the complement of the declination SD.

Let fall SO perp. to LP. Then

1. *To find the longitude.*

By Caſe 7th, of oblique ſpherical triangles,
Radius :
Coſ. SPO (comp. right aſcenſion) : :
Tan. PS (comp. declination) :
Tan. PO.

Then

Fig. Then LP + PO = LO, then
22. S. PO :
S. LO : :
Cotan. SPO (comp. right aſcen.) :
Cotan. SLP (the comp. longitude).

2. *For the latitude.*

Having found PO and LO, as before; then by Caſe 8th,

Coſ. PO :
Coſ. PS : :
Coſ. LO :
Coſ. LS, the complement of latitude.

PROB. XXXIII.

22. *The longitude and latitude of a ſtar being given; to find its right aſcenſion and declination.*

This is the reverſe of the laſt problem. Let S be the ſtar, LSD a circle of longitude, cutting the ecliptic in D; and PSA a meridian cutting the equator in A. Then in the oblique ſpherical triangle LPS, there is given the ſide LP, and the angle SLP the complement of the longitude from Aries, or the neareſt equinoctial point; and the ſide SL the complement of the latitude. To find the angle LPS or SPO, which determines the right aſcenſion, and PS the complement of the declination. Draw SO perpendicular to LP. Then

1. *For the right aſcenſion.*

By Caſe 7th, of right angled ſpherical triangles,
Radius :
Coſ. SLP (comp. longitude) : :
Tan. LS (comp. latitude) :
Tan. LO, Then LO — LP = PO.

Again, S. LO : Fig. 22.
S. PO : :
Cotan. SLP (comp. longitude) :
Cotan. SPO (comp. right aſcenſion).

2. *For the declination.*

Having found LO and PO as before; then by Caſe 8th,

Coſ. LO :
Coſ. LS (comp. latitude) : :
Coſ. PO :
Coſ. PS (comp. declination).

SCHOLIUM.

Hitherto we have given the actual ſolution of ſuch problems as have been handled relating to the ſun and ſtars. In what follows, it will be ſufficient to point out the method of ſolution, without troubling the reader with the particular proportions, they depend on.

PROB. XXXIV.

Having given the places of two Stars A *and* D, *and the diſtance of third ſtar* S *from both of them; to find the place of the Star* S. 23.

Suppoſe the places of the ſtars are referred to the ecliptic BQ, whoſe pole is L. Then in the triangle LAD, there is given the ſides LA, LD, the complements of the latitudes of the ſtars, and the included angle ALD, or BQ, the difference of longitude of the ſtars; from thence the ſide AD, and angle LAD will be found. Again in the triangle ASD, we have all the ſides, by which the angle SAD may be found. Then the angle LAS will be had, which is the difference of the angles LAD and SAD. Laſtly in the triangle LAS, there is

Fig. 23. is given the ſide LA, and the ſide LS, and the oppoſite angle LAS; whence the angle ALS will be found, the difference of longitude from A; and the ſide LS, the complement of latitude of the ſtar S.

But if BQ be the equinoctial, P the pole; and the places of the ſtars be referred to the equinoctial. Then the angle ALS will be the difference of right aſcenſion from A; and SL the complement of declination of the ſtar S.

PROB. XXXV.

23. *The right aſcenſions and declinations of two ſtars* A *and* D, *being given; to find their diſtance* AD.

Let BQ be the equator, L the pole, AB, DQ, the declinations of the ſtars; LA, LD their complements. Then ſince their right aſcenſions are given, their difference BQ is given; therefore in the oblique ſpherical triangle LAD, there is given the ſides LA, LD, and the included angle ALD, by which the diſtance AD will be found.

Cor. 1. *If the latitudes and longitudes of two ſtars be given; their diſtance will be found the ſame way.*

For then BQ is the ecliptic, ACD their difference of longitude; and AL, DL, the complements of their latitudes. And there are the ſame data as before.

Cor. 2. *If the right aſcenſion and declination of one ſtar, and declination of another be given, and its diſtance from the firſt; its right aſcenſion may be found.*

For then in the triangle ALD, the ſides AL, AD, and LD will be given; to find the angle ALD.

Cor. 3. *And the ſame thing holds good in reſpect of the longitude and latitude of the ſtars.* Fig.

PROB. XXXVI.

Having the latitude of the place, the altitude of a ſtar, whoſe right aſcenſion and declination is known, and the day given; to find the moment of time, and the azimuth. 20.

Let MZP be the meridian, S the place of the ſtar, ZS its vertical circle, PS its hour circle. Then in the triangle ZPS, there is given all the ſides, viz. PZ the complement of the latitude, ZS the complement of the altitude, and PS the complement of declination of the ſtar; from which the azimuth PZS is found, and the angle ZPS which anſwers to the right aſcenſion included between the ſtar S, and the meridian of the place MP. But ſince the day is given, the ſun's right aſcenſion is given, and therefore the difference between the ſun's, and the ſtar's right aſcenſions, is known; therefore to or from this difference add or ſubtract the right aſcenſion belonging to the angle ZPS, according as the ſun and ſtar are on the ſame or different ſides of the meridian PM; and you have the right aſcenſion contained between the meridian and the ſun; which converted into hours and minutes, the time becomes known.

Cor. *By a like proceſs, if the azimuth was given, the altitude and time may be found.*

For in the triangle ZPS, there will be given ZP, SP and angle PZS; to find ZS and angle ZPS.

PROB. XXXVII.

Given the latitude, the ſun's place, and the hour of the day; to find the angle made by the horizon and e- 24.

Fig. 24. *cliptic, or the height of the nonageſimal degree; the point of the ecliptic aſcending; the point of the nonageſimal degree; and the azimuth of the aſcending point, the point culminating, and its altitude, &c.*

Let HZRC be the meridian, HR the horizon, Z the zenith; DQ the equinoctial, P the pole, CAM the ecliptic, E the firſt point of aries, S the ſun, PSB an hour circle, A the point of the ecliptic deſcending, AN 90 degrees, and N the higheſt point of the ecliptic, ZNG a vertical circle.

The place of the ſun at S being given his right aſcenſion EB is had by Prob. 1. And the hour being given; the diſtance of the meridians, or the arch of the equinoctial BQ is given; and conſequently the complement BO. Therefore the difference of EB and BO, that is, EO is known.

Therefore in the oblique ſpherical triangle AEO, there is given the angle AEO the obliquity of the ecliptic, and AOE the complement of the latitude, and ſide EO; whence all the reſt will be found. Therefore we ſhall have,

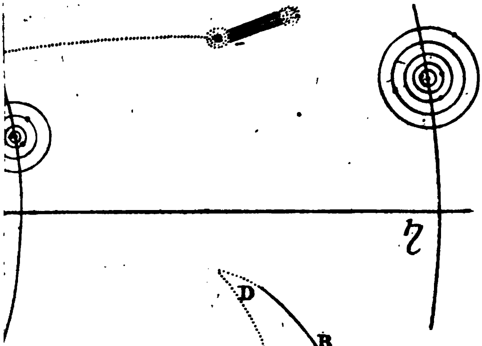
1. The angle EAO and its ſupplement SAO, which is the angle made by the horizon and ecliptic; or which is the ſame thing, the arch NG the height of the nonageſimal degree; this then is known.

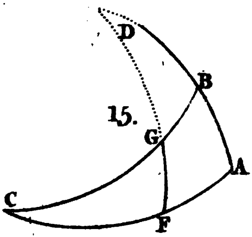
2. The ſide AE will be known, and therefore the point A of the ecliptic deſcending will be known; and conſequently the oppoſite point, or the aſcending point of the ecliptic is found.

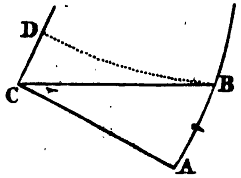
3. The ſide AO is had, which is the azimuth of the deſcending point in the weſt, or of the aſcending point in the eaſt, therefore this becomes known.

4. By adding AN or 90 degrees to AE, the arch EN is had, which gives the higheſt point, or the point of the ecliptic, that is, in the nonageſimal degree.

5. Since




15.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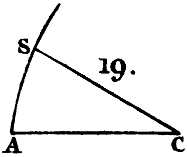
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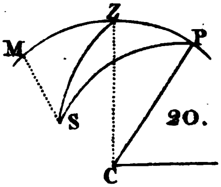
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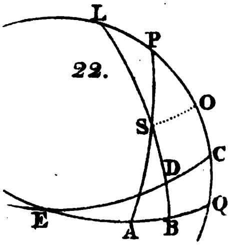

22.

Pl. IV. *pa.* 82.

UNIV

5. Since AG, OR are both quadrants, therefore GR = AO; whence GR, or the azimuth of the nonageſimal degree from the ſouth is known. Fig. 24.

6. To find the culminating point. If to AO the quadrant OR be added, we ſhall have AR. Therefore in the right angled ſpherical triangle ARM, we have the ſide AR, and the angle MAR found before; to find the hypothenuſe AM; to which add AE found before, gives EM; whence the culminating point M of the ecliptic is known.

Otherwiſe thus, in the right angled triangle EQM, EQ and <E are given to find EM, and angle M, and MQ. Whence MR, MZ are known. Or elſe,

7. In the ſame triangle, the ſide RM will be found, which is the altitude of the culminating point of the ecliptic.

8. In the ſame triangle AMR, the angle M will be found, which is the interſection of the meridian with the ecliptic, or the meridian angle.

9. To find the parallactic angle, or the angle which the vertical circle makes with the ecliptic, as ZSM. In the right angled triangle ZSN, the ſides ZN, NS are given; for ZN is the complement of NG; and SN is the complement of AS, or of $\overline{ES - EA}$; conſequently the angle ZSN or parallactic angle will be known. The ſame may be found in the triangle ZMS; where ZM, and angle ZMS are given, and MS had by the 6th article; to find ZSM.

10. In the ſame triangle the ſide ZS will be found, which is the ſun's diſtance from the vertex, or the zenith diſtance.

Cor. 1. *And to find the parallactic angle* ZIF, *or the angle which the verticle circle* ZS *makes with a leſſer circle* FIK, *at a given point* I.

Fig. 24. Let the vertical circle ZIS cut the horizon in T, and let MSA be the ecliptic, or any great circle parallel to FIK. Thro' the given point I, draw IL perp. to MSA; then the parallactic angle ZSN is known by Art. 9. Therefore in the right angled ſpherical triangle ISL, there is given the angle ISL, and the ſide IL, the diſtance of the parallel, from its great circle; whence there will be found the angle SIL, the complement of the parallactic angle KIS or ZIF.

Or thus,

If the angle SAT, and ſide AT, of the right angled ſpherical triangle AST, be known; then it will be,

As Coſ. IL : Coſ. AT : : S.SAT : Coſ. KIS the parallactic angle. For in the triangle AST (by Caſe 11th); Coſ. AT : Coſ. S : : Rad : S. A = $\frac{\text{Coſ. S}}{\text{Coſ. AT}} \times$ Rad; and Coſ. IL : Coſ. S : : Rad : S.I = $\frac{\text{Coſ. S}}{\text{Coſ. IL}} \times$ Rad. Therefore, S.A : S.I : : $\frac{1}{\text{Coſ. AT}} : \frac{1}{\text{Coſ. IL}}$: : Coſ. IL : Coſ. AT: for Coſ. S, and radius are common both.

PROB. XXXVIII.

63. *If a planet or comet* S, *be in a line with two known ſtars* A *and* B; *and in another line with two other known ſtars,* C *and* D. *To find its latitude and longitude.*

The longitude and latitude of all the ſtars A, B, C, D, are known. Therefore in the triangle APB, the ſides AP, PB, and angle P are given; to find the angle PAB. Draw AC; then in the triangle PAC, we have AP, PC, and angle P; Fig. to find AC, PAC, and PCA; then we have the 63. angle CAB. In the triangle CPD, we have CP, PD, and angle CPD; to find the angle PCD; then we have the angle ACD. In the triangle ACS, we have AC, ACS, CAS; to find AS. Laſtly, in the triangle APS, we have AP, AS; and angle PAS; to find APS the difference of longitude of A and S, and PS the complement of latitude of S.

If the interſection of the lines, or the place of the comet at S, be out of the figure; the calculation will be much the ſame.

PROB. XXXIX.

Having given the latitude of the place, the day 25. *and hour; and the latitude and longitude of a ſtar; to find its altitude and azimuth, and the point where the vertical circle cuts the ecliptic, and the angle they make.*

Let HMPR be the meridian, HR the horizon, Z the zenith, DEQ the equinoctial, P the pole, MAC the ecliptic, ZFS a vertical circle, PSB an hour circle.

From the latitude and longitude of the ſtar, the right aſcenſion and declination will be found, by Prob. XXXIII. and the day being given, the ſun's right aſcenſion is given. Whence there will be had the difference of right aſcenſions of the ſun and ſtar, or the arch of the equinoctial contained between their meridians. And the hour being given, there will be given the arch of the equinoctial contained between the meridian of the ſun, and the meridian of the place PZD. Hence by addition or ſubtraction, the arch of the equinoctial between the meridian of the ſtar, and that of the place is had which is DB, or angle DPB.

There-

Fig. 25. Therefore in the triangle PZS, there are given the ſides PZ, PS, and angle ZPS: PS being the complement of the declination of the ſtar. Whence will be found ZS the complement of the altitude, and the angle PZS or the azimuth, equal to the arch FR; then ſince OR is a quadrant, FO will be known, or the azimuth from the eaſt.

Again, FO is known, and AO will be found by the laſt Prob. therefore AF is known; alſo the angle OAG may be found by the laſt Prob. Therefore in the right angled triangle AFG, the ſide AF and angle FAG are known; to find the angle FGA, the interſection of the ecliptic and vertical circle; and AG the diſtance of the interſection from A.

Or elſe find MH and angle AMH by the laſt Prob. Then in the triangle ZGM, there are given the angle MZG the ſupplement of FR, the angle ZMG the ſupplement of AMH, and the ſide MZ the complement of HM; to find the angle MGZ made by the ecliptic and vertical; and MG the diſtance of the point of interſection G from M; and laſtly ZG, the diſtance of G from the vertex or zenith Z.

PROB. LX.

To erect a ſcheme or figure of the heavens for any time and place.

26. Let HMRC be the meridian, HR the horizon, DQ the equator, MAC the ecliptic. Having found the point of the ecliptic M that culminates at the time given, by Prob. 37, and D its right aſcenſion. To the right aſcenſion of D add 30 degrees continually, or in the equinoctial DQ, make DB, BI, IO, &c. each equal to 30 degrees, quite round; to find the points B, I, O, T, V, &c. Thro' theſe points draw the circles of poſition HBR, HIR, HTR, HVR, &c; interſecting the

ecliptic in N, K, A, L, S. Then theſe points are Fig. called *cuſps*; A is the cuſp of the firſt houſe, L 26. that of the ſecond, S of the third, C of the fourth, &c. round the globe, M of the tenth, N of the eleventh, K of the twelfth houſe. Then to find theſe points of the ecliptic, or the cuſps of the ſeveral houſes; we have the ſeveral right angled triangles ADB, ADI, ADO, &c. and the oblique triangles EBN, EIK, EOA, ETL, &c. to work by.

In the right angled ſpherical triangle HDB, there is given DH the complement of latitude, DB an arch of 30 degrees; to find the angle DBH, or angle of poſition with the equator. Likewiſe in the ſeveral right angled triangles HDI, HDO, HDT, &c. there is given in each of them the ſide HD, and in the reſpective triangles, the ſides DI, DO, DT, &c. to find the angles of poſition at I, O, T, &c.

Again in the oblique triangle EBN, there is given the ſide EB, and the angle of poſition EBN, and the angle BEN, the obliquity of the ecliptic; to find EN, the diſtance from the equinoctial point E. In like manner in the ſeveral oblique triangles EIK, EOA, ETL, EVS, &c. there is given reſpectively, the ſides EI, EO, ET, EV, &c. and the angles at I, O, T, V, &c. and in all of them the angle E. Therefore the ſides EK, EA, EL, ES, &c. will be found. And from hence are determined the points of the ecliptic, in which the cuſps of the ſeveral houſes are placed. And the cuſps on the oppoſite ſide of the globe, will be diametrically oppoſite to theſe. Thus the 7th and 1ſt, the 8th and 2d, the 9th and 3d, the 10th and 4th, the 11th and 5th, and the 6th and 12th are oppoſites. The firſt is in the eaſt part of the horizon, and is called the *aſcendant* or *horoſcope*, or the *orient*; the fourth is in the meridian below, the ſe-

Fig. 26. venth is in the weſt part of the horizon, and the tenth in the meridian above. And theſe are the poſitions of the twelve houſes at that given moment of time. But they continually change their places in the ecliptic; for different parts of the ecliptic are always coming to the horizon, and to the meridian; ſince in 24 hours it makes a whole revolution.

PROB. XLI.

27. *Having the latitude of the place, and the right aſcenſion and declination of a ſtar; to find when it riſes coſmically or with the ſun; and likewiſe when it ſets coſmically, that is, when the ſun riſes.*

Find the aſcenſional difference as in Prob. III. which add to or ſubtract from the right aſcenſion, according as it has ſouth or north declination, and you have the oblique aſcenſion.

Let HDR be the meridian, HR the horizon, DQ the equinoctial, MAC the ecliptic, S the ſtar. Then in the oblique triangle EOA, there is given the oblique aſcenſion EO, the angle EOA the ſupplement of DOH, the height of the equinoctial, or of the complement of the latitude, and the angle AEO the obliquity of the ecliptic; to find the ſide EA, and conſequently the point A of the ecliptic, that riſes with the ſtar. Therefore when the ſun is in the point A it riſes with the ſtar. Therefore find on what day the ſun is in that point of the ecliptic, and that is the day on which the ſtar riſes coſmically.

Again, add or ſubtract the aſcenſional difference to or from the right aſcenſion, according as the ſtar has north or ſouth declination, for the oblique deſcenſion EO. Then in the oblique triangle EOA, there is given EO, and angles EOA, OEA, as before, to find EA; and A is the point ſetting with the ſtar. And therefore the point oppoſite to Fig. A in the ecliptic is the place of the ſun when he 28. riſes as the ſtar ſets. And by finding the ſun's place at that time, it is known when the ſtar ſets coſmically.

PROB. XLII.

Having the latitude of the place, and the right aſcenſion and declination of a ſtar; to find when it riſes or ſets acronically, that is, at ſun ſet.

Find the point of the ecliptic A, wherein he is, 27. when the ſtar riſes coſmically, by the laſt Prob. Then it it is plain when he is in the oppoſite point of the ecliptic, he ſets when the ſtar riſes; therefore find what day the ſun is in that point of the ecliptic, and that is the day, whereon the ſtar riſes acronically.

Again, find by the laſt Prob. the place the ſun 28. is in when he ſets with the ſtar, and that is his place when the ſtar ſets acronically. Therefore find when he is in that point, and that is the time when the ſtar ſets acronically.

PROB. XLIII.

Given the latitude of a place, and the right aſcenſion and declination of a ſtar; to find the heliacal riſing and ſetting thereof.

Let G be the place of the ſun when the ſtar at 27. S riſes heliacally; that is, when it can firſt be ſeen to riſe, the ſun being ſo far below the horizon, as not to make it inviſible by its light. Thro' the zenith Z, draw the vertical circle ZIG, then in a ſtar of the firſt magnitude, IG muſt be 12 degrees, and in one of the ſecond magnitude, 13 degrees, and the moon 5 degrees. Then in the right angled triangle AIG, there is given IG, and

Fig. and the angle IAG may be found by Prob.
27. XXXVII. therefore AG will be had. And this added to EA found by Prop. XL. gives EG the ſun's diſtance from the equinoctial point E; and the ſun's place being had, the time becomes known when he is in that point, and conſequently when the ſtar riſes heliacally.

28. In like manner, if G be the place of the ſun when the ſtar S ſets heliacally, or will not be affected by the ſun's light when ſetting. Then AG, found as before, ſubtracted from the place of the ſun, found by the laſt Prob. gives the ſun's place at G; and from thence is had the time when he is in G, which is the ſame as when the ſtar is at S or ſets heliacally.

Cor. *A ſtar becomes inviſible all the time from his heliacal ſetting, to his heliacal riſing.*

For all this while it is too near the ſun, and up in the day time.

PROB. XLIV.

86. *Given the latitude of the place, and the right aſ-*
87. *cenſion and declination of two ſtars A, B; to find the time when they will both be upon one azimuth; and alſo the azimuth.*

Let Z be the zenith, P the pole, ZBA their common azimuth circle; PB, PA two meridians.

In the ſpherical triangle APB, there is given the two ſides AP, BP, and the included angle APB; to find the angle A or B. Then in the triangle ZPA, there are given the ſides ZP, AP, and angle A; to find the angle ZPA, or the time of A from noon; and the angle PZA, or the azimuth from the north. Or elſe in the triangle ZPB, there is given the ſides ZP, BP, and angle B; to

find the angle ZPB, the time of B; and PZB, the Fig.
azimuth. 86.

Cor. 1. *In the ſame triangle* ZAP, *or* ZPB, *the altitude* ZA *or* ZB *may be found, at that time.* 87.

Cor. 2. *If inſtead of the latitude, the altitude* ZA, *or* ZB *be given; the latitude* ZP *may be found, in the ſame triangle* ZPA, *or* ZPB.

PROB. XLV.

To find the annual variation of the declination and right aſcenſion of a ſtar. 84. 85.

Let KL be the equator, B its pole, GH the ecliptic, A its pole, F a ſtar, which in a year's time comes to *r*. Then ſince the latitude never changes, A*r* will be equal to AF. Thro' F and *r*, draw the circles of longitude AF*n*, A*rd*; and the meridians BF*p*, B*rq*, and draw F*r*, F*s* perp. to AF, BF; whence the $\angle rFs = \angle AFB$. Then will *nd* be the variation of the angle A, or of the longitude, which is conſtantly the ſame $= 50''.35$; and *pq* is the variation of B, or of the right aſcenſion; and *rs* is the variation of declination.

By ſimilar ſectors *nd* : F*r* : : rad : S.AF; and F*r* : *rs* : : rad : S.*r*F*s* or S.BFA. Therefore, *nd* (var. A) : *rs* (var. BF) : : rad² : S.AF × S.BFA. Whence *rs* or variation of BF $= \frac{\text{S.AF} \times \text{S.BFA}}{\text{rad}^2} \times nd = \frac{\text{S.AB} \times \text{S.ABF}}{\text{rad}^2} \times nd = \frac{\text{S.AB} \times \text{Coſ. CBF}}{\text{rad}^2} \times nd$, where AB and *nd* are given.

Again,

It will be *nd* : F*r* : : rad : S.AF,
and F*r* : F*s* : : rad : S.F*rs* or Coſ. AFB,
and F*s* : *pq* : : S.BF : rad.

there-

Fig. 84. 85. therefore $nd : pq :: \text{rad} \times \text{S.BF} : \text{S.AF} \times \text{Coſ. AFB}$, and pq or var. B $= \frac{\text{S.AF} \times \text{Coſ. AFB}}{\text{rad} \times \text{S.BF}} \times nd =$

$$\frac{\text{rad} \times \text{Coſ. AFB}}{\text{S.BF} \times \text{S.AFB}} \times rs = \frac{\text{rad}^2}{\text{S.BF} \times \text{tan. AFB}} \times rs = \frac{\text{Cotan. AFB}}{\text{S.BF}} \times rs.$$

1. *For the var. declination.*

To the conſtant log. 1.302408, add the log. coſine of the ſtar's right aſcenſion from the neareſt equinoctial point, the ſum (rejecting 10 for radius) is the log. of the variation in declination.

In the 1ſt and 4th quadrant of the ecliptic, the ſtar approaches the north pole B; in the 2d and 3d quadrants, it recedes from it.

2. *For the var. of right aſcenſion.*

Firſt, to find the angle F, ſay as Coſ. lat. of the ſtar (AF) : Coſ. its right aſcenſion (CBF) : : S.AB (23° 29′) : S.F. Then,

To the log. cotangent of F, add the log. of the variation of declination (rs); and from the ſum, ſubtract the log. coſine of the declination of the ſtar, the remainder is the log. of the variation of right aſcenſion, for a year.

The right aſcenſion always increaſes, except when the angle F is obtuſe, which happens when the ſtar is between the poles of the equator and ecliptic; and then it decreaſes.

Examp.

For the ſtar allioth, whoſe right aſcenſion is [illegible]0° 44′, and lat. 54° 20′, and declination 57° 19′.

Fig. 84. 85.

1. Var. declination.

Conſtant log. — —	1.302408
Coſ. 10° 44′ — —	9.992334
— 19″ 76 = var. decl. —	1.294742

2. For right aſcenſion.

Coſ. 54° 20′ — —	9.765719
Coſ. 10 44 — —	9.992334
S. 23 29 — —	9.600409
	19.592743
S.F, 42° 11′ — —	9.827024
Co[illegible]F. 42 11 — —	10.042769
19″ 76 — — —	1.294742
	11.337511
Coſ. 57 19 — —	9.732390
+ 40″ 38 — — —	1.605121

the variation of right aſcenſion.

PROB. XLVI.

To find the hour of the night, by the circumpolar ſtars.

Find the ſun's right aſcenſion, and ſubtract it from the right aſcenſion of the ſtar given, taken from the following table (Col. 2.); the remainder is the time of night, when that ſtar is directly under the pole ſtar; which is known by hanging up a line and plummet.

Note, The edge of a window or door ſtead, facing the north, may ſerve inſtead of a line and plummet.

If the ſtar you obſerve happens to be above the pole, add 12 hours to the time.

If you have not a table of the ſun's right aſcenſion at hand; you may make a ſhift with the following ſhort table.

A TABLE

Fig.

A TABLE *of circumpolar Stars.*

Stars	R.Aſcenſ.			var.	dif.R.A		Azimuth			mag.
	h	m	s	s	m	s	°	′	″	
Allioth	0	43	53	27	0	8	0	1	14E	2
G. Bear's tail and Benetnaſch	1	27	05	24	11	18	1	50	26W	2
L. Bear Kochab	2	32	31	0	19	6	1	32	28W	2
Dragon's back	4	38	26	20	25	30	3	34	20W	3
Lyra	6	11	57	20	17	8	3	21	14W	1
Swan's tail Deneb	8	16	40	20	16	51	3	2	43W	2
Cepheus's knee	11	11	32	23	17	40	1	25	40W	3
Caſſiopeia's hyp	12	42	37	35	0	15	0	2	10W	2
Perſeus's ſide Algenib	15	19	30	42	11	38	1	57	48E	2
Capella	17	16	50	44	17	16	3	3	47E	1
G. Bear's foot	20	53	45	43	20	50	3	28	10E	3
Lower pointer	22	58	37	37	10	56	1	34	36E	2

The firſt column is the names of the ſtars; the 2d is the ſtars right aſcenſion when under the pole ſtar; the 3d is the increaſe of right aſcenſion every 10 years; the 4th is the difference of time between the ſtars being under the pole, and under the pole ſtar; the 5th is the azimuth at that time; and the laſt is the magnitude. This Table is made for the middle of England.

The following Table is a Table of the ſun's right aſcenſion, the 10th day of every month, at the following midnight, being adapted to the mean time of taking obſervation of the ſtars.

A TABLE *of the ſun's right aſcenſion.*

Months	H.	M.
Jan. 10	19	28
Feb. 10	21	37
Mar. 10	23	27
Apr. 10	1	20
May 10	3	13
June 10	5	19
July 10	7	23
Aug. 10	9	24
Sept. 10	11	19
Oct. 10	13	7
Nov. 10	15	7
Dec. 10	17	14

To find the ſun's right aſcenſion for any day, find it for the 10th day of the month propoſed, and add or ſubtract ſo many times 4 minutes, as the day propoſed is after or before the 10th day. Or more nearly, $\overline{4-\frac{1}{18}}$ minutes, for every day.

SCHOLIUM.

That the reader may have an idea of the ſtars 88.
to be obſerved, as in the foregoing table, I have here given a map of the principal ſtars about the north pole, as follows.

P the north pole
E the pole of the ecliptic
1 Caſſiopeia's Chair
2 Caſſiopeia's hip
3 Caſſiopeia's foot
4 Perſeus's ſide
5 Capella
6 Great bear's lip
7 Great bear's foot
8 Lower pointer
9 Allioth
10 Great bear's tail
11 Little bear
12 Laſt joint-dragon
13 Dragon's back
14 Dragon's head
15 Lyra
16 Dragon's belly
17 Swan's tail
18 Cepheus's left ſhoulder
19 Cepheus's knee
20 Pole Star.

Fig.

PROB. XLVII.

89. *To find the aberration of the fixed ſtars; cauſed by the compound motion of the rays of light, and of the earth in its orbit.*

It is found by frequent obſervations of the eclipſes of Jupiter's Satellites, that light is about 8 minutes in moving from the ſun to the earth. And ſince the earth deſcribes about 1 degree (or 3600″) in a day (or 1440^{m}), therefore in 8 minutes, it will deſcribe 20″, in its orbit; and therefore the velocity of light, is to the velocity of the earth in its orbit; as radius to an arch of 20 ſeconds, or the third part of a minute, that is, as 1 to $\frac{.0002909}{3}$ or .00009697, or as 10300 to 1. That is, the velocity of light is 10300 times greater than the velocity of the earth in its orbit.

Now if AN be the way or path of a body in free ſpace, as of a ray of light; its apparent way on a moveable plane will be different. For it will be that which is made by the reſolution of the two motions of the body and plane. Thus if AN be deſcribed in any time by the body, and NF be deſcribed by (a point in) the plane, in the ſame time. As the plane moves forward in direction NF or AB, it leaves all the points of the fixed line AN behind it, all which will therefore ſeem to move backwards in the plane. Therefore make ND = NF, being taken backwards or contrary to the motion of the plane; and the body, inſtead of going to N in free ſpace, will ſeem to go to D, in the ſame time, upon the moveable plane; and therefore AD will be the apparent path of the body in that plane.

It will be the very ſame thing, if you ſuppoſe the plane fixed, and the body to have the plane's

mo-

motion communicated to it, in a contrary direction, ſo as the relative motion be the ſame as before. Thus if the body moves from B to A, in the ſame time that it would alſo move from A to N, then by that compound motion it would move along the diagonal BN of the parallelogram whoſe ſides are BA, AN, and in the ſame time. Fig. 89.

Therefore rays of light emitted from the ſtar in direction AN, will fall upon the point D of the moving plane; and therefore will fall upon the eye of the obſerver, in the direction AD. And therefore an obſerver at D will ſuppoſe the ſtar ſituated in the line DA. And conſequently, if BN be parallel to AD, and the point D tranſlated to N in the ſaid time; an obſerver at N will ſuppoſe the ſtar ſituated in the line NB; making the angle BNF or ADF leſs than ANF, the angle it would appear under, if the plane was at reſt. So that the angle of elevation BNF, above the line of direction NF, of the obſerver, is leſs than before, being taken on the ſide F, towards which the obſerver moves. And the obſerver, inſtead of ſeeing the ſtar at A its real place, will ſee it at B its apparent place. But on the contrary, if the obſerver moves from F to N and D, and B be the real place of the ſtar, its apparent place would be at A, to an obſerver at N.

The apparent place B is always in the plane of aberration, drawn thro' the way of the obſerver NF, and the line NA drawn from the obſerver to the real place of the ſtar; for AB being parallel to NF, is in the plane ADNF. And the angle BNA or NAD is the angle of aberration; by the quantity of this angle the ſtar is depreſſed, in going towards it; or raiſed in going from it.

Further in the triangle AND; it is, AN : ND :: S.ADN : S.NAD; but AN and ND being given; the S.NAD the aberration will be as the S.ADN.

Fig. 89. But becauſe AN is 10300 times greater than ND; therefore the S.NAD does not differ from its arch or angle; whence, the angle of aberration NAD or ANB is always as the ſine of the angle of the earth's way, ADN or ANF. And hence the angle of aberration ANB is greateſt, when AN is perp. to ND; and becomes nothing, when ANF is nothing.

Since AN is to ND, as radius to 20″; therefore when AN is perp. to ND, the angle NAD or ANB will be 20″, which is the greateſt it can be. In other caſes it will be, as radius to S. angle of the earth's way ANF : : ſo 20″, to the aberration, anſwering to that angle; which angle is always taken in the plane of aberration ADNF.

90. Theſe things premiſed, let BCDE be the earth's orbit, S the ſun, A or Q a ſtar at an almoſt infinite diſtance, N any place of the earth in its orbit. Thro' the ſtar A draw the circle of longitude AH perp. to the plane of the ecliptic, and draw KSBH, and ESC perp. to it, or parallel to the tangent at B. Draw the tangent N*d*, and draw NI towards the ſtar, and make NI to N*d* as the velocity of light, to that of the earth, or as 10300 to I, and draw *d*I which leads to the apparent place of the ſtar; and ſuppoſe DA, SA parallel to *d*I, NI; then DA will alſo lead to the apparent place of the ſtar. Draw SFG perp. to SN, or parallel to N*d*. Then will IN*d* be the plane of aberration. This plane continually changes its ſituation, revolving round the ſun in a year along with the tangent N*d*. And ſince AS, SF are parallel to IN, N*f*, therefore angle ASF is equal to IN*f*, and therefore ASF is equal to the angle of the earth's way. Hence the plane ASF may be taken for the plane of aberration, which continually turns round the line AS, as the earth revolves about the ſun; the line SF being always in quadrature with the earth at N.

Let the earth be at E, then the plane of aberration ASB will be perpendicular to the ecliptic; and the angle of the earth's way ASB is the leaſt that it can be, and therefore the angle of aberration the leaſt. Whilſt the earth moves to B, the angle of the earth's way, and of aberration increaſe; and at B the plane of aberration is ACS, and the angle of the earth's way ASC is a right angle, which is the greateſt it can be; therefore the angle of aberration is the greateſt poſſible. Whilſt the earth moves to C, the angles of the earth's way and aberration decreaſe again, and at C are the leaſt. And in moving to K they increaſe again to K, where they are greateſt. And from K to E they diminiſh again, where they are leaſt. Fig. 90.

It is plain then, whilſt the earth is at E moving towards N, the ſtar's apparent place is at *e* lower than A; at B moving towards F, it appears at *b* forward. When the earth is at C, the ſtar appears at *c* above A. And when the earth is at K, the ſtar is ſeen at *k*, having gone backward.

Hence it appears, that the apparent place of a ſtar deſcribes a ſmall ellipſis in a year, about the true place of the ſtar in its center, whoſe tranſverſe axis is parallel to the ecliptic; and leſſer axis perpendicular to it. This ellipſis is *bcke* anſwering to the places of the earth at B, C, K, E. And the points *b*, *c*, *k*, *e*, anſwer reſpectively to the points C, K, E, B, where the plane of aberration cuts the ecliptic, being 90 degrees before the earth, or 90 degrees behind the ſun.

To find the angle of the earth's way ASF, when the earth is at any place N. Draw the arch AG; then in the ſpherical triangle AHG, right angled at H; we have the ſtar's latitude AH, and the ſide GH, the complement of the angle NSB (NSB being the difference of longitude of the earth and ſtar); whence by Caſe 14th, of right angled ſpherical

Fig. 90. rical triangles, the hypothenuſe AG will be had, which is the meaſure of the angle ASG or ASF.

And ſince the aberration is always in the plane of aberration; therefore when the earth is at N, the plane ASG is the plane of aberration, and the aberration lies in the arch AG, from A towards G; but the contrary way when the earth is in the point of the ecliptic oppoſite to N. And therefore in the ſame triangle AHG, having AH, HG by Caſe 13, the angle GAH is had, being the angle which the line or arch of aberration makes with the circle of longitude AH. Hence are deduced the following articles.

Cor. 1. *The angle of aberration* (SAD), *to an obſerver at* N, *is always in the plane* ASF, *cutting the ecliptic at* F, 90 *degrees before the earth, or behind the ſun. And it always lies from* A *towards that way the earth is moving. And therefore a ſtar appears lower, when the earth moves towards it; and higher when it moves from it.*

For NF is a quadrant; and when the earth is moving in E, the aberration lies from A to *e*; when the earth is in B, it lies from A to *b*. When in C, it lies from A to *c*. And when in K, it lies from A to *k*.

Cor. 2. *The aberration in longitude is conſtantly* 20″; *and alſo* 20″ *in latitude, at the pole of the ecliptic. For other latitudes, it is as the ſine of the ſtar's latitude.*

Cor. 3. *If a ſtar was placed in the pole of the ecliptic, it would appear to deſcribe a ſmall circle round the pole, in a year, whoſe diameter is* 40″. *And any other ſtar would ſeem to deſcribe in a year, a ſmall ellipſis, whoſe greater axis is* 40″, *parallel to the ecliptic; and the leſſer axis perpendicular to it, and leſs*

leſs than the other in the ratio of radius to the ſine of the ſtar's latitude. Fig. 90.

Cor. 4. *When the ſtar is in the ſyziges, that is, in conjunction or oppoſition to the ſun; there is no aberration in latitude, but the greateſt aberration in longitude.*

For when the earth is at B and K, the ſtar appears at *b* and *k*.

Cor. 5. *When the ſtar is in the quadratures with the ſun; there is no aberration in longitude, but the greateſt aberration in latitude. In the mean places it partakes of both.*

For when the earth is at E and C, the ſtar appears at *e* and *c*.

Cor. 6. *Hence the aberration of a given ſtar, at a given time of the year, may be eaſily found.*

Find the difference of longitude between the ſtar and the earth (or ſun), at that time. Then ſay, as radius : ſine of that difference : : ſo the coſine of the ſtar's latitude : coſ. of the angle of the earth's way, which being found; ſay, as radius : S. earth's way : : ſo 20 ſeconds : to the aberration at that time.

And to find what angle it makes with the circle of longitude AH; ſay, as ſine of the latitude (AH) : radius : : tan. diff. longitude (GH) : tan. angle ſought (GAH).

Cor. 7. *And from hence the aberration in latitude and longitude may be found.*

For ſince the ſmall arch or line of aberration, ſuppoſe A*o* lies in the arch AG, and the angle HAG is known. It will be, as radius : to the whole aberration A*o* : : S. angle GAH : to *os* the aberration in the parallel of A : : and ſo coſ. GAH : to A*s* the aberration in latitude. And coſ. lat. AH :

Fig. 90. AH : radius : : ſo is *os* : to the aberration in longitude. Whence, coſ. AH : S.GAH : : A*o* : aberration in longitude.

Cor. 8. *Hence alſo the aberration in right aſcenſion and declination may be eaſily found.*

91. Let EQ be the equinoctial, P its pole, CL the ecliptic, K its pole, A a ſtar, KAH a circle of longitude, PAB a meridian. In the right angled ſpherical triangle CBD, it will be (by Caſe 8), rad : S.DCB : : coſ. CB : coſ. D $= \frac{\text{S.DCB} \times \text{coſ. CB}}{\text{rad}}$; and in the triangle AHD, rad : S.DAH : : coſ. AH : coſ. D $= \frac{\text{S.DAH} \times \text{Coſ. AH}}{\text{rad}}$ Therefore S.DCB × coſ. CB = S.DAH × coſ. AH. Therefore coſ. ſtar's latitude (AH) : coſ. of its right aſcenſion (CB) : : ſo ſine of the obliquity of the ecliptic (DCB) : S. angle DAH, which a circle of longitude makes with a meridian, or circle of right aſcenſion, at the ſtar. But the angle GAH is known by Cor. 6. therefore the angle DAG is known, being the ſum or difference of DAH and HAG. Let A*o* be the whole aberration; and draw *on* perp. to PB, or draw the parallel of declination *nor*. In the triangle A*no*, it will be, as rad : A*o* the whole aberration : : ſo the S.*n*A*o* : to *no* the aberration in the parallel *nr* : : and ſo coſ. *n*A*o* : to A*n* the aberration in declination. Laſtly, coſ. declination (S.PA) : radius : : ſo *no* : BF the aberration in right aſcenſion. Whence as coſ. declination (AB) : S.DAG : : A*o* : BF the aberration in longitude.

Cor. 9. *The apparent latitude of a ſtar will be leaſt when the ſun is 90° before the ſtar; and greateſt when the ſun is 90 degrees behind the ſtar. Alſo the* *apparent longitude will be leaſt, when the ſun and ſtar are in conjunction; and greateſt, when in oppoſition.* Fig. 91.

Cor. 10. *The velocity of light is 10300 times greater than the velocity of the earth in its orbit.*

Cor. 11. *This aberration by the motion of light, is quite different from the parallax of the annual orb. For by this latter, the parallax in longitude is greateſt in the quadratures, and nothing in the ſyziges. And the parallax in latitude is greateſt in the ſyziges, and none at all in the quadratures.*

SCHOLIUM.

This aberration by the motion of light will alſo affect the planets, but not in ſo regular a manner, by reaſon they are not far from us. For in this caſe we cannot make uſe of the plane ADS. But in regard to the plane IN*d*, and the places N, and I, of the earth and planet. The aberration NI*d* will be the ſame as in a fixt ſtar, in the ſame direction NI.

Dr. Bradley, who found out this aberration, found out another ſmall motion in the fixt ſtars. This motion is only apparent in the ſtars, but is really in the earth; and is cauſed by the moon acting upon the equatorial parts of the earth, by which means the pole of the equinoctial is continually changed, and cauſed to deſcribe a ſmall circle round the true pole, of 18 ſeconds diameter, in 19 years, or the time of revolution of the moon's nodes. By this means the places of all the ſtars ſeem to be changed ſo much in that time. And this motion is ſuch, that the moving pole always lies between the mean pole, and the 90 degrees of the moon's orbit on that ſide the equator, and is 9″ nearer to it than the mean pole. And the moving pole and this 90th degree, Fig. 91. or higheſt point of the orbit, always accompany one another in their revolution. I have calculated this motion in my book of Fluxions, and found the diameter of the little circle 21 ſeconds. They that want to ſee it, will find it in Cor. 3. Prob. XXVI. of that book.

If theſe two motions ariſing from the aberration of light, and the mutation of the earth's axis, ſhould conſpire together; they will produce an error of half a minute in the places of the ſtars. So that it is no wonder if the places of the ſtars, taken at different times, differ from one another, and from their mean places, which muſt be eſteemed their true places. But from what has been here laid down, theſe errors may at any time be corrected, and their true places found. I have dwelt the longer on this, becauſe it is abſolutely neceſſary to have correct places of the ſtars; for this is the very ground and baſis of all aſtronomy.

SECT. III.

The Uſe of the Globes in Aſtronomy; and the Solution of the principal Problems of the Sphere thereby.

THE ſolution of the problems of the ſphere, by trigonometry as delivered in the laſt ſection, being oftentimes intricate and tedious; and as there are not many people who will be at the pains to learn ſpherical trigonometry, and yet would be deſirous to know how to reſolve the common problems of the ſphere. Therefore mathematicians have contrived two globes, the *celeſtial*, and *terreſtrial globes*, by help of which, theſe problems may be reſolved mechanically, pretty near the truth, and with incredible eaſe and expedition. This makes the practice very delightful, and highly inſtructive, eſpecially for ſuch people as have not had time to be fully informed of the theory. And it likewiſe gives a better notion of the circles of the ſphere, and the uſes thereof in the practice of aſtronomy; when a perſon ſees how they are applied in the ſolution of problems. I ſhall therefore, for the ſake of the learner, give the ſolution of ſeveral problems, eſpecially aſtronomical ones, by help of the globes, after I have given a ſhort deſcription of them. Altho' a ſight of theſe globes will at once give a better idea thereof, than can be conveyed by a multitude of words.

A *Globe* is made perfectly round, of paper or ſome light matter. In two oppoſite points of it are fixed two pins or wires, upon which the globe turns round; theſe are called its *Poles*. And theſe two wires, produced thro' the center of the globe, repreſent its *axis*. Theſe pins or wires turn round in two ſmall holes made in a braſs circle, which ſurrounds the globe; this circle is called the *braſs Meridian*. Round the middle of the globe, equidiſtant from the poles, a circle is drawn, and divided into 360 degrees, this is called the *Equinoctial*. There is alſo another great circle drawn round the globe, and interſecting the equinoctial in an angle of $23\frac{1}{2}$ degrees, this is the *Ecliptic*. The brazen meridian is divided into degrees on the eaſt ſide, and one half is numbered both ways from the equinoctial to the two poles, and the other from the poles to the equinoctial. There is a flat wooden ring goes round the globe, called the *Horizon*, its upper plane lies even with the center of the globe; on this are ſeveral circles; the innermoſt of which is a circle of degrees. The next the ecliptic and its diviſions, with a circle of months and days; the laſt, the points of the compaſs. This horizon is ſupported by 4 feet; in it are 2 ſlits to receive the brazen meridian, which will turn about within them. An *hour circle* with 24 hours on it, is fixed to the braſs meridian at the north pole; the hours of 12 being on the upper and lower parts of the meridian; and an index goes upon the wire, which turns round as the globe turns, and ſhews the hours. There is a *quadrant of altitude*, this turns round at the zenith, fixed there by a nut and a ſcrew, where its center of motion is; it is divided into 90 degrees, from the horizon. There is a braſs ſemicircle called the *Semicircle of Poſition*, divided into 180 degrees, whoſe ends are fixed to the north and ſouth points, but may be taken off, and applied as there is occaſion. There is alſo a mariner's compaſs on a pedeſtal below the globe. The tropics and polar circles are drawn upon the globe. The equinoctial and ecliptic are divided into 360 degrees, beginning at the interſection; where the ecliptic running eaſtward, draws towards the north pole. The ecliptic is alſo divided into 12 ſigns, and each ſign into 30 degrees.

Upon the ſurface of the terreſtrial globe, are deſcribed all the countries and iſlands in the known world, and the ſeas adjoining; in their true proportions and ſituations as the places lie; every place having its proper longitude and latitude; to diſtinguiſh which, meridians are drawn thro' the poles at every 15 degrees diſtance; and parallels at every ten degrees diſtance from the equator. The firſt meridian uſed to be drawn thro' the iſle *Ferro*, or the weſtermoſt part of Africa.

Upon the ſurface of the celeſtial globe, are drawn the ecliptic and its parallels at 10 degrees diſtance; and circles of longitude thro' the poles of the ecliptic, at 10 or 15 degrees diſtance from one another. The ecliptic is divided into 12 ſigns, numbered eaſtward from the interſection of it with the equator. Upon the ſurface of this globe are drawn all the conſtellations in the heavens; with all the ſtars belonging to each, every one in its proper longitude, latitude, and magnitude, as they are in the heavens. All the celeſtial bodies being placed upon the common ſurface of the globe, will have the ſame appearance, as the ſurface of the ſtarry heaven, to an eye placed without it.

The motions peculiar to the globes are theſe. The pole may be elevated to any height, by turning the braſs meridian about, within the notches of the horizon, till the degrees required touch the horizontal plane. Likewiſe the globe may be turned round upon its axis, or upon the two poles; till it revolves thro' any number of degrees, counted upon the equinoctial, at the braſs meridian. And if the index of the dial-plate be ſet to 12, it will move along with the globe, and ſhew the hours correſponding to that motion. By help of the magnetic needle, the globe may be ſet north and ſouth, having proper regard to the variation. The quadrant of altitude ſet to the vertex, or any other point, is moveable round about, and ſo may be applied to any point of the globe. By theſe motions the following problems are ſolved.

PROB. I.

To rectify the globe, or ſet it in the ſame poſition as the earth itſelf is in.

Place the globe level upon a horizontal plane; and by the help of the box and compaſs ſet it north and ſouth.

In north latitude, raiſe the north pole above the horizon, equal to the latitude of the place; which is done by moving the braſs meridian along the ſlits in the horizon, till the upper edge cut the degrees of latitude upon the meridian; there let it reſt. If you are in ſouth latitude, you muſt raiſe the ſouth pole as much.

Reckon your latitude from the equinoctial towards the elevated pole, and that point is the zenith. There fix the quadrant of altitude with the ſcrew, that the graduated ſide may be cloſe to that point.

Turn the globe about its axis, till the place you live in comes under the braſs meridian. Then the globe is in the ſame poſition as the earth.

Bring the ſun's place in the ecliptic, to the braſs meridian, and ſet the hour index to 12 o'clock at noon. Then the globe is rectified to the place of the ſun; and is in the ſame poſition as the heavens, at 12 o'clock.

PROB. II.

To find the place, and the right aſcenſion and declination of the ſun, for any time given.

Look in the circle of months and days, for the time given; againſt it in the ecliptic circle, you will find the ſun's place. Seek his place in the ecliptic upon the globe, and bring it to the braſs meridian. Then count the degrees on the equator from the beginning of Aries to the meridian, and that is the ſun's right aſcenſion. Likewiſe reckon the degrees of the braſs meridian between the equinoctial and the ſun's place; and you have the ſun's declination; which will be north or ſouth, according as it is towards the north or ſouth pole.

Cor. *If the globe be rectified for the place, the ſun's meridian altitude may be found.*

For having the ſun's place, count the degrees from thence to the horizon, and that is his meridional height.

PROB. III.

On a given day, to find the time of ſun-riſe and ſun-ſet, the ſun's oblique aſcenſion and deſcenſion, length of day and night, and his amplitude.

Find the ſun's place upon the horizon for the given time, and ſeek it in the ecliptic on the globe. And the globe being rectified to the latitude, bring the ſun's place on the ecliptic to the braſs meridian. The globe reſting there, ſet the hour index to 12 o'clock. Then turn the globe about eaſtward till the ſun's place be in the eaſtern part of the horizon, and the index will point the hour of ſun-riſe. Alſo if the globe be turned the other way, till the ſun's place be in the weſtern horizon; the index will point to the hour of ſun-ſetting.

The length of the day is had by doubling the time of ſun-ſet, and the length of the night by doubling the time of ſun-riſe.

When the place of the ſun is brought to the eaſtern horizon, mark what degree of the equinoctial is alſo in the eaſtern horizon; and that is the ſun's oblique aſcenſion. Alſo the ſun's place being brought to the weſtern part of the horizon; the degrees of the equinoctial cut by the horizon, ſhews the oblique deſcenſion of the ſun.

Reckon the degrees between the eaſt or weſt point of the horizon, to the ſun's place, brought to the horizon; and that gives the eaſtern or weſtern amplitude; which will be north or ſouth, according as the declination is north or ſouth.

Cor. *Hence the time of ſun-riſing, and the length of the longeſt day may be found.*

For if the 1ſt point of Cancer be brought to the horizon, the hour index will point to the hour of ſun-riſing, from whence the length of day and night may be found.

PROB. IV.

Having given the hour of the day at one place; to find what o'clock it is, at any other given place.

Bring the place you are at to the meridian; ſet the index to the hour given. Then turn the globe about till the other place comes to the meridian; and the hour index will ſhew the hour, or what o'clock it is at the other place.

Cor. *The difference of the longitude of theſe two places may then be found.*

For if the difference of the times be converted ed into degrees, allowing 15 degrees for an hour; the reſult will be the difference of longitude.

PROB. V.

To find the place where the ſun is vertical, at any given time.

The time being given, the ſun's declination is had by Prob. II. bring the place where you live, to the meridian; ſet the index to the given hour of the day. Then turn the globe about till the index points to 12 o'clock at noon. Then ſeek the declination on the meridian, and the point under it, is the place where the ſun is in the zenith.

Cor. 1. *Hence all the places where the ſun is riſing or ſetting may be found.*

For if a great circle be ſuppoſed to be deſcribed at 90 degrees diſtance from the place found, it will be the circle terminating light and darkneſs upon the earth. And then the ſun is either riſing or ſetting in all the places in this circle.

Cor. 2. *And hence may be found all the places where the ſun is riſing, ſetting, culminating, &c.*

For if the globe be rectified for that laſt place, the ſun is then in the zenith; and therefore all the upper hemiſphere is enlightened, and all the lower hemiſphere is in the dark. And the horizon is the boundary of light and darkneſs, and all the places on the weſt ſide have the ſun-riſing; and on the eaſt ſide ſetting. To thoſe in the parallel of 18 degrees below the weſtern horizon, the day is breaking, and below the eaſtern, twilight is ending. To thoſe under the upper part of the meridian, it is 12 o'clock at noon. And to thoſe under the lower part of the meridian, it is 12 o'clock at night. The twilight is in all theſe places between the horizon, and the parallel of 18 degrees below it. And all the places below that, have dark night.

PROB. VI.

On a given day, to find the time of day break, or the end of the evening twilight, for a given place.

The time being given, the ſun's place is had by the circles on the horizon. Then rectify the globe for the given place, and bring the ſun's place to the meridian; and ſet the hour index to 12 o'clock. Then turn the globe about towards the eaſt, till the ſun's place be 18 degrees below the eaſtern horizon; or which is the ſame thing, till the point oppoſite to the ſun's place, be elevated 18 degrees above the weſtern horizon; which may be known by the quadrant of altitude fixt to the zenith. This done, the index will ſhew the hour of day break.

And the end of evening twilight is found by turning the globe weſtward till the ſun's place be 18 degrees below the weſtern horizon, or its oppoſite point ſo much above the eaſtern. Then the index ſhews the time when this happens.

Cor. 1. *Hence may be found the length or duration of twilight.*

For if the time of ſun riſe (found by Prob. III.) be ſubtracted from the time of day break, what remains is the length of twilight.

Cor. 2. *During that part of ſummer when the ſun's depreſſion is not ſo much as 18 degrees; twilight continues all night, or there is no dark night.*

PROB. VII.

The place of the ſun being given; to ſhew at once the length of the day, and nights in all places on the earth.

The place of the ſun being given, his declination is found, by the ſecond problem. Then if the globe be ſet ſo as that the north pole be elevated above the horizon, as much as is the declination. Then the ſun will be in the zenith; and the horizon will be the circle bounding light and darkneſs. Then it will be evident to view, that all the parallels of declination are cut by the horizon into unequal parts; whereof the part above the horizon is illuminated by the ſun; and the part under it is in darkneſs. But the upper parts of theſe parallels are the diurnal arches, and ſhew the length of the day; and the under parts are the nocturnal arches, and ſhew the length of the nights for that time of the year.

For as the earth turns round, a place ſituated in any one of theſe parallels, will be ſo long in the illuminated arch, as is the length of it; and ſo long in the dark part, as the length of the dark arch; the one is day time to that place; and the other is night. And all places in the weſtern part of the horizon, will have the ſun-riſing at the ſame time; and all places in the eaſtern part of the horizon, will have him ſetting. For becauſe the ſun ſtands ſtill, and the earth revolves towards the eaſt, the places in the weſtern horizon are advancing towards the ſun, and therefore it riſes to them. But thoſe in the eaſtern part are leaving the ſun, and therefore it ſets to them.

Take any particular place of the earth, bring it to the weſtern horizon, and ſet the hour index to 12; then turn the globe about eaſtward till the ſame place comes to the eaſtern horizon; and then the index will ſhew the length of the day.

Hence it will appear that all places, ſituated in different parallels of latitude, have their days of different lengths, being longer or ſhorter in proportion to the ſegment of each parallel, that is above the horizon. Only at the equinox they are always of the ſame length. All places in the ſame latitude, have their days of the ſame length; but they begin ſooner or later, as the places lie more eaſtward or weſtward.

Cor. 1. *Hence if the pole be continually elevated to greater altitudes, as the ſun's declination grows greater; the gradual increaſe and decreaſe of the days will appear, which are made in all places of the world; according as greater or leſſer ſegments of the parallels of latitude ſtand above the horizon.*

Cor. 2. *All places whoſe complement of latitude is leſs than the ſun's declination, have continual day; when the latitude and declination are both one way. But they have continual night, if the latitude and declination lie different ways.*

PROB. VIII.

Any particular place being given; to ſhew at once the length of the days and nights, at all times of the year.

Rectify the globe according to the latitude of the place given; then the parallels of declination between the two tropics, will be the ſeveral parallel circles which the ſun ſeems daily to deſcribe. Now it is plain, whilſt he moves ſlowly along the ecliptic; which cuts all theſe parallels; he advances gradually from one parallel to another, till he comes at laſt to the utmoſt which is the tropic. The upper parts of theſe parallels, or thoſe above the horizon, are his diurnal arches; and the parts beneath are the nocturnal arches. It is evident then, as the ſun advances towards the elevated pole, that the upper parts of theſe parallels grow greater, and conſequently that the days grow longer, till he comes to the tropic, where the days are longeſt. Then returning back, the upper ſegments continually decreaſe till he come to the equinoctial, which is biſected by the horizon, and then the days and nights are equal. Proceeding towards the other pole, which is below the horizon, the upper parts of the parallels grow leſs, and the under parts greater; and therefore the days continually ſhorten, and the nights grow longer, till the ſun comes at the other tropic, where the days are ſhorteſt, and the nights longeſt. And returning back again, the days again increaſe in the ſame manner as they decreaſed. And thus the ſeveral ſeaſons of the year are made evident to the ſight. How the length of the day is to be found, when the ſun is in any of theſe parallels, has been ſhewn before at Prob. III.

Cor. 1. *When the place is under the equinoctial, the days and nights are equal all the year round.*

For all the parallels of declination are biſected by the horizon.

Cor. 2. *In any latitude, the days in ſummer are equal to the nights in winter, when the ſun has the ſame declination. And the nights in ſummer, equal to the days in winter.*

Cor. 3. *If the place be under the polar circle; the longeſt day will be 24 hours long; and the longeſt night the ſame.*

For no part of the ſun's parallel will be then under the horizon.

Cor. 4. *If the place be ſituated within the polar circle, then in the height of ſummer the ſun never ſets for a conſiderable time; nor does he riſe for the ſame time, in the depth of winter. And this time is ſo much longer as the place is nearer the pole.*

Cor. 5. *If the place be under the pole; there will be but one day and night thro' the whole year.*

For here the horizon coincides with the equinoctial; and all the parallels, on the ſame ſide of the equinoctial, are above the horizon; and thoſe on the other ſide below it.

PROB. IX.

Given the latitude of the place, the ſun's place in the ecliptic, and his altitude; to find the hour of the day, and his azimuth.

Rectify the globe to the latitude, and zenith, and bring the ſun's place to the meridian; and ſet the hour index to 12 o'clock. Then turn about the globe and quadrant of altitude till the ſun's place in the ecliptic fall upon the given degree of altitude. Then the index will ſhew the hour. And where the quadrant of altitude cuts the horizon, is the azimuth.

PROB. X.

Given the latitude of the place, the hour of the day, and the ſun's place in the ecliptic; to find the ſun's altitude and azimuth.

Rectify the globe for the latitude, and zenith; and bring the ſun's place to the meridian, and ſet the hour index to 12. Then turn the globe about, till the index point to the given hour. There let the globe reſt; and turn the quadrant of altitude about till it falls on the ſun's place, and the point where it cuts it, ſhews the altitude. And where the quadrant cuts the horizon, you have his azimuth at that time, reckoned from the north.

If the ſun is below the horizon; take the point of the ecliptic directly oppoſite to the ſun's place; and find his altitude and azimuth, as if the ſun was there. And that altitude will be his depreſſion below the horizon; and the azimuth the ſame from the oppoſite point of the horizon.

And if the azimuth or altitude be given, the hour may be found by working backwards.

PROB. XI.

Having given the latitude of the place, the ſun's place in the ecliptic, and his azimuth. To find his altitude, and the hour of the day.

Rectify the globe to the latitude, and zenith; and ſet the index to 12 o'clock. Then turn the quadrant of altitude till it cut the horizon in the point where the azimuth is. Then turn the globe till the ſun's place fall upon the quadrant; then the graduated edge of the quadrant will ſhew the altitude, and the index will point to the hour.

Likewiſe if the latitude, the ſun's altitude and azimuth be given; his place in the ecliptic may be found, and the hour, by working backwards.

PROB. XII.

To find the latitude and longitude of a given ſtar.

Bring the ſolſtitial colure to the meridian, and fix the quadrant of altitude over the pole of the ecliptic. Then turn about the quadrant till it fall upon the given ſtar; and the arch of the quadrant comprehended between the ſtar and the ecliptic, is the latitude of the ſtar; and the degree cut on the ecliptic by the quadrant, will be the ſtar's longitude.

If the latitude and longitude of a ſtar be given; the place of the ſtar upon the globe may be found, by working backwards.

The diſtance of two ſtars may be found on the globe, by applying the quadrant of altitude to both the ſtars; and reckoning the number of degrees contained between them.

PROB. XIII.

To find the right aſcenſion and declination of a given ſtar.

Bring the ſtar to the meridian; and the degrees of the equinoctial reckoned from the firſt point of Aries to the meridian, is the ſtar's right aſcenſion. And the degrees of the meridian contained between the ſtar and the equinoctial, is its declination.

And if the right aſcenſion and declination of a ſtar be given; its place on the globe may be found, by working backwards.

PROB. XIV.

Having the latitude of the place, and the day of the month; to find the time of the riſing, ſouthing, and ſetting of a given ſtar, and the time it is above the horizon.

Rectify the globe for the latitude, bring the ſun's place for that day to the meridian; and ſet the index to 12 o'clock. Turn the globe round, till the ſtar comes to the eaſtern horizon, then the index will ſhew the time of riſing. Turn the globe till the ſtar comes to the meridian, and the index will ſhew the time of its ſouthing, or when it culminates. Turn the globe ſtill further, till the ſtar comes to the weſtern horizon, and the index will ſhew the time of the ſtar's ſetting. And the diſtance of time between the riſing and ſetting, is the time of continuance above the horizon.

PROB. XV.

Having the latitude, the day and hour, given; to find what ſtar is upon the meridian at that time. And the contrary.

Rectify the globe to the latitude; find the ſun's place for the day, and bring it to the braſs meridian, and ſet the hour index to 12 o'clock. Then turn the globe about, till the index points to the given hour; and you may ſee what ſtars are in or near the meridian at that time.

And to find the day of the year, any ſtar will be upon the meridian, at a given hour. Bring the ſtar to the meridian, and ſet the index to the hour; and turn the globe till the index points to 12 at noon; and the meridian will cut the ecliptic in the ſun's place, whence the day will be found.

PROB. XVI.

Having the latitude of the place; to find the oblique aſcenſion and deſcenſion, the aſcenſional difference, and the amplitude of a given ſtar.

Rectify the globe to the latitude, bring the ſtar to the meridian, and ſet the index to 12 o'clock. Then turn the globe, till the ſtar comes to the eaſtern horizon; and the degree of the equinoctial, which is upon the horizon at that time, is the oblique aſcenſion, and the diſtance of that point of the equinoctial from the point of right aſcenſion, is the aſcenſional difference; and the ſtar being in the horizon, its diſtance then from the eaſt point, is the eaſtern amplitude.

Turn the globe till the ſtar comes to the weſtern horizon, the diſtance thereof from the weſt point is the weſtern amplitude. And the point where the horizon cuts the equinoctial is the oblique deſcenſion.

PROB. XVII.

Given the latitude; to find the time when two known ſtars will have the ſame azimuth, and to find that azimuth.

Rectify the globe for the latitude, bring the ſun's place to the meridian, and ſet the index to 12. Then turn the globe round with one hand, and the quadrant about the zenith with the other, till both the ſtars fall upon the edge of the quadrant; then the index will point to the time, and the quadrant will cut the horizon in the azimuth of both ſtars.

Cor. *If both ſtars have one altitude, the time may be found.*

Move the globe ſo, that the ſame degree of the quadrant will paſs thro' both the ſtars; then that degree ſhews their altitude; and the index ſhews the hour when that happens.

PROB. XVIII.

Having the latitude of the place, the day of the month, and altitude of a known ſtar; to find the azimuth and hour of the night.

Rectify the globe for the latitude, and zenith; and bring the ſun's place to the meridian, and ſet the index to 12. Then turn the globe with one hand, and the quadrant of altitude with the other, keeping the ſtar to the edge of the quadrant; move them both till the ſtar falls upon the degree of altitude given. Then where the quadrant meets with the horizon, is the azimuth of the ſtar; and the index will ſhew the time of night.

And on the contrary, when the time is given, the altitude may be found.

PROB. XIX.

Having given the latitude of the place, the month and day, and the azimuth of a known ſtar; to find its altitude, and the hour of the night.

Having rectified the globe for the latitude and zenith; bring the ſun's place to the meridian, and ſet the index to 12 o'clock. Turn the quadrant of altitude round, till it cut the horizon in the given azimuth; keep it there, and turn the globe about, till the ſtar fall upon the edge of the quadrant; then the degree of the quadrant, where the ſtar is, ſhews the altitude; and the index will point to the hour.

And by the converſe of this problem, if the time be given, the azimuth may be found.

PROB. XX.

Having the latitude and longitude of any planet, to find the time of its riſing, ſouthing and ſetting.

Find the place of the planet upon the globe, by Prob. XII. and there make a mark, or ſtick a ſmall patch. Then rectify the globe, and bring the ſun's place to the meridian, and ſet the index to 12 at noon. Then turn the globe about till the mark or patch falls on the eaſtern part of the horizon, and the index will ſhew the hour of riſing; then turn the globe, till the mark comes to the meridian, and the index ſhews the ſouthing; turn the globe ſtill about, till the mark comes to the weſtern horizon; and the index will ſhew the ſetting.

PROB. XXI.

To find what ſtars never ſet, and what ſtars never riſe, in a place whoſe latitude is given.

Rectify the globe to the latitude of the place. Which being done, turn the globe round; then ſuch ſtars as never deſcend below the horizon, in a whole revolution of the globe; theſe never ſet in that latitude, but always remain above the horizon. Likewiſe theſe ſtars that never riſe above the wooden horizon, in an entire revolution of the globe; never riſe above the horizon of the place.

Thus to us living in north latitude, the ſtars about the north pole never ſet, and the ſtars about the ſouth pole never riſe; and ſo they are never ſeen by us.

PROB. XXII.

To find the time of the coſmical riſing or ſetting of a given ſtar, at a given place.

Rectify the globe for the latitude of the place, then turn the globe about, till the ſtar comes to the eaſtern part of the horizon, and note the degree of the ecliptic that is then riſing in the horizon. Look for that degree on the wooden horizon, in the circle of ſigns, and againſt it in the circle of months, you'll find the day when the ſun is in that degree of the ecliptic; and conſequently, the day when the ſtar riſes coſmically.

If the globe be turned, till the ſtar comes to the weſt part of the horizon; the degree of the ecliptic which is then riſing in the eaſtern horizon is the ſun's place, at the time of the ſtars ſetting.

This being found in the circle of degrees, gives the time in the circle of months when this happens; that is, when the ſtar ſets coſmically.

PROB. XXIII.

In a given place, to find the time of the acronical riſing and ſetting of a given ſtar.

Rectify the globe for the latitude of the place; then turn the globe about, till the ſtar comes to the eaſtern ſide of the horizon, then the degree of the ecliptic ſetting in the weſtern horizon, being found in the circle of degrees on the wooden horizon, gives the time in the circle of months, when the ſun-ſets, or when the ſtar riſes acronically.

Likewiſe turning the globe about, till the ſtar comes to the weſtern horizon; the degree of the ecliptic that ſets with the ſtar, being found in the circle of degrees, gives the time in the circle of months when the ſun ſets, or when the ſtar ſets acronically.

PROB. XXIV.

In any given place, to find the time when a known ſtar riſes and ſets heliacally.

The ſtars of ſeveral magnitudes at their riſing, require the ſun to have different degrees of depreſſion below the horizon, as follows.

Magn.	1,	2,	3,	4,	5,	6	♄,	♃,	♂,	♀,	☿,	☽
Depreſ.	12,	13,	14,	15,	16,	17	11,	10,	11,	5,	10,	5.

Rectify the globe to the latitude, and turn it about till the given ſtar comes to the eaſtern ſide of the horizon. And turn the quadrant of altitude to the eaſt, ſo that it may cut the ecliptic ſo many degrees below the horizon as its magnitude requires. But if the quadrant cannot reach ſo far, turn it to the weſt ſide, that it may cut the ecliptic, ſo many degrees above the horizon; note that point of the ecliptic ſo cut, and take the point oppoſite to it in the ecliptic; find this oppoſite point in the circle of degrees, and againſt it, in the circle of months, is the day when the ſun is ſo many degrees below the eaſtern horizon, when the ſtar riſes; and that is the time of the ſtar's heliacal riſing.

Again, bring the ſtar to the weſt part of the horizon; and turn the quadrant to the eaſt ſide, till it cuts the ecliptic at a proper number of degrees above the horizon; take the point oppoſite to this; and find it in the circle of degrees; and againſt it in the circle of hours, is the time when the ſun is in that oppoſite point, that is, when the ſun is ſo many degrees below the weſtern horizon, when the ſtar is ſetting; and then the ſtar ſets heliacally.

PROB. XXV.

To ſhew all the places on the earth where the eclipſes of Jupiter's Satellites are viſible, at any given time.

Find the place on the earth where the ſun is vertical at the time of an eclipſe, by Prob. V. bring that place to the zenith, and ſet the index to 12. Then if Jupiter riſes after the ſun, draw a line on the globe along the eaſtern ſide of the horizon, with a black lead pencil. This line paſſes over all places where the ſun is then ſetting. Turn the globe weſtwards, till it revolves thro' as many degrees, as is the difference of the right aſcenſions of the ſun and Jupiter; ſtay it there; then bring Jupiter's place to the zenith, keeping the globe from revolving. Then draw another line upon the globe along the eaſtern ſide of the horizon; then at all the places contained in the ſpace between theſe two lines, the eclipſe will be viſible. And that is from ſun-ſet to Jupiter's ſetting.

If Jupiter riſes before the ſun, bring the place, where the ſun is vertical, to the zenith. Draw a line on the weſt ſide of the globe, by the weſt ſide of the horizon. Then let the globe revolve eaſtwards, thro' an arch equal to the difference of right aſcenſions; and there ſtop the revolution, and bring Jupiter's place to the zenith. Draw another line along the weſtern ſide, by the horizon. Then the ſpace between theſe two lines will contain all the places of the earth, where the eclipſe is viſible, from Jupiter's riſing to ſun riſe.

PROB. XXVI.

To find all the cuſps of the celeſtial houſes for any given place and time.

Rectify the globe according to the latitude, bring the ſun's place to the meridian, and ſet the index to 12 o'clock. Then turn the globe about, till the index point to the time given. Then fix the ſemicircle of poſition to the meridian, at the north and ſouth points of the horizon. Then from the ſouth or culminating point of the equator count 30 and 60 degrees eaſtward; bring the ſemicircle to theſe diviſions, and where it cuts the ecliptic, will be the cuſps of the 11th and 12th houſes; bring the ſemicircle to the weſtern ſide, and reckon 30 and 60 degrees from the culminating point, on the equinoctial; and making the ſemicircle to paſs thro' theſe points; where it interſects the ecliptic, will be the cuſps of the 9th and 8th houſes. And the meridian cuts the ecliptic above in the cuſp of the 10th houſe, and below in the cuſp of the 4th houſe. And the horizon cuts the ecliptic, on the eaſt, in the cuſp of the 1ſt houſe; and on the weſt, in the cuſp of the 7th houſe. Laſtly, the 2d and 3d are oppoſite to the 8th and 9th, and the 5th and 6th oppoſite to the 11th and 12th. And ſo the cuſps of all the houſes are found, which muſt be ſet down on paper.

PROB. XXVII.

To repreſent the face of the heavens, or to ſhew the ſituation of all the celeſtial bodies for any time and place given.

Rectify the globe for the given latitude, find the ſun's place for that day, and bring it to the meridian, and ſet the hour index to 12 o'clock; likewiſe place the globe by the compaſs, that the meridian may point north and ſouth. Then turn the globe about, till the hour index point to the given time; then the globe is in the ſame ſituation as the earth and heavens. Therefore the globe reſting thus,

All ſtars in the eaſtern part of the horizon are riſing, and thoſe in the weſtern ſide are ſetting. All ſtars under the braſs meridian are ſouth or culminating, and juſt ſo they are in the heavens. All ſtars on the eaſt of the braſs meridian on the globe, are eaſtward in the heavens; and all on the weſt ſide, are weſtward in the heavens. The quadrant of altitude laid over any ſtar, ſhews its altitude and azimuth, juſt as it is in the heavens.

If the ſun ſhines upon the globe, it will illuminate it juſt as the earth is illuminated. And right lines drawn from the center of the globe thro' each ſtar, will produced paſs thro' the real ſtars which they repreſent. And if the quadrant of altitude be laid over any ſtars on the globe, its plane produced will paſs thro' the ſame ſtars in the heavens. So that the globe in this poſition is a true picture of the heavens. Thus you may obſerve the north ſtar and great bear have like ſituations. That the ſeven ſtars on the globe, and in the heavens, are alike ſituated. The like for Orion's Girdle, Caſſiopeia, Syrius, Lyra, &c. or any you have a mind to obſerve.

This problem is very uſeful for finding out the conſtellations, and the ſeveral ſtars contained in them; by obſerving how they lie in triangles, ſquares, or right lines, in reſpect of one another. By this means one may ſoon come to be acquainted with all the ſtars in heaven.

SECT. IV.

The Determination of the Magnitude and Situation of the Orbits of the Earth and Planets, their periodical Times; the Places of the Stars; and all ſuch Aſtronomical Matters as depend upon Obſervations. This contains the Elements of Aſtronomy.

DEFINITIONS.

DEF. I.

Fig. 29. THE *Orbit* of a planet N, is the curve line or tract it deſcribes about the ſun, in the center of force, or ABPDA. This tract or path of a planet, in the earth and all the planets, is an ellipſis; where the ſun at S is placed in one of the foci.

DEF. II.

The Apſides are the two points of the orbit A and P, which are placed at the ends of the tranſverſe axis AP, which is called the *line of the apſides.* The point A which is furtheſt off the ſun is called the *aphelion* or the *higher apſis.* The point P which is neareſt the ſun, is called the *perihelion* or *lower apſis.*

DEF. III.

The *Excentricity*, is the diſtance of the focus from the center, SC. This is very little in all the planets.

DEF.

Fig. 29.

DEF. IV.

The *mean diſtance* of a planet, is the line SB, drawn from the focus S, to the end B or D of the conjugate axis BD. This line SB is equal to half the tranſverſe axis AC or CP.

DEF. V.

The *periodical time*, is the time of a planet's deſcribing its orbit; till it return to the ſame point again, where it begun.

DEF. VI.

The *nodes* of any orbit, are the points where its plane cuts the plane of the earth's orbit; ſuppoſing them both produced to the fixt ſtars. 30. 31.

For aſtronomers ſuppoſe the orbit of the earth, or the plane of the ecliptic, to be a fixt plane, to which the planes of all other orbits are to be referred, as to a common ſtandard. For the planets move in orbits, whoſe planes are all different from that of the earth, and from one another. And the ſun being in all theſe planes, they muſt cut one another in lines paſſing thro' the ſun. Thus if ABG be the earth's orbit, HFI the orbit of a planet cutting the earth's orbit in the line FG. Then F and G are the *Nodes*; and FG *the line of the Nodes*.

It is evident the orbits of the planets will cut the earth's orbit in different lines, as they are all in different planes. Therefore each planet will have its own nodes, and line of the nodes. That node thro' which the planet paſſes towards the north pole, is the *Aſcending Node*; and that thro' which it moves towards the ſouth pole, is called its *Deſcending Node*.

DEF. VII.

The inclination of a planet's orbit, is the angle which the plane of it makes with the ecliptic, as LGH. This is meaſured by the arch of a circle LH, at a quadrant's diſtance from the nodes.

DEF. VIII.

Elongation of a planet, is the angle between the ſun and the planet, taken at the earth. Or the angle between the ſun and place of the planet, reduced to the ecliptic.

DEF. IX.

Parallax, is the difference between the real and apparent place of a planet; or when it is ſeen from the center of the earth, and from ſome given point in its ſurface. And the *annual parallax*, or parallax of the earth's orbit, is the difference of the places of a ſtar, ſeen from oppoſite points of the earth's orbit.

DEF. X.

The moon's *apogee and perigee*, are the points where the moon in its orbit, is furtheſt from, and neareſt to the earth. Theſe anſwer to the aphelion and perihelion of a planet.

DEF. XI.

The moon's *ſynodic revolution*, is the time of its revolution, from conjunction to conjunction with the ſun.

DEF. XII.

The *ſyziges* of the moon's orbit, is the points where a line, drawn thro' the earth and ſun, cuts it. Theſe are the points of conjunction and oppoſition.

DEF. XIII.

The *quadratures*, are the points of the moon's orbit, which are at a quadrant's diſtance from the ſyziges, or from the conjunction and oppoſition. And the *octants* are in the middle between the ſyziges and quadratures.

The matters treated of in this ſection, are ſuch as depend merely upon obſervation, or of that and a little geometrical reaſoning. Which things being ſettled, are a foundation for all calculations in Aſtronomy. And as nothing is to be done without knowing the latitude of a place, and having a meridian line; I ſhall begin with finding theſe.

PROB. I.

To find the latitude of a place.

1 *Way.*

Take a quadrant, the larger the better; then direct it to the pole ſtar, and take its altitude when it is at the higheſt, which is known by continuing your obſervations, till it begin to deſcend. And about 12 hours after, take the altitude of it when it is at the loweſt, as before for the higheſt. Then half of the ſum of theſe two altitudes, is the height of the pole, OP, above the horizon HO. And that is the latitude. 32.

For if EC be the equinoctial, HO the horizon, Z the zenith; then the arch EP is 90 degrees, and the arch ZO is 90 degrees alſo; therefore taking away ZP, the arch EZ, or the latitude, is equal to the arch PO, or the height of the pole.

Inſtead of the pole ſtar, any other will do, that is out of the power of refraction.

Fig.

2 *Way.*

33. Near the top of a wall or of a poſt, put in a pin B; and another in a block at A, near the earth, to be moved back and forward at pleaſure. Then by the right aſcenſion of the ſtars, find by a table when the pole ſtar is at its higheſt. When that happens, place the block ſo that the pins A and B may be in a direct line with the pole ſtar. If it be dark, caſt a light upon A and B with a candle. Let it reſt till day light. Then hang a line and plummet BD, from B, in a veſſel of water D, to make it reſt ſteady. Then take a long rule, and apply one end to A, where make a mark; and at the ſame time apply the other end to the point B, which point mark alſo on the ruler. Again apply the firſt mark to A, and apply the rule to the line BD, at C its neareſt diſtance, and there put in a pin. So you have the hypothenuſe AB, and the baſe AC, ſet out on your rule. Then to meaſure them, extend a thread between the marks on the rule, that you may meaſure ſtreight, by the thread. Then open the compaſſes to 6, or any number of inches exact; and meaſure the number of ſpaces, (or lengths of 6 inches) from A to B, and A to C on the rule, keeping cloſe by the thread; and what remains, take off with another pair of compaſſes, and apply to the ſcale; ſo you have the meaſures of the hypothenuſe and baſe in inches.

At 12 hours diſtance, you muſt repeat the ſame operation quite thro'. Then you will have another hypothenuſe and baſe. Then in theſe two triangles, find the angle A at the baſe of each. And half the ſum of the angles is the latitude.

Note, this muſt be done in a calm place, where there is nothing to diſturb the experiment. And the longer the diſtance AB is, the exacter will the work be.

Alſo if you know the declination of the pole Fig.
ſtar at that time, you need but find one triangle 33.
ABC. For if the ſtar was at the higheſt, you have nothing to do but ſubtract the complement of its declination from the angle at A, for the latitude; or add it, if the ſtar was at its loweſt.

3 *Way.*

If the ſun's declination be known at that time; 32.
take his meridian altitude, which you may know, by continuing the obſervation till he begins to deſcend. Then if the declination be towards the elevated pole, add it to the complement of the altitude, for the latitude of the place. But if the declination be contrary, take the difference between the declination and the complement of the altitude, for the latitude.

Cor. 1. *The height of the pole is equal to the latitude of the place; or* PO = EZ.

Cor. 2. *The height of the equinoctial* EH *is equal to the complement of the pole's elevation* PO.

For EH is the complement of EZ, or its equal PO.

PROB. II.

To draw a meridian line in any place. 34.

1 *Way.*

Upon a plain board, ſet parallel to the horizon, deſcribe a circle ABF. And upon the center C erect a ſtile or gnomon, exactly perpendicular to it, and ſo high, that the top of the ſhadow thereof may fall upon the circumference of the circle about the middle of the forenoon. Mark the point B exactly where the top of the ſhadow falls in the forenoon. Likewiſe mark the point F where the top of the ſhadow falls again on the circumference,

Fig. in the middle of the afternoon. Then thro' the
34. center C, draw the line ACD biſecting the arch BF. Then AD is the meridian required.

It will be proper to draw ſeveral concentric circles, and to make obſervation with them all, that they may confirm one another. And if the ſun happens to be clouded in one, it may anſwer in another. And will be beſt to make the obſervation about the ſolſtices, when the ſun does not alter his declination ſenſibly. And the ſummer ſolſtice is to be preferred.

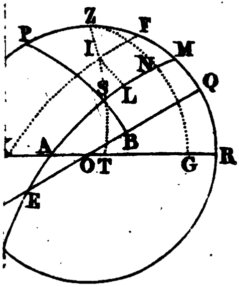
It is evident the ſun is higheſt when in the meridian; and at equal diſtances therefrom, has equal altitudes. Therefore when the diſtances DB, DF are equal, the ſhadows CB, CF will be equal, and therefore the altitudes equal. And the contrary.

2 *Way.*

35. Hang up two threads and plummets AB, CD, at a good diſtance, in veſſels of water, to keep them ſteady; of which CD is moveable towards the left and right, upon a pin C. Wait till the pole ſtar E, and the ſtar Alliot F (in the great bear's rump), come into the ſame plumb line AB, to an eye placed at I. At that inſtant (or rather before) move the thread CD alſo into the ſame line; ſo that the thread CD may hide the thread AB, and the pole ſtar E from the eye at I. Then the plane ABCD is the plane of the meridian; and where it interſects the horizontal plane, is the meridian line. And the ſame may be done with the ſtar called Caſſiopeia's hip.

Note, to take away the ſtars rays, look thro' a ſmall hole in a thin plate. And this muſt be performed in a calm place.

36. If you would have a meridian drawn in ſome other place, let the threads and plummets AB, CD, remain; and hang up two others *ab*, *cd*, in the



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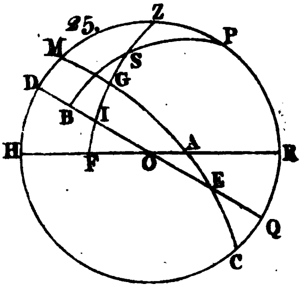

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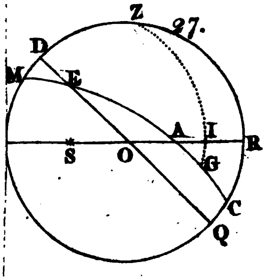

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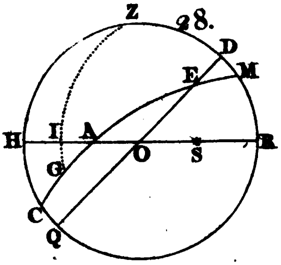

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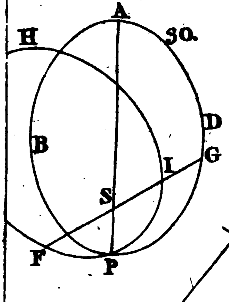

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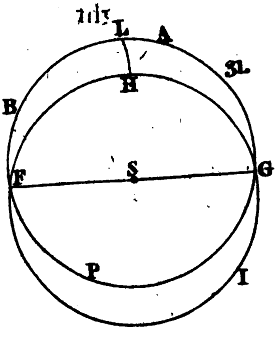

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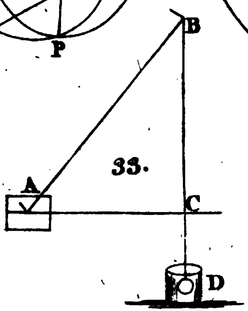

Pl.V. pa. 134.

Fig. 36. the place propoſed, letting *ab*, be moveable upon a pin at *a*. Then wait till any ſtar as G comes into the plane ABCD, to the eye at H; and at that inſtant, move the thread *ab*, till the ſame ſtar G fall in the plane *abcd*, to the eye at *b*; then *abcd* is the plane of the meridian. This is beſt done by help of an aſſiſtant.

This method will in time deviate a little from the truth, occaſioned by the ſtars changing their places; but that change is very inconſiderable in a few years.

3 *Way*.

If you have a clock with minutes and ſeconds. Find the northing of the ſtar Allioth F, by the aſtronomical tables; and wait till you find the pole ſtar E and F, in a plumb line. At that inſtant, ſet your clock to the ſaid time of northing. And next day at 12 o'clock, you may draw a meridian line, by the ſhadow of a plumb line hung in the ſun. Or you may find the time of ſouthing of any other ſtar G, and the clock remaining as before, when ſhe ſhews the time of ſouthing, place the threads *ab*, *cd*, ſo that the line G*b* may paſs thro' them both. Then *abcd* will be in the plane of the meridian.

SCHOLIUM.

37. If any one is deſirous to know how much the azimuth of theſe ſtars differs from the north, when in a perpendicular line; he may find it thus by ſpherical trigonometry. Let Z be the zenith, P the pole, $\ast$ the north ſtar, and S any other known ſtar.

Then in the triangle $SP\ast$, there are given two ſides SP, $P\ast$, and the included angle at P, to find the angle at $\ast$.

Again, in the triangle $ZP\ast$, there will be given two ſides ZP, $P\ast$, and an oppoſite angle at $\ast$; to find the other oppoſite angle $PZ\ast$, the azimuth from the north, as required.

Fig. 37. And likewiſe the time or the angle ZP ✱ or ZPS, may be found. And thus the meridian line may be corrected at pleaſure.

PROB. III.

To find the obliquity of the ecliptic.

1 *Way.*

Take the ſun's meridian altitude at both the ſolſtices. Subtract the leſſer from the greater, and take half the difference, and this is the obliquity of the ecliptic. This is ſuppoſing the ſun is on the ſame ſide of the zenith, at both obſervations; but if it lie different ways, add the complements of the two latitudes together, and take half the ſum for the obliquity of the ecliptic, or the angle it makes with the equinoctial. For this half difference or half ſum is the meaſure of that angle. If one of the obſervations be made when the ſun is low, regard muſt be had to the refraction, and that altitude muſt be diminiſhed as much as the refraction comes to.

2 *Way.*

If you have the latitude of the place, take the meridian height of the ſun at the ſummer ſolſtice, ſubtract the complement of this from the latitude of the place; or which is the ſame thing, ſubtract the height of the equinoctial from this meridian altitude; what remains is the ſun's greateſt declination, or the obliquity of the ecliptic.

If the obſervation happens not to be made preciſely when the ſun is in the ſolſtice; yet that makes no ſenſible difference; for if the moment of the ſolſtice happens to be at midnight, which is 12 hours diſtance, yet the error will be only 4 ſeconds.

To get the obliquity of the ecliptic with ſufficient exactneſs; obſervations ſhould be made with great care every year, for a long time, and with large Fig. and accurate inſtruments. By comparing the obſervations of the ancients with the modern ones, it appears that the obliquity has been continually decreaſing; and it is found now to be ſcarce 23 degrees 29 minutes.

PROB. IV.

To find the time of the ſun's being in the equator.

Having the latitude of the place, and conſequently the height of the equator; watch carefully, and make obſervation of the ſun's meridian altitude, on theſe days when it is equal to the height of the equator, or nearly ſo. Then if it happen that on any day at noon, the obſerved meridian altitude is juſt equal to the height of the equator, or the complement of the latitude; then the ſun is in the equator that day at noon. But if it is greater or leſſer, you muſt obſerve another day either before or after; ſo that on one day the meridian altitude may be leſs, and the day after, greater, than the height of the equinoctial; or alſo that on the firſt day it may be greater, and on the next day leſs. Then you have no more to do, but to divide the time between them (which is 24 hours) in proportion to that exceſs and defect; and taking the time required, nearer to the time, where there was the leaſt difference of declination. For the time required is evidently between the lines of the two obſervations.

By a like method, the time may be found, when both the altitudes are leſs, or both greater than the height of the equinoctial. And then the time of the equinox will be after or before both the obſervations.

Care muſt be taken that the refraction be allowed

Fig. ed both in the meridian altitudes, and in the height of the equinoctial; or elſe in neither of them.

PROB. V.

To find the length of the year.

Find the time of the ſun's entering into the equator by the laſt Prob. either at the vernal or autumnal equinox. Again, the next year, find in like manner the time of his entering the ſame equinox. Then the diſtance of time between them is the length of the year. This is called the *tropical year*, becauſe in that time all the ſeaſons return again in the ſame order for ever. This is ſomething leſs than the ſydereal year; or the earth's periodic time, occaſioned by the preceſſion of the equinoxes.

But ſince the taking two years which ſucceed one another, may be liable to ſome error, and therefore not exact enough; it will be better to take two ſuch equinoxes as are at the greateſt diſtance of time from one another; ſo that the error, divided into ſo many parts may become inſenſible. Then this time divided by the number of years or revolutions, will give the time of one. By this means the length of the tropical year is found to be 365 days, 5 hours, 48 minutes, 57 ſeconds.

Cor. 1. *Hence the time of a ſydereal year, or the earth's periodic time may be found.*

For this is the time of (the earth or) the ſun's departing from any ſtar, and returning to the ſame again. And is found like the former, by taking the time between leaving any ſtar and returning to it again, for a great many years; and dividing it by the number of years, gives the length of one year.

Or it may be found by comparing the place of a fixed ſtar, where it was many years ſince, with its preſent place; and from thence finding the annual preceſſion of the equinoxes. This arch reduced to time, and added to the tropical year, gives the ſydereal year. This preceſſion of the equinoxes is found to be 50 ſeconds yearly. And from thence the ſydereal year is 365 days, 6 hours, 9 minutes, and 14 ſeconds.

Cor. 2. *By the ſame rule, may the time between the vernal and autumnal equinox be found; and likewiſe between the autumnal and the vernal equinox.*

For by taking the times between two ſucceeding ſolſtices, as directed above, it appears by many obſervations, that the time from the vernal to the autumnal equinox, is 186 days, 12 hours, 29 minutes, 30 ſeconds. And from the autumnal to the vernal equinox only 178 days, 17 hours, 19 minutes, 30 ſeconds. So that the ſummer half year, exceeds the winter by 7 days, 19 hours, and 10 minutes.

PROB. VI.

To determine the time of either ſolſtice. 38.

Let AD repreſent a very ſmall part of the tropic, and AL a like part of the ecliptic; draw AH perpendicular to AD. Let the ſhade of the top of a tower or a high wall, fall upon the line AH placed north and ſouth; and let the extreme points of the ſhadow be obſerved three times about the ſolſtice, or within 5 or 6 days thereof, and let theſe points be accurately marked, in the line AH. Suppoſe theſe points of the ſhadow to be at F, G, H. Draw FI, GK, HL, parallel to AD, to cut the ecliptic in I, K, L; and draw IB, KC, LD perpendicular to AD.

Now the motion of the ſun in the ecliptic, about the time of the tropics, is very near equable for a few

Fig. 38. few days. And ſince the part of the curve AL is very ſmall, and very nearly coincides with a circle or parabola; therefore the ſubtenſes BI, CK, DL, of the angle of contact DAL, are as the ſquares of the lines AB, AC, AD. For this is the property of the parabola.

Therefore ſuppoſe A to be the point of the tropic; then BA will repreſent the time of moving thro' IA, CA the time in KA, and DA the time in LA. And there are given the differences of the times when the obſervations were made; that is, there is given DC, and CB and DB; and alſo FG and FH. Then put AB $= x$, AF or BI $= y$, DB $= a$, CB $= b$, FH $= c$, FG $= d$; then will AH or DL $= c + y$, AG or CK $= d + y$, AC $= b + x$, AD $= a + x$. Then we ſhall have BI (y) : CK $(d + y)$: : AB² (xx) : AC² $(\overline{b + x}^2.)$ Whence $dxx + yxx = bby + 2bxy + xxy$, and $dxx = bby + 2bxy$, whence $y = \frac{dxx}{bb + 2bx}$.

Again, BI (y) : DL $(c + y)$: : AB² (xx) : AD² $(\overline{a + x}^2)$; therefore $cxx + yxx = aay + 2axy + xxy$, and $cxx = aay + 2axy$, whence $y = \frac{cxx}{aa + 2ax} = \frac{dxx}{bb + 2bx}$; and $\frac{c}{aa + 2ax} = \frac{d}{bb + 2bx}$, and multiplying, $bbc + 2bcx = aad + 2adx$, reduced $x = \frac{aad - bbc}{2bc - 2ad}$, which is the time between the laſt obſervation at F and the tropic, which here is ſuppoſed to be after that obſervation; but if x come out negative, then the tropic will happen before the obſervation at F. And if G or H lie upward from F, d or c will be negative.

To make theſe obſervations to the beſt advantage, the top of the building that caſts the ſhade, ſhould be very ſtreight and even, ſo as to caſt a true ſhade. If it is not, ſomething may be placed upon it that will do it. And the plane ALH on which the ſhadow falls, ſhould not be too oblique to the ſun's rays. And becauſe the penumbra is apt to confuſe the ſhadow, ſo that its termination cannot be diſtinctly made; therefore it will be proper to tranſmit the ſun's light thro' a ſolar teleſcope, and then the limit of the ſhadow will be exactly had. Fig. 38.

Cor. *If the obſervations are made at times equidiſtant; then* $x = \frac{4d - c}{2c - 4d}b$.

For then BC = CD, or $b = a - b$, and $a = 2b$; therefore expunging a, $x = \frac{4bbd - bbc}{2bc - 4bd} = \frac{4bd - bc}{2c - 4d}$.

PROB. VII.

To find the excentricity of the earth's orbit, and its form.

1 *Way.*

If the motion of the ſun be obſerved in the ecliptic, it will be found ſometimes to move ſlower and ſometimes faſter; and ſometimes to appear bigger and ſometimes leſs. Therefore make ſeveral obſervations, when he is about the aphelion, and take his apparent diameter, ſo often, till you find it the leaſt poſſible. At half a year's diſtance, when he is about the perihelion, obſerve his apparent diameter again ſeveral times, till you find it at its greateſt; theſe diameters muſt be taken very exactly.

Then thro' any point S, which ſuppoſe the focus of the orbit, or the place of the ſun, draw a line PSA, and from S, take SP to SA, as the leaſt 39.

apparent

Fig. 39. apparent diameter of the ſun, to its greateſt; and A and P will be aphelion and perihelion of its orbit. Then make SC and CF equal to half the difference of SA and SP; and F will be the other focus, and C the center. And the ellipſis ABPD deſcribed to the foci S, F, will be the form of the orbit. For the diſtance is reciprocally as the apparent diameter.

2 *Way.*

Obſerve with a teleſcope and micrometer what the apparent motion of the ſun is when ſloweſt, and (half a year after) how much the apparent motion is when ſwifteſt. Take SA to SP, as the ſquare root of the ſwifteſt motion, to the ſquare root of the ſloweſt. And A and P will be the aphelion and perihelion. And the reſt will be found as before.

3 *Way.*

40. Suppoſe ADPB the earth's orbit, S the ſun in the focus, F the other focus, AP the line of the apſides. Let B, C, D be three places of the earth obſerved in the ecliptic; found by obſerving 3 places of the ſun which are oppoſite to them. With the radius FM equal to AP, and center F, deſcribe the circle MRGY. Draw FBR, FCZ, FDY; and SB, SC, SD, and alſo SR, SZ, SY. Then the angles BSC and CSD are known, being meaſured by the arches of the ecliptic. But the angles deſcribed about the focus F are nearly proportional to the times, and the periodic time of the earth being known, by Prob. V. the angles BFC, CFD will be known. Now ſince FB + BR or FR = AP = FB + SB, therefore BR = SB, and the angle BRS = angle BSR, and FBS = 2BSR. But AFB = ASB + FBS = ASB + 2BSR, whence 2BSR or 2FRS = AFB — ASB. And by the ſame reaſoning 2FZS = AFC — ASC. And

by adding, 2FRS + 2FZS = BFC — BSC. Fig. 40. Therefore the ſum FRS + FZS is known, ſince BFC and BSC are known. But FRS = RFA — RSA, and FZS = ZFA — ZSA; then by addition FRS + FZS = RFZ — RSZ = a known angle; and ſince RFZ or BFC is known, RSZ will be known. And by the ſame reaſoning RSY will be known, and conſequently ZSY.

Draw the lines YSG, GR, and RZ and ZG. Then aſſuming ZS any number as 1000; in the triangle ZSG, there are given the angles ZSG, ZGS, and ſide ZS, to find SG. For ZSG = ſupplement of ZSY, and ZGS = $\frac{1}{2}$ ZFY. Alſo in the triangle SGR, are given the angle RSG (ſuppl. RSY), and SGR ($\frac{1}{2}$YFR), and ſide SG, to find SR. Again in the triangle SZR, there are given SZ and SR and angle ZSR, to find the ſide ZR, and angle SZR. Alſo in the triangle ZFR, there is given, angle ZFR, and ſide ZR; to find ZF, which is equal to AB, the greater axis of the orbit, and angle RZF. This ſubtracted from RZS leaves FZS known. Laſtly, in the triangle FZS, we have FZ, ZS and angle FZS, to find the angle ZSF which ſhews the poſition of the apſides; and ſide SF, which is double the excentricity; whence the orbit is eaſily deſcribed. And in the ſame triangle is found the angle ZFS, and CFA, and therefore AFB. Whence if the time between the obſervations at B and C, be divided in proportion to the angles BFA and AFC, the time will be known when the earth is in its aphelion at A.

SCHOLIUM.

Becauſe the angles AFC, AFB, &c. are not accurately as the times. The place of the earth C at any time, may be corrected thus, make as the leſſer axis of the orbit is to the greater; ſo is the tangent of the angle AFC, to the tangent of an

Fig. 40. angle, which is more correct for the place of C; and the like for B. But in the 2d and 3d quadrants, or in the arch DPO, the angle muſt be diminiſhed.

4 *Way*.

41. The periodic time of the earth and of Mars being known. Let ABDE be the earth's orbit, M the planet Mars. Obſerve the exact time when Mars is in oppoſition to the ſun, the earth being at A; then the place of Mars muſt be obſerved. After Mars has performed his period (of 687 days), he will come again to the ſame place at M. But the earth will only be at B in its orbit, from whence, the ſun will be obſerved in the line BS, and Mars in the line BM. And at that time obſerving the places of the ſun and Mars, all the angles of the triangle MBS will be had; and ſuppoſing MS any number as 1000, the ſide SB may be found, as alſo the angle BSM. Again, when Mars has finiſhed another revolution, and the earth is at C; the line SC may be found the ſame way, as alſo the angle CSM. By the ſame method when Mars has finiſhed a third revolution, and the earth come to D, the line SD, and angle DSM will be found the ſame way. Then having the three points B, C, D; the ellipſis may be deſcribed by Prop. LXXXV. B. I. Conic Sections. Whence the tranſverſe axis and excentricity will be had, and alſo the place of the aphelion A, and perihelion P.

By theſe methods, the earth's excentricity is found to be 166; ſuppoſing the mean diſtance, or half the tranſverſe axis to be 10000.

PROB.

Fig. 40.

PROB. VIII.

To find the place of the earth's aphelion, or the poſition of the apſides; and the time when it is in the aphelion.

Obſerve two places of the ſun at a good diſtance from one another; where the apparent motion is the ſame, or where the apparent magnitude is the ſame. Let theſe places be C and B. Biſect the arch BC, by the line AF. Then A is the aphelion, if the magnitude or motion was decreaſing towards A; otherwiſe it is the perihelion. Whence the place of the aphelion is had; and the poſition of the apſides AP. And if the times be noted when the earth was in C and B; the time in the middle between them, is the time when the earth is in the aphelion A.

2 *Way.*

The poſition of the apſides AP, may be had by either the 3d or 4th method of the laſt Prop. and likewiſe the time when the earth is in the aphelion.

By theſe methods the earth's aphelion is now found to be when the ſun is in ♋ 8 deg. 52 $\frac{1}{2}$ min.

SCHOLIUM.

Hence by comparing the place of the aphelion at preſent with its place ſeveral years ſince, its motion will be known for that time. Therefore dividing the angle of motion, it has deſcribed in that time, by the number of years, the quotient will ſhew the annual motion of the aphelion; which is little more than a minute of a degree in a year.

Note, the time of the earth's moving or revolving from aphelion to aphelion, is called the *anomaliſtic Year*.

PROB. IX.

To find the declination of a ſtar.

To do this rightly, the obſerver ought to have a large quadrant fixed in the plane of the meridian, with an index moving round its center, furniſhed with teleſcopic ſights. Then when the ſtar comes into the plane of the meridian, take its meridian altitude; and if the ſtar be towards the equinoctial, and on the north ſide of it, ſubtract the height of the equinoctial, and that of the ſtar from one another, and the remainder is the north declination. But if the ſtar is on the ſouth ſide of the equinoctial; then the remainder is its declination ſouth.

Again, if the ſtar be towards the pole, take the difference between its height, and the height of the pole, and that is the complement of the ſtar's declination north, if it is the north pole; and ſouth if it be the ſouth pole.

PROB. X.

To find the right aſcenſion of a ſtar.

1 *Way.*

Having a good pendulum clock with minutes and ſeconds. Let the clock be ſo adjuſted, that the hand may go thro' the 24 hours in a ſydereal day, or in the time that a ſtar leaves the meridian and comes to it again; which is ſomething leſs than a natural day, and is 23 hours, 56 minutes, 4 ſeconds. Then obſerve when the ſun is in the meridian, and ſet the clock, at that inſtant, to the time of the ſun's right aſcenſion, computed by Prob. I. Sect. II. Then obſerve when the ſtar comes to the meridian, and at that inſtant mark the time ſhewn by the clock; and that will be the ſtar's right aſcenſion in time. Fig.

Or you may ſet the clock to 12, when the ſun is in the meridian; and note the time when the ſtar comes to the meridian; and add this time to the ſun's right aſcenſion in time; and you have the ſtar's right aſcenſion in time.

Then the right aſcenſion in degrees may be found, by allowing 15 degrees for an hour, and 1 degree for 4 minutes.

2 *Way.*

By having the right aſcenſion of any ſtar; obſerve when that ſtar comes to the meridian, and then ſet the clock to its right aſcenſion. Again, obſerve when the ſtar required comes to the meridian, and note the time, and that will be the ſtar's right aſcenſion in time.

Or you may ſet the clock to 12, when the known ſtar comes to the meridian, and mark the time when the unknown ſtar comes to the meridian; and that time, added to the right aſcenſion of the known ſtar, gives the right aſcenſion of the unknown one.

Cor. 1. *Hence by having the right aſcenſion of one ſtar, the right aſcenſion of the reſt may be found.*

Cor. 2. *Having the right aſcenſion and declination of any ſtar; its longitude and latitude will be known.*

This is calculated by Prob. XXXII. Sect. II.

Cor. 3. *From the longitude of the ſame ſtar at different times, the apparent motion of the fixed ſtars may be found; or the preceſſion of the equinoxes.*

Thus comparing the places of the fixt ſtars, as deſcribed by the ancients, with their places at preſent, the progreſſive motion will be known, which

divided

Fig. divided by the number of years, will give the motion for one year, which is 50 ſeconds of a degree.

Cor. 4. *From hence the longitude of any ſtar being known at any one time, its longitude may be found at any other time.*

For it is only adding ſo many times 50″ as is the number of years after; or ſubtracting them for preceding years.

SCHOLIUM.

By this method the longitudes and latitudes of all the fixt ſtars are found, and made into a catalogue; which being once eſtabliſhed, and ſet in their proper places, ſerve for ſo many land-marks for eſtimating the places and diſtances of all other celeſtial bodies. If there were no ſtars to reckon the places of the planets by, their ſituations could never be known; and therefore we may juſtly account the having a true catalogue of the ſtars, to be the baſis of all aſtronomy.

PROB. XI.

To find the periodic time of a planet.

Let the planet be obſerved ſeveral times, till at laſt it have no latitude, for it is then in the node; note the time when this happens. Alſo let it be obſerved again about the ſame place of its orbit, and mark the time when it has no latitude, and it is then in the ſame node. Then the time between theſe obſervations is the periodic time of the planet.

If, inſtead of one period, you take two points of time, at a great number of years diſtance from one another, when the planet was in the ſame node; and divide the interval of time, by the number of revolutions; the periodic time will be exactly determined.

For the orbit of any planet, being out of the plane of the earth's orbit, the planet is never in that plane, but when it is in the nodes, where the planes interſect; and therefore it is in the ſame node only once in every revolution; that is, when it comes to the ſame point of its orbit. Fig.

And to know how many revolutions there has been in any great length of time, that there may not happen a miſtake of one revolution; it will be ſufficient to know the periodic time in round numbers, which may be done thus.

Obſerve the time when the planet is in oppoſition to the ſun, and likewiſe the time when he is next in oppoſition; then having its place at both times, there is had the arch he deſcribes in the time between the obſervations. Then it will be as that arch to the whole circumference; ſo the time elapſed, to the periodic time; which would be exact if the orbits were circles, from which they do not differ much.

But for an inferior planet, take the two obſervations when the planet begins to be ſtationary. And in that time it has deſcribed its whole orbit, together with an arch or angle equal to what the earth has deſcribed in the mean time; which is known, from the ſun's place at the two obſervations. Then it will be, as the whole circumference + that angle, to the time elapſed; ſo the circumference, to the periodic time, nearly.

PROB. XII.

To find the nodes of a planet's orbit; and the planet's diſtance from the ſun at that time.

Let ABD be the earth's orbit, FNQ that of the planet, S the ſun, NSQ the line of the nodes. This method ſuppoſes that the places and diſtances 42.

Fig. 42. of the earth from the ſun, are known at the times of obſervation; and alſo the arch of the ecliptic deſcribed in that time.

Therefore take two obſervations, when the planet is in the node N; let the places of the earth at theſe times be A and B. Then in the triangle ASB, there is given the ſides BS, AS, and included angle ASB; to find the ſide AB, and angles at A and B. And the angles SAN and NBS being known by obſervation, we ſhall have the angles NAB, and NBA. Again, in the triangle ABN, there are given the angles, and the ſide AB; to find NA the planet's diſtance from the earth. Laſtly, in the triangle NAS, we have the ſides NA, AS, and the angle NAS; to find NS the diſtance of the planet from the ſun, when it is in the node at N; and angle ASN, which determines the place of the node; and the poſition of the line of the nodes NSQ. This calculation will be the ſame for an inferior planet, *mutatis mutandis.*

Cor. *Hence the motion of the nodes of a planet may be known, by comparing their preſent places with the places of the nodes, found out by the obſervations of the ancients.*

PROB. XIII.

To find the inclination of a planet's orbit.

1 *Way.*

43. Let *n*PF be the orbit of a planet, EQN the earth's orbit, N*n* the line of the nodes. The place of the nodes of the planet is known by the laſt Prop. and therefore the time will be known when the earth is in one of the nodes at N. At that time let the planet be at P. From P let fall PO perpendicular to the plane of the ecliptic *n*OL. And when the earth is at N take the latitude PO

of the planet, and the diſtance O*n* from the oppoſite node *n*, as ſeen from the earth at N. Then in the right angled ſpherical triangle *n*PO, there are given the ſides *n*O, and OP found by obſervation, to find the angle P*n*O the inclination of the orbit required. Fig. 43.

For draw from the ſun at S, the line S*p* parallel to NP, and *po* parallel to PO, and draw S*o*, which will be parallel to NO. Therefore the triangles PNO, *pSo* will be ſimilar. Therefore the latitude PO ſeen from N, is equal to the latitude *po* ſeen from S, being both ſeen under equal angles. Likewiſe the angle ON*n* is equal to the angle *o*S*n*; that is, the longitude O*n* ſeen from N, is equal to the longitude *on* ſeen from S. Therefore there are the ſame data in the triangle *n*OP as in the triangle *nop*, to find the inclination O*n*P of the orbit.

2 *Way*.

Let the orbit of the planet be PDG, that of the earth BEQ, N*n* the line of the nodes. At the time when the earth is at Q in the line of the nodes, obſerve accurately the longitude of the planet at P, from the ſun, or the angle OQS, and alſo its latitude PQO. Then it will be as the ſine of the angle SQO, to radius; ſo tan. latitude PQO, to the tan. of the inclination of the orbit. 44.

For PO being perp. to the plane of the ecliptic; draw PQ, OQ; and draw OB perp. to N*n*; then the angle PBO is the inclination of the orbit. In the right angled triangles QPO, and BPO, we have OB : S.OQB : : QO : radius : : PO : tan. PQO. And alternately, S.OQB : tan. PQO : : OB : PO : : radius : tan. PBO.

SCHOLIUM.

If it happen that the ſun is in the node, and the planet 90 degrees from the ſun. Then the latitude

Fig. 44. of the planet ſeen from the earth, is equal to the inclination of the orbit.

For imagine B to be in the earth's orbit, and PBS a right angle, or PB perp. to BS; then PBO is the inclination.

PROB. XIV.

To find the orbit of a planet, the place of its aphelion, and the time when it is in the aphelion.

1 *Way.*

Let *n*PN be the planet's orbit, AEQ the earth's, S the ſun, N*n* the line of the nodes. This method ſuppoſes the earth's motion in its orbit to be known; and likewiſe the inclination of the planet's orbit.

Let the planet be at P in oppoſition to the ſun, the earth being at E. Draw PO perpendicular to the ecliptic, and draw SEO; and O is the planet's place reduced to the ecliptic. Then obſerve the quantity of the angle PEO, the planet's latitude ſeen from the earth. And ſince the ſun's place is known, the planet's place being oppoſite to it is known, and therefore its diſtance from the node O*n* is known. Therefore in the ſpherical triangle *n*PO, the ſide O*n*, and angle O*n*P are known; therefore the ſide P*n* will be known, and PO, which is the meaſure of the angle PSO, the latitude of the planet ſeen from the ſun. Likewiſe SE is known being the earth's diſtance from the ſun. Therefore in the triangle PSE having all the angles and the ſide SE; the ſide SP, the planet's diſtance from the ſun, is known. Therefore we have the planet's place in its own orbit.

In like manner, by getting two more obſervations, of the ſame planet, in a poſition oppoſite to the ſun; we ſhall have two more places D and G of the planet in its orbit. Hence we have the magnitude and poſition of three lines SP, SD, SG, drawn from the focus of an ellipſis; and then the curve may be deſcribed by Prop. LXXXV. B. I. Conic Sections; and that will be the orbit of the planet, from which all other requiſite may be found. It is evident this method can only be applied to the ſuperior planets. Fig. 44.

2 *Way.*

Let GHT be the earth's orbit, APE the planet's orbit, S the ſun, DSN the line of the nodes. Let the planet be at P, whilſt the earth is in any place T. Draw PB perpendicular to the plane of the ecliptic. Draw TB meeting with the line of the nodes in N. Draw SB; and draw TC parallel to BP, and TD perpendicular to DN, and draw CD; and the angle TDC is the inclination of the orbit. 45.

Let the angle BTP be obſerved, the planet's latitude at the earth; and the angle BTS its diſtance from the ſun in the ecliptic. Then 1. In the triangle NTS, we have given, the ſide ST the diſtance of the earth and ſun; and the angle TSN, the earth's diſtance from the node, and the angle NTS or BTS; to find NT. 2. In the right angled triangle TSD, we have TS, and TSD; to find TD. 3. In the right angled triangle TDC, we have TD and TDC; to find TC. 4. In the right angled triangle TCN, we have TC, NT; to find the angle TNC. 5. In the triangle NTP, we have the angle NTP or BTP, and PNT or CNT, and the ſide NT; to find TP. 6. In the right angled triangle PTB, we have TP, and angle PTB; to find BT, BP. 7. In the triangle TSB, we have TB, TS, and the angle BTS; to find SB, and angle TSB, which is the place of the

Fig. 45. the planet reduced to the ecliptic. 8. In the right angled triangle PBS, we have the ſides PB, BS; to find SP, the planet's diſtance from the ſun; and angle PSB, the planet's latitude ſeen from the ſun. 9. In the right angled ſpherical triangle APB, we have PB and angle PAB, to find AB and AP. So the planet's place in his orbit is known, and his diſtance from the ſun. And if by this method we find two more places of the planet in its orbit, and the diſtances from the ſun; we ſhall have three lines given in magnitude and poſition, drawn from the focus of an ellipſis; and therefore the curve may be deſcribed by Prop. LXXXV. B. I. Conic Sections, which will be the planet's orbit.

3 *Way*.

46. Let BKL be the earth's orbit, S the ſun, A the planet, P the point in the ecliptic where it is cut by a line AP perpendicular to it. Suppoſe the earth at K, let the angle PKS be obſerved, being the difference between the longitude of the ſun and of the planet, ſeen from K. After the planet has compleated its period, it comes to the ſame point P again; at which time, let the earth be in L. At that time obſerve the angle PLS, or the difference of the longitude of the ſun and planet. Then having the times of obſervation, there will be given the places of the earth in the ecliptic at K and L, and conſequently the angle LSK, and alſo the ſides LS, SK; therefore in the triangle LSK, there will be given the ſides LS, SK, and included angle LSK; to find LK and the angles SKL, and SLK. And having the angles SKP, and SLP; the angles PKL and PLK will be known. In the triangle PLK, we have the angles, and the ſide LK; to find PL. In the triangle PLS, we have the ſides PL, LS, and included angle PLS; to find LSP, the longitude of the planet from the

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earth; and SP, the planet's diſtance from the ſun. Fig. 46. And the place of the node being known, the diſtance of the planet from the node in the plane of the ecliptic, is alſo known. But PA being perpendicular to the ecliptic, and if A be the place of the planet in its orbit: then in the right angled triangles APL and APS, it will be as tan. ALP, to the tan. ASP; ſo is SP to LP; whence the angle ASP will be known, and from thence AS. And laſtly, the diſtance of A from the node will be found as in the laſt method. If by this means two more places of the planet be found, there will be given the focus, and 3 points in the curve, to deſcribe an ellipſis, by Prob. LXXXV. B. I. Conic Sections, for the orbit of the planet.

Then the poſition of the tranſverſe axis of the orbit, will ſhew the place of the aphelion; and from thence the time will be known; or the place of the aphelion may be known, by having ſeveral obſervations; and from thence finding the time of paſſing from the aphelion to the perihelion; and the time of going back to the aphelion; which is done by help of the obſerved places. Then theſe times will be equal, when the point is in the aphelion; and muſt be corrected by taking new points for the aphelion, till it happen ſo. But the exact points may be better found by interpolation, from the ſeveral obſervations, by Prop. V. of the differential method.

PROB. XV.

To find the greateſt elongation of an inferior planet. 47.

Let ABD be the orbit of the planet, E a point in the earth's orbit, S the ſun. From the point E, draw the line EB to touch the orbit ABD of the inferior planet; and the angle SEB will be the planet's

Fig. 47. net's greateſt elongation. Therefore draw SB to the point of contact B, and it will be perpendicular to the tangent EB; whence in the right angled triangle ESB, there are given ES and SB, the diſtances of the earth and planet from the ſun; to find the angle SEB the elongation.

For a line drawn from E to any other point of the half orbit ABD, beſides B, will fall within the angle SEB; ſince that half orbit is wholly included between the lines ED and EB.

If you would find the greateſt elongation experimentally; you muſt take the elongation SEB, by an inſtrument, ſeveral times, when it is about the extremity of its orbit; and the greateſt angle contained between the ſun and the planet, is that ſought. But in elliptical orbits, the ſame muſt be repeated ſeveral times, when the earth and planet are in different parts of their orbits; and the greateſt obſerved angle is that required.

PROB. XVI.

To find the parallax of a planet.

1 Way.

48. Let AD be the earth, C its center, P the planet; and let the planet's diſtance CP, from the center of the earth be given. Then ZAP is the complement of the apparent altitude, ZCP the complement of the true altitude. Then it will be, as the planet's diſtance from the center of the earth CP : to the earth's radius AC : : ſo the coſine of the apparent altitude, S.ZAP : to the ſine of the parallax.

For draw AF parallel to CP. Then angle FAP is equal to the angle APC. But ZAF is equal to ZCP, the true zenith diſtance, and ZAP is the apparent zenith diſtance; and their difference FAP or its equal APC is the parallax. But in the triangle CAP, it is CP : S.CAP or ZAP : : CA : S.CPA or PAF the parallax. Fig. 48.

2 *Way*.

If the diſtances of two planets or ſtars, having the ſame apparent altitude, be known, and the parallax of one of them.

Let P and G be the planets in the line APG; then APC is the parallax of P, and AGC the parallax of G. Therefore in the triangle CPG, we have CP, CG, and an angle oppoſite, ſuppoſe G; to find the other oppoſite angle. Therefore diſtance CP : diſtance CG : : S.CGP : S.CPG or CPA; that is, the ſines of the parallaxes are reciprocally as the diſtances from the earth's center.

3 *Way*.

Let S be the ſtar or planet, whoſe parallax is ſought. Obſerve it when it is in the ſame vertical circle with any two fixt ſtars, A, B. Obſerve again when the ſame two ſtars come into a poſition parallel to the horizon at *a* and *b*, and let the planet be come to *s*. Then with an inſtrument meaſure the altitude of *a* or *b*, and likewiſe the altitude of *s*; and the difference of theſe altitudes is the parallax. 49.

For the real place of the ſtar S is ſome where in the line AB, and therefore it is alſo ſomewhere in the line *ab*, and therefore its altitude is the ſame as that of *a* or *b*. Therefore the parallax is the difference of the altitudes of *a* and *s*, or of *b* and *s*.

4 *Way*.

Let S be the ſtar or planet; obſerve its diſtance from any fixt ſtar B, which is in the ſame vertical circle ZSB; and meaſure the diſtance SB with an inſtrument. Then obſerve again when the ſame two ſtars have equal altitudes above the horizon, Fig. 49. at *b* and *s*; and then take the diſtance *bs*. This diſtance will be very near the true diſtance of the ſtars B and S. Therefore the firſt diſtance BS ſubtracted from the latter diſtance *bs* (when B is below S), gives the parallax. Or the latter diſtance ſubtracted from the former, when B is above S, gives the parallax.

5 *Way*.

50. The parallax may be found by obſerving the azimuth and altitude of the ſtar or planet. Let HZO be the meridian, EQ the equinoctial, HO the horizon, Z the zenith, P the pole, S the ſtar, ZSB a vertical circle paſſing thro' it. Obſerve the altitude BS, and the azimuth BO; and mark the moment of time when theſe obſervations are made. Then obſerve the moment of time that the ſtar comes to the meridian; and then you have the diſtance of time from the obſervations. Convert this into degrees, allowing only 23 hours, 56 minutes to 360 degrees (which is the time of the earth's rotation to the ſame ſtar), and you have the arch ED or angle EPA, ſuppoſing PAD an hour circle. Therefore in the ſpherical triangle ZPA, we have the angle ZPA, and angle PZA equal to BO, and the ſide ZP the co-latitude; to find the ſide ZA the complement of the altitude; this ſubtracted from ZS, known by obſervation; the remainder AS is the parallax.

6 *Way*.

This is to be done with a teleſcope with croſs hairs in the focus. Direct the teleſcope to the planet, and turn it round till its motion is along one of the croſs hairs, which repreſents part of the planet's parallel circle; and the other hair perpendicular to it, will repreſent its hour circle. Obſerve the time when the planet comes to this hour circle; there fix the teleſcope, and then take its altitude;

tude; then obſerve the time when ſome fixt ſtar, Fig.
whoſe right aſcenſion is known, comes to the ſame 50.
hour circle. The difference of time between the
planet and ſtar coming to this hour circle, turned
into degrees (allowing 360 degrees to 23 hours, 56
minutes), gives the difference of right aſcenſions
of the planet and ſtar; and ſo the apparent right
aſcenſion of the planet is known. Again, when the
planet comes to the meridian, obſerve it with the
teleſcope, and note the time; and when the ſtar
comes to the meridian, note the time of that. Then
the difference of the times reduced to degrees as
before, gives the true difference of right aſcenſions,
ſo the true right aſcenſion of the planet will be
known. Therefore let HO be the horizon, HZO 51.
the meridian, Z the zenith, P the pole, A the true
place of the planet, S its apparent place, ZSB a
vertical circle. Then in the triangle ZPS, we have
ZP, ZS, and angle ZPS to find the angle PZS. In
the triangle ZPA, we have ZP, angles ZPA,
PZA; to find ZA, which taken from ZS, gives
AS the parallax.

If the planet have a proper motion of its own, its true place will be always changing; and therefore the change of place muſt be computed for the time of the obſervations. This is done by obſerving its place when in the meridian, twice; and thence the change of place is had for 24 hours. And therefore the place at the times of obſervations will be had by proportioning the motion, according to the times.

Here the angle ZPS ſhould be about 90 degrees, to have APS the greateſt poſſible.

7 *Way*.

This requires two obſervers in different places 52.
of the earth, and can be applied to none of the planets
but Mars in oppoſition to the ſun, or to Ve-
nus

Fig. 52. nus in the ſun's diſk. Let PERQ be the earth, PR its axis, EQ the equinoctial, S the planet Mars in oppoſition to the ſun, and if near the perihelion, 'tis better. And this will be beſt done when the ſun is about the equinoxes. Let there be taken two places F, G, one in north latitude, the other in ſouth latitude, and the further from the equinoctial the better, and nearly in the ſame meridian, or rather ſo placed, that the line FG drawn from one to the other may be nearly perpendicular to the orbit of Mars. By this means we have a greater baſe to work upon. Then let the two obſervers pitch upon ſome fixt ſtar as A, which Mars in his motion comes very near at that time; and the nearer the better. And having two good inſtruments perfectly alike, furniſhed with micrometers, and being ſituated at F and G; let them obſerve for ſeveral nights ſucceſſively about midnight, the places of Mars at B and C, as it paſſes by the ſtar A; and take the diſtances AB and AC every night, during his tranſit by this ſtar. Theſe obſervations are to be continued till the diſtances begin to encreaſe, and no longer; for then he is paſt the ſtar. From this number of obſervations, the neareſt diſtance of Mars from the ſtar A may be found, as obſerved from the places F and G; at leaſt they may be found by interpolation. Let theſe neareſt diſtances be AB and AC, then we have the difference BC, or the angle BSC or FSG. And from the ſituation of the places F and G, the length and poſition of FG will be known; and by theſe, FS may be found. And laſtly, the angle which the radius of the earth ſubtends at the diſtance FS, or the horizontal parallax of Mars will be known.

If, inſtead of Mars, Venus be obſerved in the body of the ſun; then her neareſt diſtances from either limb of the ſun muſt be taken, whoſe difference will give the angle at Venus, ſubtended by FG, the reſt as before; ſo the parallax of Venus will be obtained. Fig. 52.

The parallax of Mars, when neareſt the earth, has been found 25, 27 and 30 ſeconds at different times.

Cor. 1. *A ſtar or planet appears lower than it really is by the quantity of the parallax; which is greater the lower the ſtar is; and therefore the horizontal parallax is the greateſt.*

Cor. 2. *The parallax is equal to the angle which that radius of the earth, where the obſervation is made, ſubtends at the planet.*

Cor. 3. *The parallaxes of two planets are as the coſines of the apparent altitudes directly, and their diſtances from the earth's center reciprocally.*

For when the diſtance is given, the parallax is as the ſine of the zenith diſtance (by method 1.) and if the apparent altitude be given, the parallax is reciprocally as the diſtance, by method 2. and therefore is in a compound ratio, when neither is given. Here the parallax being very ſmall, one may take the parallax itſelf for the ſine of the parallax.

Cor. 4. *The parallax of a planet being known, its diſtance may be found.*

For this is only working backward, ſaying as ſine of the parallax, to the earth's radius; ſo S. zenith diſtance, to the planet's diſtance.

Cor. 5. *Having the parallax of any of the planets; the diſtances of all the planets from the ſun may be known, in diameters of the earth, or any ſort of meaſure.*

For the diſtances of the planets from the ſun and from one another, are known in ſome aſſumed meaſure, Fig. ſure. And by the parallax of a planet, the true diſtance of the earth from it, is known. And therefore all the other diſtances will be known by proportion.

PROB. XVII.

Having the parallax of a planet, and its poſition in reſpect of the ecliptic and circles of longitude, &c. to find the parallax of latitude and longitude; or the parallax of right aſcenſion and declination.

53. Let HO be the horizon, EC the ecliptic, Z the zenith, P the pole of EC, S the apparent place of the planet, ZSB a vertical circle drawn thro' the planet, A the true place of the planet, PAL, PSI, circles of longitude. Draw AD parallel to EC. Then AS is the parallax of altitude, SD the parallax of latitude, LI the parallax of longitude; L being the true place of the ſtar reduced to the ecliptic, and I the apparent place.

Then having AS and the angle ASD, in the ſmall right angled plane triangle ASD; the parallax of latitude SD will be had, and the ſide AD. Therefore in the triangles PAD, PLI, it will be, as S.PD : radius : : AD : LI the parallax of longitude. And whether it is to be added to, or ſubtracted from the longitude at L, will eaſily appear, by the poſition of the circles EC, ZB, PL.

But if EC be the equinoctial, P its pole; then SD will be the parallax of declination, and LI the parallax of right aſcenſion.

Cor. 1. *If the circles* PS, ZS *coincide*; SD = AS, *and* AD = 0. *That is, the parallax of longitude or right aſcenſion, is nothing, and the parallax of latitude or declination, is equal to the parallax of altitude.*

Cor. 2. *If the circles* EC, ZS *coincide. Then* $AS = LI$, *and* $SD = 0$. *That is, the parallax of longitude or right aſcenſion (as the caſe is) is equal to the parallax of altitude. And the parallax of latitude or declination is nothing.* Fig. 53.

PROB. XVIII.

To find the periodic time of the moon, her orbit and parallax.

1. The moon's *periodical time* is known by obſerving the diſtance of time between her conjunction with any fixed ſtar, and her next return to the ſame ſtar again. But this being different at different times; if the time be taken, between her conjunction with any ſtar, obſerved ſeveral years ſince; and her conjunction with the ſame ſtar in this age; and that time be divided by the number of her revolutions; it will give her mean periodic time, which is 27 days, 7 hours, 43 minutes, 12 ſeconds; and is ſomething more than her return to the ſame point of the ecliptic, which is $27^d\ 7^h\ 43^m\ 5''$.

2. The time of the moon's *ſynodic revolution*, or the time of her revolving from conjunction to conjunction with the ſun, is found the ſame way, by obſervation; and in a great number of years is found, by dividing the diſtance of time by the number of theſe revolutions; and is $29^d\ 12^h\ 44^m\ 3''$.

3. To find the parallax, proceed by the methods laid down in Prop. XVI. or by this method, which is peculiar to the moon. In an eclipſe of the moon, obſerve when the horns of the moon are in the ſame vertical circle, and at that moment take the altitude of both the horns; then half the ſum is the apparent altitude of the moon's center. And we ſhall have the true altitude of her center thus.

Fig. 53. the place of the ſun in the ecliptic at that time is known, and therefore the place of the center of the earth's ſhadow, which is oppoſite to it, is known; and the altitude of the moon at that time is equal to the altitude of the center of the ſhadow, becauſe they are both in a line parallel to the horizon. Therefore finding the altitude of that point of the ecliptic, in which the center of the ſhadow is; from which ſubtracting the apparent height of the moon, and the remainder is the parallax. Thus her mean horizontal parallax is 57′ 12″.

54. 4. To find her diſtance. Let AD be the earth, whoſe center is C, and A the place of obſervation. Let F be the place of the moon. Find her parallax AFC by Art. 3. and apparent diameter. Then in the triangle AFC, we have the angle F the parallax, and angle CAF the ſupplement of ZAF her apparent zenith diſtance; to find her apparent diſtance AF. Then if G be another place of the moon, whoſe diſtance is ſought. Let the moon's apparent diameter at G be obſerved, and her altitude BAG. Then it will be, as her apparent diameter at G, to her apparent diameter at F; ſo is AF to AG. Then in the triangle CAG, having the ſides AG, AC, and angle CAG (= the right angle CAB + altitude BAG), the ſide CG will be found, or her diſtance from the earth's center = 60.3 of the earth's radii, at her mean diſtance.

5. To find the tranſverſe axis of her orbit, and excentricity. Find her diſtances from the earth's center, by a great number of obſervations, continued thro' her period, as in Art. 4. Out of theſe ſelect the greateſt and leaſt, and the ſum of them is the greater axis of her orbit; and half the difference is the excentricity. Thus the mean excentricity is 55, the mean diſtance being 1000.

Note, the moon will be nearly at the greateſt and leaſt diſtances from the earth's center, when the apparent diameters are the greateſt and leaſt, Fig. taken about the ſame altitude. And about theſe times you muſt find the ſeveral diſtances from the earth's center, in order to avoid needleſs trouble.

6. To find the place of the moon's apogee. Obſerve two places of the moon, when ſhe has the ſame apparent diameter, and the ſame altitude; biſect that arch to find the intermediate place; for the apſis of her orbit paſſes thro' it, and therefore the poſition of the apſis is known. And that point is the apogee where the apparent diameter is leaſt. By comparing two ſucceſſive revolutions, the motion of the apogee will be known. Or rather by dividing the whole ſpace gone thro' in a long time, by the number of revolutions, gives the time of one; thus the apogee makes a revolution in conſequentia, to the ſame point of the ecliptic, in 8 years, 309 days, 8 hours, 20 minutes.

7. To find the place of her nodes. Obſerve the latitude of the moon ſeveral times till you find it to be nothing, when the parallax is allowed for; and that is the place of her node. If the moon be coming northward, it is the aſcending node; if ſouthward the deſcending node. If the places of the moon be found thus, when ſhe is in two ſucceeding nodes; the arch deſcribed by the moon in that time, compared to a ſemicircle, will ſhew the motion of the nodes. Or rather, it being known by diſtant obſervations how far either node has changed its place in a great number of years, the time of one revolution will be found. Thus the moon's nodes are found to be retrograde; and make a revolution to the ſame point of the ecliptic in 18 years, 224 days, 5 hours.

8. To find the inclination of the moon's orbit. Obſerve her latitude ſeveral times when ſhe is near a quadrant from her node; and the greateſt latitude is the inclination of her orbit, allowing for

Fig. the parallax. Thus by many obſervations, the inclination of her orbit is ſometimes 5° 0′, and ſometimes 5° 18′; and always between theſe limits.

SCHOLIUM.

Frequent obſervations have diſcovered to us that the orbit of the moon is continually changing. For tho' the moon's orbit is nearly an ellipſis, where the earth is in one of the foci; yet it neither keeps its magnitude nor poſition. For,

1. When the earth is in its aphelion, the moon moves faſter, and performs her revolution in leſs time. But when the earth is in the perihelion, the moon moves ſlower, and is longer in making a revolution.

2. When the moon is in the ſyziges, ſhe moves ſwifter; and in the quadratures, ſlower.

3. When the moon is in the ſyziges, the apſides move forward, but in the quadratures they go backward. But in a whole revolution of the moon, they move forward.

4. The excentricity of her orbit increaſes as the line of the apſides approaches the ſyziges, where it is greateſt; and diminiſhes as the apſides approach the quadratures, and is there the leaſt.

5. The line of the nodes is moſtly moving backwards; except when the moon is in the quadratures, and then the nodes ſtand ſtill; or when the nodes are in the ſyziges, where they alſo ſtand ſtill; and between the node and neareſt quadrature, they go forward.

6. The inclination of the orbit increaſes from the moon's ſyziges to the quadratures; and decreaſes from the quadratures to the ſyziges. And in the paſſage of the nodes from the quadratures to the ſyziges, the inclination increaſes; but in their paſſage to the quadratures, it is diminiſhed.

PROB.

PROB. XIX.

To find the refraction of a ſtar.

The earth being ſurrounded with a body of air, called the Atmoſphere, thro' which the rays of light come to the eye, from all the celeſtial bodies. And ſince theſe rays come out of a vacuum, and fall obliquely upon this body of air; they will by the laws of optics be refracted in lines approaching nearer to the perpendicular of the refracting ſurface. Let AD be the earth, ſurrounded by the atmoſphere PQR; Let F be a ſtar whoſe rays paſs 55. thro' the atmoſphete QA to the obſerver at A. Now ſince the atmoſphere is denſer near the earth, and grows rarer by degrees as its height grows greater. Therefore at the top Q it is but a very little refracted, and as the air grows denſer, the nearer the ray approaches the earth; the ray will be more and more refracted, and gradually bent into a curve QA. By this means the ſtar comes to the eye in the direction GA, which is a tangent to the curve at A; and ſo the ſtar, inſtead of being ſeen at F where it really is, appears to be at G, ſituated in the line AG. For this reaſon a ſtar always appears higher or more elevated above the horizon, by the refraction, than it really is. And ſince the refraction is made in a plane which is perpendicular to the ſurface of the atmoſphere, or to the ſurface of the earth, which plane paſſes thro' the center of the earth; therefore both the true and apparent place, F and G are in the ſame vertical circle, whoſe plane paſſes thro' A the place of the obſerver, and the center of the earth C. It is plain alſo, that the refraction will be the ſame at all diſtances from the earth, when the altitude is the ſame; for from whatever part of the line FQ, the

Fig. 55. rays proceed; they will be refracted along the curve QA to A. Likewiſe we know from optics, that if a ray of light paſs thro' ſeveral refracting mediums, it will ſuffer the ſame refraction, as if it paſſed immediately out of the firſt into the laſt; therefore,

It has been confirmed by experiments, that light falling upon a medium of air of a mean denſity, at a very ſmall angle of inclination, will make the refracted angle half a degree; but this will vary according to the denſity of the air. Aſtronomers commonly make the horizontal refraction to be 33 minutes. Here the angle of incidence is 90 degrees. Therefore to find the refraction at any other altitude, it will be by the laws of Dioptrics, as radius : Coſ. 33′ (ſine of refraction) : : coſine of the true altitude of a ſtar : to the coſine of the apparent altitude by refraction. Then the true altitude taken from the apparent altitude, gives the refracted angle, which is the refraction of the ſtar.

For the ſines of incidence and refraction are in a given ratio, and the coſine of the true altitude, or the ſine of the zenith diſtance, is here the ſine of incidence. And the coſine of the apparent altitude, or the ſine of the apparent zenith diſtance, is the ſame as the ſine of refraction.

56. Having the horizontal refraction, to find it at any other altitude. Let ADBG be a circle, C the center, and let the diameter AB repreſent a part of the ſurface of the denſe atmoſphere. Draw DCG perpendicular to it. And let the horizontal ray AC be refracted in the line CT, and the ray PC in the line CZ. Produce PC to N, and draw TO, NM, ZV, PR, perp. to DG; and ZF, TI parallel to it. Let AC $= r$, CI or OT $= a$, PR or MN $=$ C, VZ $= c$, VC $= s$, $t =$ tangent of VCZ $=$ GL, $z =$ NZ the refraction, $r =$ AC. Then by the principles of Dioptrics, $r : a :: C : c$, and by diviſion, $r - a : a :: C - c : c$, whence $C - c$ or

$FN = \frac{r-a}{a}c$. But becauſe ZN is a very ſmall Fig. 56. arch, FZN may be looked upon as a plane triangle, ſimilar to CVZ, whence CZ (r) : CV (s) : : ZN (z) : FN $\left(\frac{r-a}{a}c\right)$. Therefore $SZ = \frac{r-a}{a}cr$, and $z = \frac{r-a}{a} \times \frac{rc}{s} = \frac{r-a}{a}t$. But becauſe $\frac{r-a}{a}$ is a given quantity, z will be as t; that is, the refraction at any altitude will be nearly as the cotangent of the altitude, or as the tangent of the zenith diſtance.

This rule may deviate a little in very ſmall altitudes, becauſe the rays near the horizon are not regularly refracted; for the rays coming parallel to the horizon, and falling on the atmoſphere, will cut its curve ſurface in ſome ſmall angle. That is, the part of the atmoſphere, where a horizontal ray falls, cannot ſtrictly be looked upon as parallel to the horizon, but will vary the more as the elevation is leſs; and this cauſes the failure.

2. *To find the refraction by obſervation.*

Take the altitude of a fixed ſtar, when it is at its loweſt point, under the pole; and again at about 12 hours diſtance, take its altitude, when at its higheſt, and clear of refraction. Then having the latitude of the place, you'll find the diſtance of the ſtar from the pole at each obſervation; take the leſſer diſtance from the greater, and the remainder is the refraction of the ſtar at the leſſer altitude, when under the pole.

For at the greater height of the ſtar, it is ſuppoſed clear of refraction; and therefore you have its true diſtance from the pole. But at the leſſer height you have its apparent diſtance. So their differenee muſt needs be the refraction.

Fig. 56. In this obſervation, it is beſt to chuſe ſuch a ſtar as comes pretty near the zenith, either northward or ſouthward; and then you may be ſure its refraction is not ſenſible. But if you ſuſpect, that it has ſome ſmall refraction at its higheſt obſervation, it may be found out by the proportion before given; by ſaying, as tan. greater zenith diſtance : tan. leſſer zenith diſtance : : ſo the refraction at the lower altitude : to the refraction at the higher. By this the lower refraction will be corrected, by ſubtracting this laſt from it, provided the lower altitude be not leſs than 8 or 10 degrees.

After this method, by the obſervation of different ſtars, the refraction for all altitudes may be found very accurately; and a table made of them for uſe. Thus the refraction at 10 degrees altitude is found to be about 5 minutes; and in the horizon from 30′ to 37′.

Cor. 1. *Refraction makes a ſtar appear higher than it is; contrary to the parallax, which makes it appear lower.*

Cor. 2. *The refraction decreaſes from the horizon to the zenith (like the parallax). And therefore the horizontal refraction is greateſt, and in the zenith it is nothing.*

Cor. 3. *All the ſtars, &c. have the ſame refraction at the ſame height, whatever their diſtances may be.*

Cor. 4. *The ſun and moon appear of an oval figure near the horizon, by reaſon of the refraction.*

For the under ſide being more refracted than the upper ſide, the perpendicular diameter will be leſs than the horizontal one.

Cor. 5. *Hence by the refraction, the ſun, moon, and ſtars, riſe ſooner, and ſet later than they ought to do, by their proper motion.*

For they are really below the horizon, when they appear to be in it, by the quantity of the horizontal refraction. Fig. 56.

Cor. 6. *By the variableneſs of refraction, it happens that the tops of mountains, or high towers, are at ſome particular times viſible, and at others inviſible.*

For as the denſity of the air changes, the degree of refraction changes. And theſe changes happen ſometimes in a few hours, eſpecially within 3 or 4 degrees of the horizon. And thus it comes to paſs, that the top of a mountain may be ſeen in the morning or at night, and be inviſible at noon; in the ſame place. And thus the lower part of a tower will be ſeen in the morning, and the upper part at noon, thro' a teleſcope, remaining fixt all the while. And thus ſometimes the top of a remote tower has been viſible above the top of one nearer, and ſometimes inviſible. So great a change there is in the denſity and refraction of the air within 2 or 3 degrees of the horizon, cauſed by vapours ariſing from the earth, rarefied and condenſed by heat and cold; that it is no wonder if the refraction, ſo very near the horizon, be ſo variable and ſubject to change. Whilſt at greater altitudes, where the atmoſphere is not diſturbed by ſuch cauſes, it is more regular and uniform.

SCHOLIUM.

Beſides this effect which the atmoſphere has, of refracting the rays of light; it has another effect, of reflecting them. By this means the twilight is produced; and this light is ſeen long before the ſun riſes, and continues long after ſun-ſet. For the height of the atmoſphere is ſo great, that it is able to reflect ſome of the ſun's rays to us, when he is 18 degrees below the horizon. From which time it grows gradually lighter till the ſun riſes, and the

Fig. 56. twilight diminiſhes gradually after ſun-ſet, till it vaniſh into total darkneſs.

PROB. XX.

To find the ſun's parallax and diſtance.

1 *Way.*

This is done by finding in the firſt place, the parallax of ſome of the planets; or perhaps of a comet if it come near enough. And the parallax of this being known, the ſun's parallax will become known. For the parallaxes of two celeſtial bodies are reciprocally as their diſtances. And the proportional diſtance of the planets from the ſun, and from one another, are known; whence the parallax of any one being known, that of any other, or of all the reſt, will be known.

For this purpoſe the planet Mars when in oppoſition to the ſun will be fitteſt to be made uſe of; becauſe at that time he comes neareſt the earth, and his parallax is then the greateſt. Therefore find his parallax at that time by ſome of the methods laid down in Prob. XVI. Then having his parallax, and the proportional diſtances of Mars and the ſun, from the earth at that time; the ſun's parallax is found by the foregoing rule. By this method Mr. Flamſteed found the parallax of Mars leſs than 30″; and from thence determines the ſun's parallax not to be above 10″.

2 *Way.*

This method depends upon finding the parallax of Venus, in her tranſit over the ſun's diſk, a thing which very ſeldom happens, but may be put in practice when it does happen, provided an obſerver be in a proper place upon the globe; and the nearer the equinoctial the better, becauſe the arch de-

Fig. 56. ſcribed in any time by the diurnal motion, is the greater. This is done by finding the apparent time of the tranſit, by a good pendulum clock, and the true time by calculation. This is beſt and moſt exactly performed by two obſervers, upon different places of the globe whoſe longitudes differ about 180 degrees. But as few people care to go to far diſtant countries in order to make obſervations; I ſhall ſhew how it may be done by one alone, who happens to be in a fit place for it.

57. Let GQI be half the diſk of the ſun. E and F the earth in its orbit, V and X Venus in her orbit. And let ACPDB be the path of Venus over the ſun's diſk. Find the horary motion EF of the earth in its orbit, and VX the horary motion of Venus in her orbit, from the aſtronomical tables, by the theory of the earth and Venus. Then we ſhall have the angles FDE and XDV. Put FDE $= p$, XDV $= q$. VE or XF $= a$, VC or XD $= b$. And the lines EVC, FXD, being drawn from the centers E and F, thro' Venus at V and X, then CD will be the horary motion of Venus in the ſun's diſk. To find which, draw DV, and sXB, touching the earth in s; then FDs is the ſun's horizontal parallax, put FDs $= x$ the horizontal parallax. And we have the $\angle$EDV $= q - p$.

Now in the triangle DEV, VE (a) : VD (b) :: EDV $(q - p)$: DEV or DEC $= \frac{q-p}{a}b$, the angle ſubtended by CD, which is the apparent horary motion on the ſun's diſk.

Watch the time carefully when Venus enters the ſun's diſk at A, and note the time; and alſo the time of the internal contact. Do the like at the end B of the tranſit; likewiſe take the greateſt diſtance of Venus PQ from the ſun's limb, in the middle of the tranſit; take alſo the ſun's diameter GI or zSQ, and that of Venus. Then we ſhall have

Fig. 57. have $SP = SQ - PQ$, and from thence AB, the path deſcribed. Then ſay, as CD : 1 hour or 60^m : : $AB : \frac{60AB}{CD}$, the minutes wherein the whole path would appear to be deſcribed from the center of the earth.

But the rotation of the earth being from weſt to eaſt, and Venus tranſit being from eaſt to weſt; ſhe will appear to go ſooner off the diſk. For while the earth moves from E to F, the place of the obſerver is moved thro' the arch *no*. And therefore, Venus inſtead of being obſerved at D will be obſerved at B in the line *o*XB. The time of deſcribing DB, then is owing to the parallax.

Put d = earth's diameter, c = *no* the chord of the arch deſcribed during the tranſit. Then in the triangle DX*s*, X*s* (a) : XD (b) : : x (the horizontal parallax) : angle D*s*X or D*s*B $= \frac{bx}{a}$. But becauſe *no* is leſs than the diameter, this angle muſt be diminiſhed in that ratio, therefore $d : c : : \frac{bx}{a} : \frac{bcx}{ad}$, the angle which BD ſubtends by half the chord *no*. And this muſt be doubled, for it is the ſame both at the beginning and end of the tranſit, as it cauſes it to begin later and end ſooner. Therefore 2BD or $\frac{2bcx}{ad}$ is the diminution of the length cauſed by the parallax of the chord *no*. Then to find the time wherein this is deſcribed; it is, as CD : 60^m : : $\frac{2bcx}{ad} : \frac{120bcx}{ad \times CD}$, the contraction of the time, by the parallax. But we have the time by obſervation, that the center of Venus ſeems to deſcribe the whole path AB, for it is nearly equal to the diſtance of time of the inner contact at the beginning, and the outer contact at the end, let this time in minutes $= t$; then $t + \frac{120bcx}{ad \times CD} = \frac{60AB}{CD}$. Then Fig. 57. $adt \times CD + 120bcx = 60ad \times AB$. Put for CD its equal $\frac{q-p}{a}b$, and let $AB = n$, then $adt \times \frac{q-p}{a}b + 120bcx = 60adn$, or $120bcx = 60adn - bdt \times \overline{q-p}$; and $x = \frac{60adn - bdt \times \overline{q-p}}{120bc} = \frac{d}{2c} \times : \overline{\frac{a}{b}n - \frac{t}{60} \times \overline{q-p}}$; where n is a ſmall angle expreſſing the ſun's chord AB.

If the chord *no* is not at right angles to DF or CE as the calculation ſuppoſes; that is, if the ſun is not in the meridian, at the middle of the tranſit; then c muſt be diminiſhed in the ratio of radius to the coſine of the meridian's diſtance from the middle of the tranſit.

And if that chord drawn in the ſphere is not parallel to the orbit of Venus, then c muſt be further diminiſhed in the ratio of the radius to the coſine of inclination.

If this method be carefully managed, it will give the parallax to great exactneſs. And to perform it, there is required a good pendulum clock with ſeconds, to note the time of the tranſit; and good inſtruments, with teleſcopic ſights to take the diameters of the ſun and of Venus, and to take her furtheſt diſtance from the ſun's limb, to have her neareſt approach to the center.

3 *Way.*

But if another obſervation can be made in the oppoſite part of the ſame meridian, or near it, where the immerſion and emerſion can be ſeen (which is Dr. Halley's method); the parallax may be determined

Fig. 57. mined to far greater exactneſs, and with leſs calculation. For ſince on the oppoſite ſide of the earth, the revolution is made from eaſt to weſt, the ſame way that Venus moves; it is plain the time of the tranſit will be prolonged, by reaſon of the parallax. Therefore if the time of the tranſit be exactly obſerved in this laſt place as well as in the former; it muſt needs be longer than the firſt obſerved time. And from the difference of the times the parallax will eaſily be found.

Let $T =$ this laſt obſerved time of the tranſit, C the chord of the arch deſcribed in the time of the tranſit, by the place of the obſerver. Then proceeding as before, we ſhall have this equation $T - \frac{120bCx}{ad \times CD} = \frac{60AB}{CD}$, and $adT \times CD - 120bCx = 60ad \times AB$, or $adT \times \frac{q-p}{a} b - 120bCx = 60adn$, and $db \times \overline{q-p} = \frac{120bCx + 60adn}{T}$. Put this value of $db \times \overline{q-p}$ into the equation found before, $120bcx = 60adn - bdt \times \overline{q-p}$. Then $120bcx = 60adn - t \times \frac{120bCx + 60adn}{T}$, and $120bcTx + 120bCtx = 60adnT - 60adnt$. Whence $x = \frac{adn}{2b} \times \frac{T-t}{cT + Ct}$, the parallax.

SCHOLIUM.

The ſame method might be made uſe of in Mercury's tranſit over the ſun, but that his parallax is ſo ſmall, one can make nothing of it. For he being nearer the ſun than Venus, and conſequently further from the earth; his true and apparent place upon the ſun's diſk almoſt coincide. So that no difference of the times of his real and apparent motion over the ſun's diſk can be diſtinguiſhed. Fig. 57. Otherwiſe, as his tranſits happen far oftener than thoſe of Venus, they would have afforded many opportunities for this purpoſe. For there are no leſs than ſix tranſits of Mercury before this century is out, viz. in 1769, 1776, 1782, 1786, 1789 and 1799. But there are only three tranſits of Venus before the commencing of the next thouſand years; and thoſe are in 1769, on June 3d, at 11 o'clock at night; in 1874, on Dec. 8th, at 4 o'clock in the morning; and in 1996, on June 8th, at 2 o'clock. So that if that in 1769 be let paſs; no further opportunity can be expected in this age. And even that will not be viſible in this country.

I muſt take notice, that the path of the planet ſeen by an obſerver, is not exactly the ſame as her path ſeen from the center of the earth which we here calculate; and this is alſo owing to her parallax. But whenever this method is put in execution, theſe trifling errors may eaſily be corrected.

PROB. XXI.

To find the diſtance of ſome of the fixed ſtars.

If this problem is to be done at all, it muſt be 58. after ſuch a manner as this. Chooſe two ſtars S and T, a greater and a leſſer, which appear exceedingly near together, or if poſſible in one line AST; when the earth (or the ſun) is about 90 degrees diſtant from them in the ecliptic; that is, when the difference of longitude of the ſun and the ſtars then obſerved, is about a quadrant. And out of ſuch a number of ſtars as the heavens afford, ſome will be found to anſwer this poſition. Now a greater and a leſſer ſtar are to be choſen, becauſe in all probability, they are at very unequal diſtances from the earth. The earth being then at A, in its orbit ADB, you muſt, with the beſt teleſcope you can

Fig. 58. get, obſerve diligently the ſtars S and T, in or near the line AST, and take their diſtance if they have any, as near as poſſible. Then at the diſtance of half a year, or when the earth is at B, in the oppoſite point of its orbit, obſerve the ſame two ſtars again, with the ſame inſtrument; which then will appear further at a diſtance from one another, and be ſeen under the angle SBT; therefore let the angle SBT be meaſured as before. But if you can find no difference in theſe angles, then the earth's orbit has no ſenſible parallax.

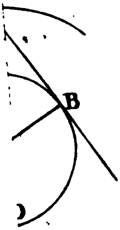
Now ſince the ſtar T is ſuppoſed to be very ſmall, it will probably be at a very great diſtance beyond S. And the angle SBT will be very near equal to the angle ASB, which is the parallax of the earth's orbit at the nearer fixt ſtar, if S and T are in a line from A.

But if S and T are not in a line, but make an angle at A, then that angle muſt be taken from the angle at B, if they can be meaſured or eſtimated; and the difference will be nearly the parallax of the annual orbit ADB, or the angle ASB; which being known, the ratio of AB to AS will be known.

If the ſtars S and T be taken near the pole of the ecliptic, one may make obſervations in any two oppoſite points of the ecliptic, or in as many places as you will, to find what alteration appears in the two ſtars.

If the ſtars are not very different in magnitude, the angles at B and T may be eſtimated by the diſtances BS and ST, and thoſe by the magnitude of the ſtars; from thence the angle ASB will be nearly known. But by all obſervations that have hitherto been made, thoſe angles are ſo ſmall as not to be had by any inſtruments.

The immenſe diſtance of the fixt ſtars is alſo evinced from this; that when their diameters are



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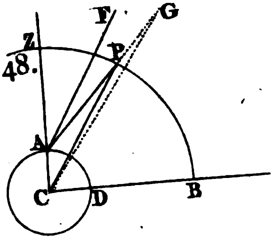

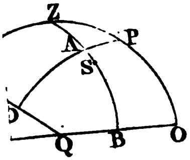

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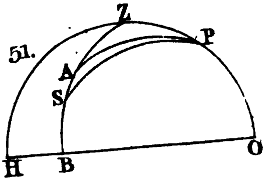

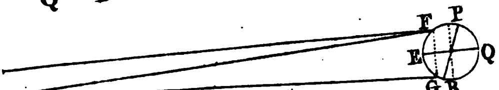

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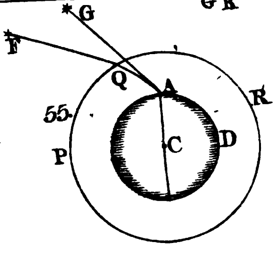

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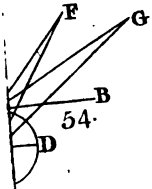

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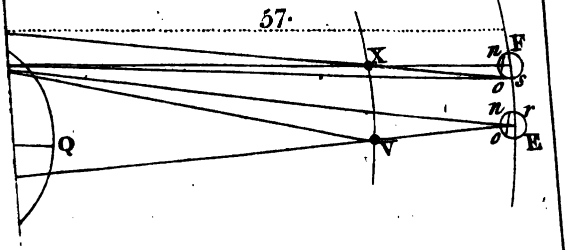


Pl. VII *pa.* 178.

magnified 100 or 120 times, the greateſt of them Fig.
appear only as points without any magnitude. 58.

SCHOLIUM.

It is extremely probable, that the annual parallax of the ſtars is quite inſenſible, and cannot be found to be of any magnitude by the beſt contrived inſtruments. For ſuppoſing ſome of the fixt ſtars to be in reality as big as the ſun, as the ſun is one of that kind of bodies. And if any ſtar, viewed thro' a teleſcope that magnifies 100 times, ſhould appear under an angle of half a minute, which is a ſenſible magnitude; the true apparent diameter would not exceed 18 thirds; which is leſs than the 6000th part of the ſun's apparent diameter. And then the ſtar's diſtance would at leaſt be 6000 times more than the ſun's. But as none of the ſtars appear to have any magnitude thro' a teleſcope; of conſequence the ſtar's diſtance is far more than 6000 times the diſtance of the ſun.

Mr. Flamſteed, by 7 year's obſervation, found the diſtance of the pole ſtar from the north pole, to be greater, about the ſummer ſolſtice, than about winter, by 40 or 50 ſeconds; and ſo concluded that the parallax of the earth's orbit was ſo much. But when this matter is rightly conſidered; that difference is found to ariſe from the compound motion of the earth in its orbit, and of the rays of light.

PROB. XXII.

To find the orbit of a comet. 59.

This is the moſt troubleſome piece of work in the whole practice of Aſtronomy. For the neceſſary data for determining their orbits, cannot be had here, as may for determining thoſe of the planets. The reaſon is, that none of them is viſible

Fig. 59. ſible for its whole revolution, but only for a very ſmall part of it. And therefore a comet is very ſeldom ſeen in conjunction or oppoſition to the ſun; and ſeldom at its node, in the plane of the ecliptic. So that it is but in a ſmall part of their orbits, that one can make any obſervations upon them; and no man has the opportunity of ſeeing the ſame comet twice, as they return not till after a great number of years. Therefore a comet cannot be obſerved twice in the ſame node, nor in any one place; ſo that we are deſtitute of thoſe methods of obſerving, which we can make uſe of to find a planet's orbit.

Under theſe difficulties, we are forced to call in all the helps we can get; and after all, it cannot be done by any direct method. For it is of ſo complicated a nature, that it can only be ſolved by frequent trials. And to this end, different people have invented different methods of proceeding; and all of them aſſume an orbit at pleaſure, and where they find the obſerved places do not agree, they correct it after, by repeated trials, till with great labour, they at laſt get it to correſpond with the obſervations. And that is when the computed places agree with the obſerved ones. The method I ſhall here purſue is rather a method for proving the truth of an orbit already found out, than a method for finding one. Tho', like all the reſt, it may be corrected time after time, till it be as perfect as poſſible.

As all the planets move in ellipſes, approaching very near to circles; ſo the comets, at leaſt thoſe that return, revolve in very long, and excentric ellipſes, approaching very near to parabolas. And in that ſmall part of the orbit, in which a comet is viſible, there is no ſenſible difference. Therefore we are to ſuppoſe that any comet moves in a parabola about the ſun, placed in the focus thereof.

And therefore what we have to do is to find the magnitude and ſituation of a parabola, in reſpect of the ecliptic; ſo that a comet moving therein, may have all its computed places, the ſame as thoſe obſerved from the earth. And the ſeveral elements of a comet's orbit are theſe, Fig. 59.

1. The place of the node.
2. The inclination of the plane of its orbit.
3. The perihelion diſtance from the ſun.
4. The place of the perihelion in the comet's orbit.
5. The time of its being in the perihelion.

Before one can attempt to calculate the orbit of a comet, a ſufficient number of obſervations ought to be taken, of the places of the comet. Out of which three ſelect ones muſt be taken, ſuitable to the purpoſe; by help of which this buſineſs is to be effected. Therefore,

1. As ſoon as ever the comet is ſeen to appear, begin to make your obſervations. And every night about the ſame time which muſt be exactly noted, obſerve the place of the comet, among the fixt ſtars, by taking its diſtance from two known ſtars, and continue to do this as long as the comet appears; for notwithſtanding all your care, there will happen many cloudy and rainy nights, when no obſervations can be made; and therefore no opportunities ſhould be miſſed.

2. At the times you are obſerving the places of the comet, take its apparent diameter, if it can be done. By this means you may form a judgment of the diſtance of it at different times; and particularly, you'll know from its greateſt and leaſt diameters, when it is neareſt to, or fartheſt from the earth.

3. Obſerve alſo its degree of motion, and its brightneſs, and the length of its tail. For when you find it moves ſwifteſt or appears brighteſt, you

Fig. 59. may judge it to be about the perihelion; tho' the tail will grow longer for a while after it is paſt the perihelion.

4. With all the diligence imaginable, obſerve the place of the comet when it has no latitude, and the exact time when that happens, for then it is in the node. But if that time cannot be had exactly, make ſeveral obſervations as near as you can to the time, both before and after. By help of which obſervations, you may find the exact time and place by the method of interpolation, deſcribed in Prop. VII. of the differential method.

5. After you have obſerved the place of the comet among the fixt ſtars, you muſt calculate its longitude and latitude, at that time. And the ſame muſt be done for all the obſervations. Then all the latitudes and longitudes, muſt be ſet down regularly in a table, with the times of their happening reduced to mean time.

6. The place of a comet among the fixt ſtars may eaſily be found at any time by extending a thread thro' the comet and ſome two known ſtars; which fall in a right line with it; and then extending another thread croſs of the firſt thro' two other ſtars running in a line. For the point of interſection of the threads, being the place of the comet, becomes known, and the longitude and latitude thereof is eaſily found.

Theſe things being laid down, we muſt proceed to the calculation. It is evident both by obſervations, and the laws of centripetal force, that the planets, and alſo the comets, deſcribe areas proportional to the times in their reſpective orbits.

7. Therefore ſuppoſe NAC to be the parabolic orbit of a comet, PD its axis, S the focus where the ſun is, and P the perihelion. And let A, B, C, be three obſerved places of the comet in its orbit, taken at a good diſtance from one another.

Fig. 59.

Let RGI be the earth's orbit; and let the earth be at G, H, and I, when the comet is at A, B, and C. Put V = time between the firſt and ſecond obſervation A and B, and W the time between the ſecond and third B and C. From the focus S, draw SA, SB, SC. And from A, B, C, let fall the perpendiculars A*a*, B*b*, C*c*, upon the plane of the ecliptic; and *cba*N is the projection of the orbit CBAN, upon the ecliptic. Let 1 = mean diſtance of the earth from the ſun, d = parameter of the earth's orbit, r = latus rectum of the parabola CAN, or SP = $\frac{1}{4}r$, SA = x, SB = y, SC = z. S and t = the areas deſcribed by the earth in the times V and W, which are known.

8. Then (by Cor. 2. Prop. XVI. centrip. forces) it will be,

$\sqrt{d} : \sqrt{r} :: S : s\sqrt{\frac{r}{d}}$ = area deſcribed by the comet in the time V. And,

$\sqrt{d} : \sqrt{r} :: t : t\sqrt{\frac{r}{d}}$ = area deſcribed by the comet in the time W.

$$\text{But the area ASP} = \frac{r+2x}{12}\sqrt{rx-\tfrac{1}{4}rr},$$

$$\text{and area BSP} = \frac{r+2y}{12}\sqrt{ry-\tfrac{1}{4}rr},$$

$$\text{and area CSP} = \frac{r+2z}{12}\sqrt{rz-\tfrac{1}{4}rr}.$$

Whence (1) the area BSA =

$$\frac{r+2y}{12}\sqrt{ry-\tfrac{1}{4}rr}-\frac{r+2x}{12}\sqrt{rx-\tfrac{1}{4}rr}=s\sqrt{\frac{r}{d}}.$$

(2) area CSB =

$$\frac{r+2z}{12}\sqrt{rz-\tfrac{1}{4}rr}-\frac{r+2y}{12}\sqrt{ry-\tfrac{1}{4}rr}=t\sqrt{\frac{r}{d}}.$$

Fig. (See my book of Fluxions, Ex. 8. Prob. X. Sect.
59. II. for theſe areas).

Again, in any parabola PABC (fig. 61.), if an ordinate CD be drawn to the axis PD, thro' any of the points as C, and CS be drawn to the focus S. Then (Cor. 2. Pr. I. B. III. Conic Sections) DS + 2SP = CS, or DS = CS − $\frac{1}{2}r$. But by trigonometry CS : rad (1) : : DS : $\frac{DS}{CS}$ or $\frac{CS - \frac{1}{2}r}{CS}$ = coſ. CSD. Thus we ſhall have coſ. ASD = 1 − $\frac{r}{2x}$ or $\frac{r}{2x}$ − 1, coſ. BSD = 1 − $\frac{r}{2y}$, coſ. CSD = 1 − $\frac{r}{2z}$.

9. Then to find the place of the node, and the inclination of the orbit, near the truth. Take the two obſervations neareſt the node, as A and B, And transferring them to a ſeparate figure (as fig. 60.) let *ban* be the ecliptic, BAN the comet's orbit. Draw the great circle *b*A. Then in the right angled ſpherical triangle *ba*A, right angled at *a*, we have *ba*, the difference of longitude of B and A, and *a*A the latitude at A; to find the ſide *b*A, and angles *ab*A and *b*A*a*. Whence the angle B*b*A is known, B*ba* being a right angle. Then in the ſpherical triangle *b*BA, we have the ſides *b*B, the latitude at B, and *b*A lately found, and angle B*b*A; to find the angle BA*b*. Then *b*AB + *b*A*a* taken from 180 degrees, leaves *a*AN. Therefore in the right angled triangle A*a*N, having the angle *a*AN, now found; and *a*A the latitude at A; to find *a*N, the diſtance of the node from *a*; and the angle AN*a* the inclination of the orbit. Theſe are near enough the truth, to begin the computation with.

10. Now if we had any two of the quantities *r*, *x*, *y*, *z*, the two equations in Art. 8. would find

the reſt; and the earth being at G when the comet Fig. is at A; draw S*a*, and draw *af* perp. to SN, alſo 59. draw A*f*. Firſt then, from the quick or ſlow motion of the planet at P, you muſt judge of the perihelion diſtance SP, and alſo of its diſtance from the node. Therefore ſuppoſe SP, SA, and the angle PSN to be given. Then we have $\frac{r}{2x} - 1$ the coſine of the angle ASP, therefore that angle is given, to which add the angle PSN; and then the angle ASN is known. Therefore in the plain triangle AS*f* right angled at *f*, we have the angle AS*f*, and ſide AS; to find A*f*, and S*f*. Then in the triangle A*af*, right angled at *a*, we have A*f* now found, and angle A*fa*, the inclination of the orbit; to find A*a*, and *af*. Again in the right angled triangle S*fa*, there are given S*f*, *fa*; to find S*a*, and angle *f*S*a*, and its ſupplement RS*a* will be known. But having the place of the earth in its orbit at G, we have the angle RSG to which add RS*a*, and we have the ſum GS*a*. Therefore in the triangle GS*a*, we have GS the earth's diſtance from the ſun, S*a* found before, and the angle GS*a*; to find G*a*, and angle SG*a*, the longitude of the comet from the ſun at the firſt obſervation. Laſtly, in the right angled triangle G*a*A, we have G*a* and A*a*, to find the angle AG*a*, the latitude of the comet at the firſt obſervation. If theſe agree with the obſerved longitude and latitude, it is very well; if not, we muſt begin the computation anew, by aſſuming ſome other values for *r*, *x*, and angle PSN; which we muſt do by conſidering whether they muſt be bigger or leſſer, to cauſe the alterations required.

11. To proceed then, having aſſumed nearer values for the lines *r*, *x*, and angle PSN; the computation muſt be repeated. But in order to eſtimate the values of *r*, and *x*; find the place of the

earth

Fig. 59. earth at E, when the comet was obſerved to be in the node at N. At that time the elongation from the ſun, or the angle SEN is known; ſo that if we can judge nearly the poſition of SN, its length will be nearly known. Thus r, x, and angle PSN being again aſſumed, the computation muſt be repeated as before, thro' all the triangles, AS*f*, A*af* S*fa*, GS*a*, and G*a*A; to find A*f*, S*f*; A*a*, *af*; S*a*, and laſtly G*a*; and from thence the angles SG*a*, AG*a*; which angles muſt be compared with the obſerved longitude and latitude of the comet at A. When thus after ſeveral computations you come near the truth, you may find out the errors at laſt, by the rule of falſe.

12. The values of r and x being had, the value of y is found by the 1ſt equation, Art. 8. And then we have $1 - \frac{r}{2y}$ the coſine of BSD, whence BSR is known. Then the computation muſt be carried on for the point B, as it was before, for the point A; by calculating in the triangles BS*d*, B*b*D, S*db*, HS*b*, H*b*B; to find B*d*, S*d*, B*b*, *bd*, S*b*, H*b*, and the angles SHB the comet's longitude from the ſun, and BH*b* the latitude; which are to agree with the obſerved longitude and latitude at B. But if they do not agree, either r and x muſt be altered in a given ratio, or the angle PSN muſt be taken different; and the whole computation muſt be gone over again ſeveral times, if there is occaſion; till at laſt the obſerved and computed longitudes and latitudes are found to agree.

13. Having thus got the value of y, the value of z will be found from the ſecond equation, Art. 8. whence is had $1 - \frac{r}{2z}$ the coſine of the angle CSD, whence is had CSR. Then the ſame computations muſt be applied to all the triangles in the

figure C*cls*, as was before to thoſe in the figures Fig. B*bd*S, and A*af*S; till at laſt, you get the angles 59. SI*c*, the comet's longitude from the ſun; and *c*IC the comet's latitude at ſea; which muſt agree with theſe obſerved, or elſe the whole computation muſt be repeated, till they do agree.

14. If one of the obſervations be taken when the comet is in the node at N; it will very much facilitate the computation. For at that time, it will have no latitude; and the longitude or elongation from the ſun is the ſame as the obſerved diſtance from him (SEN) at that time. Alſo coſ. NSP or NSD $= 1 - \frac{r}{2SN}$; and area NSP $= \frac{r + 2SN}{12} \sqrt{r \times SN - \frac{1}{4}rr}$, which is proportional to the time of its deſcription.

And moreover, if the comet be in the perihelion at another of the obſervations, the computation will be eaſier ſtill. Therefore try to get theſe two places by Art. 3. and 4. and the times correſponding. All which may be had near the matter, from ſeveral obſervations, by the method of interpolation.

For let N, A, B, be the three obſerved places of the comet from the earth at G, H, I; N the node, A the perihelion. Let fall A*a*, B*b* perp. to the plane of the ecliptic. And compleating the figures SA*af*, and SB*bd*, as in Art. 7. And putting SN $= x$, SB $= y$, SA $= p$, then we ſhall have as in Art. 8. area ASN $= \frac{2p + x}{3} \sqrt{px - pp} = s\sqrt{\frac{r}{d}}$, and area ASB $= \frac{2p + y}{3} \sqrt{py - pp} = t\sqrt{\frac{r}{d}}$, and coſ. ASN $= \frac{2p}{x} - 1$, and coſ. ASB $= \frac{2p}{y} - 1$. Aſſume the angle SNG or ſide SN, ſuppoſe the angle SNG; and SGN being known by

Fig. 59. by obſervation, and the ſide GS; then SN, and the angle GSN will be known. Then putting for SN or x, its value in the equation $\frac{2p+x}{3}\sqrt{px-pp} = S\sqrt{\frac{r}{d}} = S\sqrt{\frac{4p}{d}}$, and the value of p will be had from this equation. From this value of p or SA, compute the value of y in the equation $\frac{2p+y}{3}\sqrt{py-pp} = t\sqrt{\frac{4p}{d}}$; then having p, x, and y, we ſhall have the angles ASN and ASB, by having their coſines. Then in the figure A*a*S*f*, aſſuming the inclination of the orbit A*fa*, you muſt compute all the lines A*f*, S*f*, A*a*, *af*, S*a*, and H*a*, as in Art. 10. And from thence the angles SH*a*, AH*a*, the longitude and latitude of the comet; which are to be the ſame with the obſerved longitude and latitude; but if they are not, the inclination A*fa* muſt be altered till they do agree. Then in like manner in the figure B*bd*S, all the lines B*d*, S*d*, B*b*, *bd*, S*b*, and I*b* muſt be computed as in Art. 12. and then the angles SI*b* and BI*b*, the longitude and latitude of the comet. And theſe muſt alſo agree with the obſervations at I, but if they do not, the computation muſt be performed over again from the beginning, by altering the value of the angle SNG. For the values of the two angles SNG and A*fa* or B*db*, muſt be of ſuch determined magnitudes, that the computed longitudes and latitudes at A and B, may be the ſame as the obſerved ones. And the operations muſt be repeated, by altering their values a little, time after time, till at laſt they prove ſo, and then the orbit is known. And this is the eaſieſt way it can be done. But it may happen that the perihelion is not rightly taken; and then it muſt be altered a little, in the courſe of the computation, before all things will agree. It may not be amiſs at the firſt trial, to ſuppoſe the comet's orbit perpendicular to Fig. the ecliptic; for then the error will ſooneſt be made 59. to appear, and is ſoon corrected.

Cor. 1. *If* P *be the perihelion,* S *the ſun in the focus,* $p =$ SP *the perihelion diſtance,* B *any place of the comet, its diſtance* SB $= y$, *and* PSD *the axis of the parabola. Then the coſine of the angle* BSD $= 1 - \frac{2p}{y}$; *or the coſ.* PSB $= \frac{2p}{y} - 1$.

For (by Art. 8.) coſ. BSD $= 1 - \frac{r}{2y} = 1 - \frac{4p}{2y} = 1 - \frac{2p}{y}$, becauſe $4p = r$.

Cor. 2. *The area* PSB $= \frac{2p + y}{3}\sqrt{py - yy}$.

For that area $= \frac{r + 2y}{12}\sqrt{ry - \frac{1}{4}rr}$ (by Art. 8.) $= \frac{4p + 2y}{12}\sqrt{4py - 4pp} = \frac{2p + y}{3}\sqrt{py - yy}$.

Cor. 3. *Hence more comets will be ſeen in the hemiſphere next the ſun than in the oppoſite one.*

For a comet cannot be ſeen till it be ſtrongly illuminated by the ſun, and therefore muſt be within a certain diſtance of him. And if a ſpherical ſurface be deſcribed at that diſtance about the ſun, and if it be cut by a plane paſſing thro' the earth; that ſegment of it, in which the ſun is, will be greater than the other; and therefore more comets will be ſeen in it. Alſo comets will be ſeen at a greater diſtance towards the ſun, where they are ſtrongly illuminated, than in the oppoſite points, where they are weakly illuminated; and therefore more of them will be ſeen towards the ſun.

SCHOLIUM.

The apparent way of a comet is eaſily traced upon a celeſtial Globe. For if the place of the comet among the fixt ſtars be obſerved every night, and marked on the globe; then a line drawn thro' all theſe places, will repreſent the path of the comet among the ſtars. And thus a great circle drawn thro' two places of the comet upon the globe, will nearly ſhew the way it has to go. And this circle being drawn till it interſects the ecliptic, will ſhew the place of the nodes. And the angle it makes with the ecliptic, will be the inclination of its orbit.

If the length of the tranſverſe axis of a comet's orbit was known, its periodic time would be known; and then the times of their return might be predicted. But as they are a long time in making their periods, ſome of them being ſeveral hundred years in revolving; and as we have no obſervations of former ages above 200 years, that can be depended on, it happens, that the return of above one or two of them cannot be predicted with any certainty. Therefore the finding out their ſeveral periods muſt be the work of future ages.

Here follows a table of the principal comets that have been hitherto duly obſerved.

A TABLE

Year of Appearance.	Aſcending Node.			Inclination of the Orbit.			Place of the Perihelion.		
	°	′	″	°	′	″	°	′	″
1337	84	21	0	32	11	0	37	59	0
1472	281	46	20	5	20	0	45	33	30
1531	49	25	0	17	56	0	301	39	0
1532	80	27	0	32	36	0	111	7	0
1556	175	42	0	32	6	30	278	50	0
1577	25	52	0	74	32	45	129	22	0
1580	18	57	20	64	40	0	109	5	50
1585	37	42	30	6	4	0	8	51	0
1590	165	30	40	29	40	40	216	54	30
1593	164	14	15	87	58	0	176	19	0
1596	312	12	30	55	12	0	228	16	0
1607	50	21	0	17	2	0	302	16	0
1618	76	1	0	37	34	0	2	14	0
1652	88	10	0	79	20	0	28	18	40
1661	82	30	30	32	35	50	115	58	40
1664	81	14	0	21	18	30	130	41	20
1665	228	2	0	76	5	0	71	54	30
1672	297	30	30	83	22	10	46	59	30
1677	236	49	10	79	3	15	137	37	5
1680	272	2	0	60	56	0	262	39	30
1682	151	16	30	17	56	0	302	52	45
1683	173	23	0	83	11	0	85	29	30
1684	268	15	0	65	48	40	238	52	0
1686	350	34	40	31	21	40	77	0	30
1698	267	44	15	11	46	0	270	51	15
1699	141	45	35	69	20	0	212	31	6
1702	189	25	15	4	30	0	138	41	3
1706	13	11	40	55	14	10	72	29	10
1707	52	46	35	88	36	0	79	54	56
1718	128	43	0	30	20	0	121	30	0
1723	14	16	0	49	59	0	42	52	20
1729	310	32	37	76	58	4	322	40	0
1737	226	22	0	18	20	45	325	55	0
1739	27	25	14	55	42	44	102	38	40
1742	185	38	29	66	59	14	217	35	13
1743	78	21	15	2	19	33	92	41	45
1744	45	46	11	47	5	18	197	10	0
1747	327	18	50	79	6	20	277	2	0
1748	232	52	15	85	27	0	215	0	50

In this Table the 1ſt column contains the Years of appearing. The 2d, the longitude of the aſcending node, reckoned in the comet's orbit. The 3d, the inclination of the orbit, to the plane of the ecliptic. The 4th, the place of the perihelion in the comet's orbit. The 5th, is the perihelion diſtance, ſuppoſing

Of COMETS.

Perihel. Diſtance.	Log. Perihelion Diſtance.	Mean time of the Perihelion.				Motion.
		M	d	h	m	
.40066	—1.609236	June	2	6	25	Retrograde
.54273	—1.734584	Feb.	28	22	23	Retrogr.
.56700	—1.753583	Aug.	24	21	18	Retrogr.
.50910	—1.706803	Oct.	19	22	12	Direct
.46390	—1.666424	Apr.	21	20	3	Direct
.18342	—1.263447	Oct.	26	18	45	Retrogr.
.59628	—1.775450	Nov.	28	15	0	Direct
1.09358	0.038850	Sept.	27	19	20	Direct
.57661	—1.760882	Jan.	29	3	45	Retrogr.
.08911	—2.949940	July	8	13	39	Direct
.51293	—1.710058	July	31	19	55	Retrogr.
.58680	—1.768490	Oct.	16	3	50	Retrogr.
.37975	—1.579498	Oct.	29	12	23	Direct
.84750	—1.928140	Nov.	2	15	40	Direct
.44851	—1.651772	Jan.	16	23	41	Direct
1.02575	0.011044	Nov.	24	11	52	Retrogr.
.10649	—1.027309	Apr.	14	5	15	Retrogr.
.69730	—1.843476	Feb.	20	8	37	Direct
.28059	—1.448072	Apr.	26	0	37	Retrogr.
.00612	—3.787106	Dec.	8	0	6	Direct
.58328	—1.765877	Sept.	4	7	39	Retrogr.
.56020	—1.748343	July	3	2	50	Retrogr.
.96015	—1.982339	May	29	10	16	Direct
.32500	—1.511883	Sept.	6	14	33	Direct
.69129	—1.839660	Oct.	8	16	57	Retrogr.
.74400	—1.871570	Jan.	3	8	23	Retrogr.
.64590	—1.810165	Mar.	2	14	13	Direct
.42581	—1.629218	Jan.	19	4	23	Direct
.85974	—1.934368	Nov.	30	23	30	Direct
1.02654	0.011380	Jan.	3	23	39	Retrogr.
.99865	—1.999414	Sept.	16	16	11	Retrogr.
4.2614	0.629552	June	14	10	57	Direct
.22282	—1.347960	Jan.	19	8	21	Direct
.67358	—1.828388	June	6	10	0	Retrogr.
.76568	—1.884049	Jan.	28	4	39	Retrogr.
.83501	—1.921690	Dec.	30	20	26	Direct
.22249	—1.347325	Feb.	19	8	4	Direct
2.1985	0.342128	Feb.	20	7	11	Retrogr.
.84067	—1.924624	Apr.	17	19	26	Retrogr.

the mean diſtance of the earth to be 1. The 6th, the logarithm of the perihelion diſtance. The 7th, the mean time, at London, of the comet's being in the perihelion. The 8th, the motion of the comet, whether direct or retrograde.

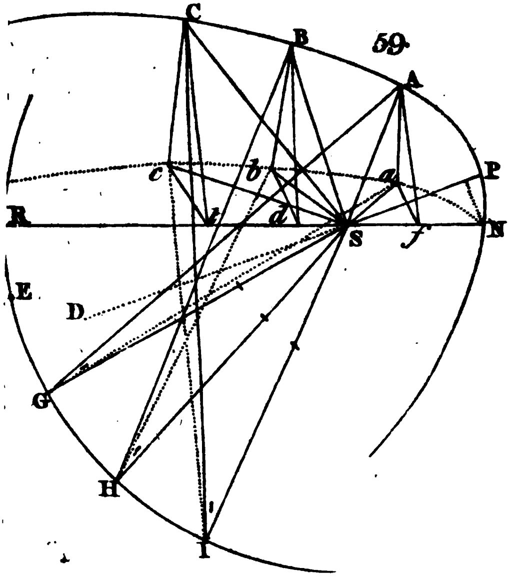
Fig.

SECT. V.

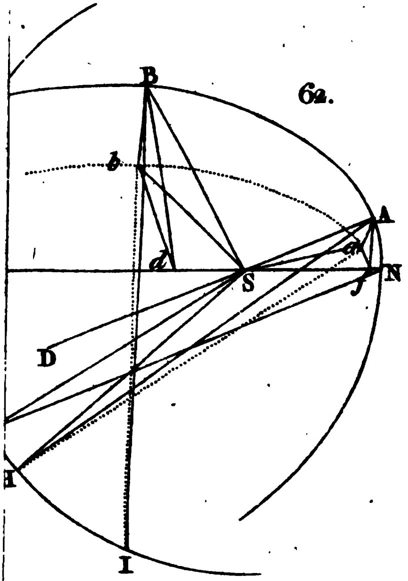
The Theory of the Primary Planets and Comets. The Calculation of their Places. The Equation of Time. The Times when the Planets are ſtationary; when they tranſit the Sun; when in Conjunction. Their Quantities of Matter, Diameters, Denſities, Figures, Weight on their Surfaces; and their Actions upon one another.

HAVING in the laſt Sect. determined the orbits of the planets, and all things relating thereto, viz. their periodic times, the tranſverſe axis and excentricity of their orbits, the inclinations of their orbits, the places of the node and aphelion, and the time of paſſing thro' the aphelion. In what follows, we muſt ſuppoſe all theſe things known, which are the neceſſary data to proceed upon, in calculating the places of the planets in their ſeveral orbits, for any given time. And in order to lay down proper rules for this purpoſe, we muſt have recourſe to the laws of Centripetal Force, before laid down, on which all their motions depend. Now it is evident from the moſt accurate obſervations, 1. That all the planets, amongſt which is the earth, move in elliptic orbits about the ſun, placed in the focus thereof. 2. That each planet in its own orbit, deſcribes areas about the ſun, proportional to the times of deſcription, included between

59.



62.



Pl. VIII. *pa*. 194.

UNIV OF

tween right lines drawn from the ſun to the planet. Fig.
3. That in different planets, the ſquares of the periodical times are as the cubes of the tranverſe axes of the orbits; or the cubes of the mean diſtances of the planets, from the ſun. From whence it follows, by the laws of Centripetal Force, that their forces of gravitation towards the ſun, by which they are retained in their ſeveral orbits, are at different diſtances, reciprocally as the ſquares of theſe diſtances. And this is the grand law by which all the bodies in the univerſe act upon one another. And therefore whatever has been demonſtrated, in the doctrine of centripetal forces, upon the ſuppoſition of this law, muſt be taken for granted, in all aſtronomical theories or calculations whatever.

DEFINITIONS.

DEF. I.

The *true anomaly* of a planet is the angle ASN 64.
at the ſun S, comprehended between the aphelion A, and the planet at N in its orbit. Or from the perihelion, for a comet.

DEF. II.

The *mean anomaly*, is an angle ASG, taken proportional to the time of the planets moving from the aphelion A to the place N in its orbit. An arch of a circle, or the area ANS of the ellipſis, may alſo be taken for the mean anomaly.

This angle ASG increaſes uniformly with the time, or with the area ANS. And therefore the motion of SG round the center S, is called the *mean motion.*

DEF. III.

The equation is the angle NSG, or the difference between the true and mean anomaly. This alſo is called the *Proſthaphæreſis.*

Fig. 64.

DEF. IV.

Heliocentric place, *heliocentric longitude*, and *heliocentric latitude* of a planet, is the place of a planet, or its longitude, and latitude, as ſeen from the ſun.

DEF. V.

Geocentric place, *geocentric longitude*, and *geocentric latitude* of a planet, is the place of a planet, or its longitude, and latitude, as it appears from the earth.

DEF. VI.

65. If S be the ſun, E the earth, P the planet, and PB perp. to the ecliptic. Then in the triangle EPS, or EBS reduced to the ecliptic; the angle of *commutation* is the angle at the ſun ESP or ESB, contained between the earth and the planet. The angle of *elongation* is the angle at the earth PES or BES, contained between the ſun and the planet. And the *parallax of the orbit* is the angle at the planet, or at its place in the ecliptic, EPS or EBS, contained between the ſun and earth.

Here AE is the earth's orbit, NPC the planet's orbit, NB its orbit reduced to the ecliptic, HSN the line of the nodes, N the aſcending node.

DEF. VII.

The *Curtate Diſtance* of a planet from the ſun, or the earth, is the line SB or EB, drawn from the ſun, or the earth, to the point where the perpendicular from the planet, cuts the ecliptic. This is the diſtance of the planet from the ſun, or the earth, reduced to the ecliptic.

DEF. VIII.

The *Argument of latitude*, is the angle at the ſun NSP, between the planet and its aſcending node, reckoned in the planet's orbit, NPC.

DEF.

Fig. 65.

DEF. IX.

The *Radix*, is the place of a planet at ſome remarkable point of time, as the birth of Chriſt, &c. From this point, all the motions are calculated as from a fixt foundation.

DEF. X.

Apparent or ſolar time, is the time as it is ſhewn by the ſun, and proceeds ſometimes faſter and ſometimes ſlower.

DEF. XI.

Mean or aſtronomical time, is the time that goes uniformly and equably forward, without any acceleration or retardation. This is called *True Time.*

DEF. XII.

The *Equation of time*, is the difference between the mean and the apparent time.

DEF. XIII.

Aſtronomical day or year, is that uſed by aſtronomers. The day begins at the noon of the preceding day, and ends at the noon of the following day; and the hours are reckoned from 1 to 24. And the year begins and ends on the laſt day of December at noon. Aſtronomical time is 12 hours before the vulgar time. And theſe beginnings will be different, in different places, according to the difference of longitude.

PROP. I. *Prob.*

Having given either the true anomaly, or the diſtance of a planet from the ſun; to find the other, and the time it will be there, as alſo the mean anomaly. 66.

1 *Way.*

The periodical time, and the orbit of the planet, are ſuppoſed to be known, as alſo the time of

Fig. 66. its being in the perihelion. Let PNAB be the orbit, S the ſun, C the center, P the perihelion, A the aphelion; and put ſemitranſverſe CP = a, ſemiconjugate CB = c, excentricity CS = n, the diſtance of the planet SN = y, its periodic time = t, coſ. angle PSN = z. Then (Cor. 3. Prop. LXXII. B. I. Conic Sections) we have SN or $y = \frac{CB^2}{CA + z \times CS}$, or $y = \frac{cc}{a + zn}$. And (by Ex. 7. Prob. X. Sect. II. Fluxions) we have the area PSN $= \frac{.017453ca\varphi}{2} - \frac{c}{2}\sqrt{-cc + 2ay - yy}$, where φ = degrees in the arch whoſe coſine is $\frac{a - y}{n}$. But (Prop. XI. Centripetal Forces) the area is as the time; and the area of the whole ellipſis is = $3.1416ca = 180ca \times .01745$; then it will be, as whole area : t : : area PSN : time from the perihelion $= \frac{.017453ca\varphi t}{2 \times 180ca \times .01745} - \frac{ct\sqrt{-cc + 2ay - yy}}{2ac \times 3.1416}$, or the time of deſcribing SPN $= \frac{Dt}{360} - \frac{t\sqrt{-cc + 2ay - yy}}{2a \times 3.1416}$, putting D (inſtead of φ) for the number of degrees in the arch whoſe coſine is $\frac{a - y}{n}$ and radius 1. Therefore if y be given, the laſt equation gives the time when the planet arrives at N; and ſince $y = \frac{cc}{a + zn}$; therefore $ay + zny = cc$, and $z = \frac{cc - ay}{ny}$; therefore the angle PSN, or the true anomaly is given. Or if z or the true anomaly be given, we have y, and the time of arriving at n.

And if half the periodic time be added to, or ſubtracted from, this time; we ſhall have the time Fig. of its moving from the aphelion. 66.

Again, for the mean anomaly, as the whole area, to 360 degrees; ſo any part of the area PSN, to the mean anomaly. That is, 3.1416ca : 360 : : $\frac{.017453ca\varphi}{2} - \frac{c}{2}\sqrt{-cc+2ay-yy} : \frac{.017453\varphi}{2\times 3.1416} \times$

$$360 - \frac{360\sqrt{-cc+2ay-yy}}{2\times 3.1416a} =$$

$$\varphi - \frac{\sqrt{-cc+2ay-yy}}{.01745\varphi}$$ becauſe 180 × .01745

$$= 3.1416 = \varphi - \frac{57.29578}{a}\sqrt{-cc+2ay-yy}.$$

Therefore the mean anomaly is,

$$D - \frac{57.29578\sqrt{nn-\overline{a-y}^2}}{a},$$ reckoning from the perihelion.

Cor. 1. *The coſine of* D *is* $\frac{n+za}{a+zn}$.

For Coſ. $D = \frac{a-y}{n} = \frac{a-\frac{cc}{a+zn}}{n} =$

$$\frac{aa+2an-cc}{n\times\overline{a+zn}} = \frac{nn+zan}{n\times\overline{a+zn}} = \frac{n+za}{a+zn}.$$

Cor. 2. D *is the number of degrees in the arch whoſe ſine is* $\frac{\sqrt{-cc+2ay-yy}}{n}$.

For that ſine is $\sqrt{1-\frac{\overline{a-y}^2}{nn}} =$

$$\sqrt{\frac{nn-aa+2ay-yy}{nn}} = \frac{\sqrt{-cc+2ay-yy}}{n}.$$

Cor. 3. *If the planet in moving from the perihelion, be paſt the aphelion; then the equation before given,* Fig. 60. *given, ſhews the time in which it will come to the perihelion again.*

For the angles, and diſtances, are alike on both ſides the axis AP; and therefore the time is the ſame. That is, the time in nP is the ſame as in PN.

Cor. 4. *Hence a table may eaſily be made for finding the time when a planet will be in any place of its orbit.*

Take the angle PSN ſucceſſively equal to 1, 2, 3, 4, &c. degrees, to 180; then its coſine z is had, and from thence y, and laſtly the time when it will be at the diſtance y. The table being thus conſtructed to 180 degrees, will ſhew the time and the diſtance for any given true anomaly; and this by a direct method. And *vice verſa*, having the time given, you may by the ſame table find the diſtance, and the true anomaly, correſponding thereto in their ſeveral columns; taking a proportional part as there is occaſion.

But if you would rather chuſe to have a table where the time increaſes uniformly, you may eaſily transform the preſent table, into ſuch a one, by finding the angle PSN, and diſtance SN, from the time in this table; which is eaſily done by taking parts proportional. And thus a table may be compoſed, where any one of the quantities may be made to increaſe uniformly.

Example.

Suppoſe PNA be the earth's orbit, and the ſun's true anomaly ASN $= 60°$, and PSN $= 120°$; here $a = 1$, $c = .99857$, $n = .01691$, $z = -\frac{1}{2}$, then $y = 1.00824$, and $\frac{a-y}{n} = -.48725 =$ Coſ. 119.16, and D $= 119.16$, and $\frac{D}{360} = .331$, and $\frac{\sqrt{-cc+2ay-yy}}{2a \times 3.1416} = .00235$; therefore

.331

$.331 - .00235 \times t$ or $.32866 \times 365^{d}\ 6^{h}\ 9'\ 14''\frac{1}{2}$ Fig.
$= 120^{d}\ 0^{h}\ 59'\ 41''$ from the perihelion; or it wants 66.
$62^{d}\ 14^{h}\ 4'\ 56''$ of the aphelion. But in 1767, the
ſun is in the aphelion June $30^{d}\ 10^{h}\ 30^{m}\ 36^{s}$
ſubtract it from 60 14 4 56

gives April 30 20 . 25 40 the mean time required. But it is better to have the hours, minutes, ſeconds of time in decimals of days; and the minutes and ſeconds of degrees, in decimals of degrees.

Alſo the mean anomaly is $D - 57.295 \times .01477 = 119.16 - .84 = 118.32 = 118^{\circ} : 19'$.

2 *Way*.

Suppoſe the time to be reckoned from the aphelion A. About the tranſverſe AP, deſcribe the ſemicircle AQP, and thro' the planet's place at N, draw QNH perp. to AP. Put $CA = a$, $CB = c$, $CS = n$, $SN = y$, ſine of the true anomaly $S.ASN = s$, coſ. $ASN = z$, $t =$ periodic time. Then (Cor. 3. Prop. LXXII. B. I. Conic Sections) 67.

$y = \frac{cc}{a - zn}$. And Rad (1) : S.HSN (s) : : SN (y) : $NH = Sy$. And (Prop. XIX. ib.) CB (c) : CA (a) : : NH (sy) : $QH = \frac{asy}{c}$. Let $d =$ number of degrees in the arch (AQ) whoſe ſine is $\frac{sy}{c}\left(\frac{QH}{a}\right)$, to the radius 1; then $AQ = a \times .017453d$. But $AQS = \frac{1}{2} AQ \times AC + \frac{1}{2} CS \times QH = \frac{1}{2} a \times .017453ad + \frac{1}{2} n \times \frac{asy}{c} = \frac{.017453aad}{2} + \frac{nasy}{2c}$.

Alſo the area of the ſemicircle $AQP = \frac{3.1416aa}{2}$; and of the whole circle, $3.1416aa$; and of the ellipſis AQPB, $3.1416ca$. But $a : c : :$ circle : ellipſis

Fig. 67. lipſis : : AQS : ANS = $\frac{.017453cad}{2} + \frac{nsy}{2}$. And ellipſis $(3.1416ca)$: ANS $\left(\frac{.017453cad}{2} + \frac{nsy}{2}\right)$: : t : time of deſcribing ANS = $\frac{.0174533dt}{2 \times 3.1416} + \frac{nsyt}{2 \times 3.1416ca} = \frac{dt}{360} - \frac{nt}{2a \times 3.1416} \times \frac{sy}{c}$.

Therefore having given any angle ASN for the true anomaly; z and s will be known, and conſequently y, and from thence d; and therefore the time of deſcribing ANS will be found.

And for the mean anomaly; as the area of the ellipſis, to the area of ANS; ſo 360 degrees, to the mean anomaly. That is, $3.1416ca : \frac{.01745cad}{2} + \frac{nsy}{2} : : 360 : \frac{.01745d}{2 \times 3.1416} \times 360 + \frac{nsy}{2ca \times 3.1416} \times 360$. Therefore the mean anomaly is $= d + \frac{57.29578n}{a} \times \frac{sy}{c}$, degrees.

Cor. 1. *The ſine of d is* $\frac{sc}{a - zn}$.

For it is $\frac{sy}{c} = \frac{s}{c} \times \frac{cc}{a - zn} = \frac{sc}{a - zn}$.

Cor. 2. *The mean anomaly is = degrees in the arch* AQ + $\frac{n}{a}$QH, or AL, making QL = $\frac{n}{a}$QH.

Example.

In the earth's orbit, we have $a = 1$, $n = .01691$, $c = .99857$, and let the true anomaly ASN = 60°; then $s = \sqrt{\frac{3}{4}}$, $z = \frac{1}{2}$, whence $y = 1.0082$, and $\frac{sy}{c} = .87324$, whence $d = 60.84$. Then $\frac{d}{360} = .1690$,

.1690, and $\frac{n}{2a \times 3.1416} \times \frac{sy}{c} = .00235$; and Fig. 67. .1690 + .00235 = .17135, and $.17135 \times t = .17135 \times 365^{d}\ 6^{h}\ 9^{m}\ 14^{s}\tfrac{1}{2} = .17135 \times 365.256 = 62.5865 = 62^{d}\ 14^{h}\ 4^{m}\ 47^{s}$; which happens on Sept. $1^{d}\ 0^{h}\ 35^{m}\ 23^{s}$; 1767.

And for the mean anomaly; $d = 60.84$, and $\frac{57.29n}{a} \times \frac{sy}{c} = .846$, and $60.84 + .846 = 61.686 = 61^{\circ}\ 41'\ 9''$.

In the uſe of theſe rules, it will be beſt to work by logarithms; for many of the quantities remain conſtantly the ſame, in the ſame orbit, and create little trouble.

SCHOLIUM.

By either of the foregoing rules, the mean anomaly is found directly from the true anomaly, accurately true. But the true cannot be found from the mean anomaly, for there is nothing given to find it by; nor any method, but by gueſſing, and the methods of approximating; and therefore it is unnatural to ſeek the true from the mean. Hence, tables of the mean anomaly ought firſt to be calculated from the true; and theſe tables may afterwards be put into a different form, if any body thinks proper to do it.

But if any one ſhould think fit to calculate the true from the mean anomaly being given; it is evident that he muſt ſtill work the ſame way as if the true was given. For he muſt aſſume the true anomaly by ſuppoſition, as near as he can, and from thence calculate the mean, by the rule as it lies before him. And if it does not anſwer to that given; he muſt try again, by making a nearer ſuppoſition. And at laſt by help of the errors, he will find the true one ſought.

PROP. II.

68. *If* N *be the place of a planet in its orbit*, S *the ſun in the focus; then the planet's angular motion about the other focus* F : *is to its mean motion* : : *as the rectangle* ACE *of the ſemiaxes* : *is to the rectangle* FNS *of its diſtances from the foci.*

About the focus F, with the radius CA, deſcribe the ſemicircle VEI. Let N*n* be a very ſmall part of the orbit, and thro' N, *n*, draw FNO, F*no*; and from *n* let fall perpendiculars upon FN, SN, which will be equal to one another, becauſe the angles ON*n* and SN*n* are equal (by Prop. X. B. I. Conic Sections); call this perpendicular, $\dot{x}$. With the ſame radius CA and center S, deſcribe the circle EDK, and draw SN, S*n*, cutting the circle in D, and *d*. Let VM be the mean anomaly when the planet is at N; and V*m*, when it is at *n*. For the mean motion may be reckoned about any center F, C, or S.

Let AC $= a$, CE $= c$, CF $= n$, SN $= y$, FN $= v$. Then if N*n* repreſent the velocity of the planet in its orbit, O*o* will be its velocity reduced to the circle VEI, or its angular motion about F; and D*d* its angular motion about S; and M*m* its mean motion about F. Alſo let arch VM $= M$, VO $= F$, KD $= \dot{S}$, $p = 3.1416$; then $pa =$ VEI, $\frac{pac}{2} =$ area of the ſemiellipſis AEP, and $\frac{y\dot{x}}{2} =$ area NS*n*.

Since the mean motion is as the area deſcribed in the ſame time; we ſhall have $\frac{pac}{2}$ (area AEP) : $\frac{y\dot{x}}{2}$ (area NS*n*) : : pa (VMI) : M*m* $= \frac{y\dot{x}}{c} = \dot{M}$. And by ſimilar ſectors, y (SM) : $\dot{x}$: : a (SD) : D*d* $=$

$a\dot{x}$

Fig. 68.

$\frac{a\dot{x}}{y} = \dot{S}$. And v (FN) : $\dot{x}$: : a (FO) : Oo = $\frac{a\dot{x}}{v} = \dot{F}$. Therefore $\dot{S}$, $\dot{F}$, and $\dot{M}$ are to one another as $\frac{a\dot{x}}{y}$, $\frac{a\dot{x}}{v}$ and $\frac{y\dot{x}}{c}$. And $\dot{M} = \frac{y}{c} \times \dot{x} = \frac{y}{c} \times \frac{y\dot{S}}{a} = \frac{yy}{ca}\dot{S}$, and alſo $\dot{M} = \frac{y}{c} \times \dot{x} = \frac{y}{c} \times \frac{v}{a}\dot{F} = \frac{vy}{ca}\dot{F}$. But the velocities are as the fluxions of the ſpaces, whence $\dot{F}$ (Oo) : $\dot{M}$ (Mm) : : $ca : vy$.

Cor. 1. *The angular motion round* F *is equal to the mean motion, where the rectangle* SNF *is equal to the rectangle* ACE. *And then there is the greateſt equation between* F *and* M.

For then $\dot{M} = \frac{vy}{ca}\dot{F} = \dot{F}$.

Cor. 2. *From the aphelion, where* ca *is greater than* vy, *the motion round* F *is greater than the mean motion; and when* $vy = ca$, *they become equal; afterwards when* vy *is greater than* ca, *the motion round* F *is ſlower than the mean motion; and the angles* (F *and* M) *draw towards an equality, then* M *leads till they meet again at* P.

For the motion of FM from A is ſlower than that of FO. And it loſes ground, till ſuch time as $vy = ca$, when their motions are equal; afterwards FM moves faſter than FO, and comes nearer and nearer to it, till they coincide, where FM takes the lead.

Cor. 3. *Univerſally, the angular motions at* S *and at* F, *and the mean motion; are as* $\frac{1}{SN^2}$, $\frac{1}{SNF}$ *and* $\frac{1}{ACE}$; *the planet being in any place as* N.

For

Fig. 68. For $\dot{S}$, $\dot{F}$ and $\dot{M}$ are as $\frac{a\dot{x}}{y}$, $\frac{a\dot{x}}{v}$ and $\frac{y\dot{x}}{c}$, or as $\frac{ay\dot{x}}{yy}$, $\frac{ay\dot{x}}{vy}$ and $\frac{ay\dot{x}}{ac}$. But $y\dot{x}$ is a given part of the area, therefore we have $\dot{S}$, $\dot{F}$ and $\dot{M}$, as $\frac{a}{yy}$, $\frac{a}{vy}$, and $\frac{a}{ca}$; that is as $\frac{1}{yy}$, $\frac{1}{vy}$ and $\frac{1}{ca}$.

Cor. 4. *The angular motion of a planet round the higher focus* F, *is greater than the mean motion, near the apſides* A *and* P; *and leſs than the mean motion at and about* E *the mean diſtance; and is the ſame at equal diſtances from the apſes* A *and* P.

For near A and P, the rectangle vy is leſſer than ca, and near E it is greater than ca; and is equal to it when SN or $y = a \pm \sqrt{aa - ca}$. And ſince the rectangle vy or SNF is the ſame at equal diſtances from E, or from A and P; therefore in two ſuch places the angular velocities round F, are equal.

Cor. 5. *The motion of the planet about the ſun at* S, *is equal to the mean motion, when* $SN^2 = AC \times CE$.

For $\dot{M} = \frac{yy}{ca}\dot{S}$, and when $yy = ca$, $\dot{M} = \frac{ca}{ca}\dot{S} = \dot{S}$.

Cor. 6. *The motion round* F *is equal to the motion round* S, *when* SN = NF.

For $\dot{S} : \dot{F} :: \frac{1}{yy} : \frac{1}{vy} :: vy : yy :: v : y$; therefore when $v = y$, $\dot{S} = \dot{F}$.

Example.

69. Suppoſe AEPB to be ſuch an orbit, that C being the center, F the higher focus, S the focus where the ſun is, AC = 10, BC = 8, CF = 6,

Fig. 69.

half the latus rectum LF = 6.4, AF = 4, PF = 16. Then $\sqrt{aa - ca} = \sqrt{20} = 4.472136$, and $y = 14.472$ or 5.528, when $ca = vy$. Therefore if you make SG and SH = 5.528, and SI and SK = 14.472. The points G, I, K, and H will be the four points of the orbit, where the motion of the planet round F, will be equal to the mean motion. And in theſe points, are the greateſt equation, of the angle F. In the arches AI, AK, PG, PH, the motion about F is greater than the mean motion, and the greater, the nearer A or P. Between I and G, and H and K, the motion about F is leſs than the mean motion, and the leſs, the nearer to E and B. But the equations at I and K are greater than the equations at G and H. For the planet ſtays longer in the arches AI, and KA, than in the arches GP and PH; and therefore the angle F gains more in the former than in the latter. And that gain is the equation, for they are together at A and P. They are alſo together at ſome place between E and G, and alſo between H and B. For F having a greater equation to loſe in paſſing thro' IG, than it has in paſſing thro' HK, the planet will be paſſed E before it be loſt. And likewiſe the equation H will be loſt before the planet arrives at B. Hence, the mean motion and angle F, are together at A; in paſſing thro' AI the mean motion is left behind; in paſſing thro' IG, the mean motion goes paſt F; and F advancing from G to P, overtakes the mean motion again at P. In like manner, thro' PH, the mean motion is behind F; but overtakes it near B, and goes paſt it; and at laſt F overtakes the mean motion again, at A.

If you make SD and SQ = $\sqrt{ac} = \sqrt{80} = 8.944$, then the planet's motion at the points D and Q, about the focus S, will be equal to the mean motion. Laſtly, at the points E and B, the motions round S and F are equal.

Scho-

Fig. 69.

SCHOLIUM.

From the irregular increaſing and decreaſing of the angle F, in the courſe of the planet's motion; it cannot be a proper medium for finding out the mean anomaly. For before this can be done, a certain equation muſt be found out to adjuſt it by. But it is far more difficult to find out this equation for F, than to find out the equation of the orbit, or that for the angle FSN, or the true anomaly; and therefore I ſet this angle aſide as uſeleſs for that purpoſe. Indeed, when the orbits are not very excentric, it will not vary a great deal; but this is by meer chance; for that variation cannot be computed without a great deal of labour, which is a certain ſign, that it has no relation to either the mean or true anomaly. And the correction given by Bullialdus only mends it in ſome places; but in others, makes it worſe; and therefore does not anſwer the purpoſe.

PROP. III.

70. *If* AEP *be the orbit of a planet,* AQP *the circumſcribing circle;* N *the place of the planet,* QNH *perpendicular to the tranſverſe* AP. *And if* QC *be drawn to the center, and* NF, NS *to the foci; then will* CF : SN — CA : : *radius* : *coſ.* ACQ, *called the excentric anomaly.*

Let CA = a, CF = n, SN = y, coſ. QCH = x. Then (by Cor. 1. Prop. V. B. I. Conic Sections) CF × CH = CA × $\overline{\mathrm{SN} - \mathrm{CA}}$, that is n × CH = $a \times \overline{y - a}$. But in the triangle QCH, rad (r) : QC (a) : : S.CQH or coſ. QCH (x) : CH = $\frac{ax}{r}$. Whence $n \times \frac{ax}{r}$ = n × CH = $a \times \overline{y - a}$, and

nx

$\frac{nx}{r} = \overline{y-a}$, or $nx = r \times \overline{y-a}$. Therefore $n : r :: y - a : x$. Fig. 69.

Cor. 1. y *or* SN $=$ CA $+$ CS $\times$ *coſ.* QCH, *radius being* 1.

For $r \times \overline{y-a} = y - a = nx$, and $y = a + nx$.

Cor. 2. $\text{SH} = \frac{\text{AC} \times \text{SN} - \text{CE}^2}{\text{CS}} = \text{CS} + x \times \text{CA}$.

For $\text{SH} = n + \text{CH} = n + \frac{ax}{r} = n + \frac{a}{r} \times \frac{r}{n} \times \overline{y-a} = n + \frac{a}{n} \times \overline{y-a} = \frac{nn + ay - aa}{n} = \frac{ay - cc}{n}$.

PROP. IV.

The aphelion diſtance AS : *ſemiconjugate* CE :: *tan.* ½ *the excentric anomaly* ACQ : *tan.* ½ *the true anomaly* ASN. 70.

Let SA $= d$, the radius $= 1$, the reſt as before. Then from the laſt Prop. we have $y = n + nx$, $\text{SH} = n + ax$. Produce SP till SO = SN, and draw ON, then the angle HON $=$ ½ HSN. And ſince $x =$ coſine of QCH, therefore (by Schol. Prop. II. B. I. Trigonometry) $\sqrt{\frac{1-x}{1+x}} =$ tan. ½QCH. And in the triangle NOH, OH : rad :: NH : tan. NOH. But $\text{OH} = y + \text{SH} = n + nx + n + ax = d + dx$; and $\text{NH} = \frac{c}{a}\text{QH} = \frac{c}{a}\sqrt{aa - \text{CH}^2} = \frac{c}{a}\sqrt{aa - aaxx} = c\sqrt{1 - xx}$. Therefore $d \times \overline{1+x} : 1 :: c\sqrt{1-xx}$: tan. NOH $=$

Fig. 70.

$$= \frac{c\sqrt{1-xx}}{d\times\overline{1+x}} = \frac{c\sqrt{1-xx}}{d\sqrt{\overline{1+x}\times\overline{1+x}}} = \frac{c}{d}\sqrt{\frac{1-x}{1+x}}$$

$= \frac{c}{d} \times$ tan. $\frac{1}{2}$QCH. Whence $d : c :: $ tan. $\frac{1}{2}$QCH : tan. NOH or $\frac{1}{2}$NSH.

Cor. 1. *As* $\sqrt{AS}$ *the aphelion diſtance* : $\sqrt{PS}$ *the perihelion diſtance* : : *tan.* $\frac{1}{2}$ACQ *the excentric anomaly* : *tan.* $\frac{1}{2}$ASN *the true anomaly.*

For (by Prop. III. Ellipſis) AS × SP = CB². And ſince $d : c ::$ tan. $\frac{1}{2}$QCH : tan. $\frac{1}{2}$NSH; that is, AS : CB : : tan. $\frac{1}{2}$QCH : tan. $\frac{1}{2}$NSH; therefore $\sqrt{AS^2} : \sqrt{CB^2}$ or $\sqrt{AS \times SP}$: : tan. $\frac{1}{2}$QCH : tan. $\frac{1}{2}$NSH : : $\sqrt{AS} : \sqrt{SP}$.

Cor. 2. *As ſin. true anomaly* ASN : *ſine excentric anomaly* ACQ : : *ſemiconjugate* CE : *diſtance* SN.

For rad (1) : QC (a) : : S.QCH : QH = $a \times$ S.QCH; and $a : c ::$ QH : NH = $c \times$ S.QCH. And in the triangle SNH, S.NSH : NH or $c \times$ S.QCH : : rad (1) : SN = $\frac{c \times \text{S.QCH}}{\text{S.NSH}}$; and S.NSH : S.QCH : : c : SN.

Cor. 3. *As ſine of half the true anomaly* ASN : *ſine of half the excentric anomaly* : : $\sqrt{SP}$ *the perihelion diſtance* : $\sqrt{SN}$ *the diſtance of the planet.*

For $x =$ coſ. QCH, and (by Schol. Prop. II. B. I. Trig.) $\sqrt{\frac{1-x}{2}}$ = ſine of $\frac{1}{2}$QCH. In the triangle NHS, SN (y) : SH ($n + ax$) : : rad (1) : coſ. NSH $= \frac{n+ax}{y}$. Therefore (Trig. ib.) the ſine of $\frac{1}{2}$NSH $= \sqrt{\frac{1}{2} - \frac{n+ax}{2y}} = \sqrt{\frac{y-n-ax}{2y}}$.

$$=\sqrt{\frac{a+nx-n-ax}{2y}}=\sqrt{\frac{a\times\overline{1-x}-n\times\overline{1-x}}{2y}}$$ Fig. 70.

$$=\sqrt{\frac{\overline{a-n}\times\sqrt{\overline{1-x}}}{2y}}.$$ Therefore ſine $\frac{1}{2}$NSH : ſine $\frac{1}{2}$QCH : : $\left(\sqrt{\frac{\overline{a-n}\times\overline{1-x}}{2y}} : \sqrt{\frac{1-x}{2}} : : \sqrt{\frac{a-n}{y}} : \sqrt{1} : :\right) \sqrt{a-n} : \sqrt{y}$.

PROP. V. *Prob.*

From the excentric anomaly given; to find the mean, and the true anomaly; and alſo the diſtance. 71.

Let ANP be the orbit, AQP the circumſcribing circle. Take the excentric anomaly ACQ, any angle you pleaſe; from that we have the arch AQ, the ſine QH, and the area AQC, and the triangle QCS, and the area AQS. Then take the area of the ſemicircle, to the area AQS; as 180 degrees, to a fourth, which is the mean anomaly. For the area AQS is to the ſemicircle AQP; as the area ANS, is to the ſemiellipſis ANP; that is as 180, to the mean anomaly.

Again for the true anomaly, we have given QH, and from thence NH; HC is alſo given being the coſine of AQ, then we have HS; therefore in the right angled triangle SNH, we have the ſides SH, NH; to find the angle NSH of the true anomaly. And in the ſame triangle the diſtance SN will be had. And therefore the mean and true anomaly and the diſtance, thus found agreeing to the ſame aſſumed excentric anomaly ACQ, will correſpond with one another.

Cor. 1. *Hence, by means of the excentric anomaly, as many mean and true anomalies may be found as*

Fig. 71. *we pleaſe, and the correſponding diſtances; and put into a table for uſe. Or ſuch a table may be put into a more commodious form, if there is occaſion.*

For aſſuming the angle ACQ or the arch AQ, ſucceſſively of 1, 2, 3, &c. degrees; the correſponding anomalies and diſtances may be found, which being put into a table; then at any time, by finding the mean anomaly in the table, the correſpondent true anomaly will be had; or at leaſt may be found by taking a proportional part as is uſual in ſuch caſes.

Cor. 2. *Hence, the mean anomaly is found at any time, by dividing the ellipſis* AEPB, *by the line* SN, *ſo that the whole area of the ellipſis, be to the area* ANS; *as* 360 *degrees, to the mean anomaly. Or by ſaying, as the periodical time of the planet, to the time of its moving from the aphelion; ſo is* 360 *degrees to the mean anomaly.*

PROP. VI. *Prob.*

To compoſe tables for the ready calculating the places of the planets.

Altho' the place of a planet may be found from the foregoing propoſitions, by dividing the area of an ellipſis in proportion of the periodical time to the given time. Yet as this requires a great deal of calculation, aſtronomers have conſtructed tables, by help of which a great part of this labour is ſaved; and the place of a planet found with little trouble.

In order to the conſtructing theſe tables, the orbit of the planet muſt be given, and its periodical time, and the time when it is in the aphelion; the place of the node, and inclination of the orbit.

In the firſt place the radix of the motion muſt be ſettled; which done, the mean motion, or mean anomaly muſt be calculated by the foregoing propoſitions, for years, days, hours and minutes, after this manner. As the periodical time, to 365 days; ſo is 360 degrees, to the planet's mean anomaly for a year. And as the periodic time to 1 day; ſo 360 degrees, to the anomaly for 1 day. The anomaly for 1 day being had, by continual addition of it, the anomaly for 2, 3, 4, 5 days, to 31 will be had. Alſo adding the anomaly for a year to itſelf gives the anomaly for 2 years; and adding again, gives the anomaly for 3, and added again gives that of 4 years; but becauſe every fourth year is leap year, and contains 366 days, the motion of a day muſt be added to the 4th year, and ſo for every fourth year; till you come at 20; from which will be had the motion for 40, 80, 160, 200, 300 years, &c. and ſo on to thouſands, or as far as you will. In all which, when the amount exceeds 360 degrees, 360 muſt be thrown out. Theſe mean anomalies are to be regularly diſpoſed in tables, correſponding to the particular times. You muſt alſo have the mean anomalies correſponding to the beginning of every month.

In another table, to all the degrees of the mean anomaly, you muſt have the degrees of the true anomaly correſponding; or elſe the equations, to find the true anomaly by; and alſo the log. of the diſtance from the ſun. Inſtead of this table, you may have a table of the planet's heliocentric places, with the curtate diſtances from the ſun, and the planet's heliocentric latitude.

Having the inclination of the orbit, a table muſt be conſtructed to all the degrees of the argument of latitude; containing the heliocentric latitude, and the reduction, and curtation. Theſe are calculated by plain and ſpherical trigonometry. For from the inclination of the orbit, having the hypothenuſe, which is the argument of latitude, you

Fig. may find the perpendicular, which is the heliocentric latitude; and alſo the baſe. Then the difference between the hypothenuſe and baſe is the reduction. Then the planet's diſtance from the ſun being given, and the angle at the ſun, or the heliocentric latitude; we have the hypothenuſe and an angle at the baſe, in a right angled plain triangle, to find the baſe; then the difference between the hypothenuſe and baſe is the curtation. Theſe muſt all be calculated for every degree of the argument of latitude, and put into a table.

Upon theſe principles, ſeparate tables muſt be calculated for every planet. After the ſame manner, are tables to be calculated for the ſun or the earth, excepting the table for the argument of latitude; for the ſun, never deviating from the ecliptic, has no latitude.

Several authors, calculating by different methods, make uſe of different tables; but the conſtruction of them, will appear eaſy enough from the nature of them, and from what has been delivered before.

PROP. VII. *Prob.*

Having given the periodical times of the planets; to find the tranſverſe axes of their orbits, or their proportional diſtances from the ſun.

Tho' ſeveral methods have been laid down in Prob. XIV. Sect. IV. for performing this; yet when the periodical times are truly determined, it may be done with more eaſe, and greater certainty from the laws of centripetal force, Prop. XVII. Therefore, make as the ſquare of the periodical time of the earth, to the ſquare of the periodical time of any planet; ſo the cube of the tranſverſe axis of the earth's orbit, to the cube of the tranſverſe axis of the planet's orbit; and ſo the cube of the mean diſtance of the earth from the ſun, to the Fig. cube of the mean diſtance of the planet from the ſun. And the mean diſtance of the earth being taken to be 1 or 10000, the proportional mean diſtances of the planets from the ſun are had, or the diameters of their orbits. And theſe muſt be increaſed according to Prop. XXII. centripetal forces.

Cor. *Hence the periodic times of the planets, their mean diſtances from the ſun, and excentricities, will be as follows.*

Planets	periodic times			mean diſtances	excentricities
	d	h	′		
Saturn	10759	7	0	95380	54298
Jupiter	4332	12	20	52010	25051
Mars	686	23	30	15237	1415
the Earth	365	6	9	10000	168
Venus	224	16	50	7234	52
Mercury	87	23	15	3871	810

SCHOLIUM.

It appears by comparing together the obſervations made in ancient times and the preſent; that the periodic time of Jupiter is increaſing, and that of Saturn decreaſing. For when theſe two planets are in conjunction, by their great magnitudes and attractive forces, they act upon one another, and diſturb one another's motions; by which means Saturn is ſomething retarded, and Jupiter accelerated, in their reſpected orbits.

PROP. VIII. *Prob.*

To find the equation of time; or the difference between the true and apparent time.

As all uniform motions are proportional to the times wherein theſe motions are performed; there-

Fig. fore having the times given, the motions will be known; or having the motions given the times will be known. Therefore the motions and times may be taken for the meaſures of one another. But ſince the motion of the earth in its orbit (or which is the ſame, of the ſun about the earth) is not uniform; the apparent or vulgar time will not be uniform or equable. For the earth moving in an ellipſis, goes ſometimes with a quicker and ſometimes with a ſlower motion. Again, as the ſun's apparent motion is not in the plane of the equator, but in an orbit inclined to it in an angle of 23½ degrees. Therefore it will ſeem to move unequally therein, ſometimes making a greater diurnal motion along the equinoctial, and ſometimes a leſſer; ſo there are two cauſes to produce the inequality of natural days; the elliptic orbit of the earth, and the obliquity of it to the equator.

72. Let ANP be the earth's orbit, S the ſun in the focus; make the area ASB = area PSD; then the areas ASB, PSD are deſcribed in equal times; and ſince AS is greater than PS, therefore the arch PD is greater than AB. Whence the earth moves ſlower in the aphelion A, and quicker in the perihelion P; and varies all the way from A to P, increaſing its motion to P; ſo that at any place N the angular motion is reciprocally as SN^2, (by Cor. 3. Prop. II. centr. forces). Suppoſe AB and PD to be deſcribed in the times of the earth's rotation; when the earth has made an entire rotation about its axis, the meridian of any place will paſs thro' the ſame ſtar, as it did at the beginning when it was at A. But the earth in the ſame time having really (or the ſun apparently,) moved thro' an arch of the ecliptic equal to AB, the ſame meridian does not paſs thro' the ſun, it being then parallel to AS; therefore the earth muſt continue its revolution about its axis thro' an angle equal to ASB,

before it can overtake the ſun. Again, the earth paſſing thro' PD in the time of its rotation; the meridian, being then parallel to PA, it will not paſs thro' the ſun at S, but the earth muſt continue its rotation thro' the angle PSD, before the meridian overtakes the ſun. But the angle PSD being greater than ASB; and the earth's diurnal motion being uniform; the meridian of any place will be longer of overtaking the ſun, at P, than at A; or the ſun will be longer coming to the meridian at P, than at A; that is, the days will be longer when the earth is in the perihelion, than in the aphelion. And therefore, ſince the perihelion happens near the middle of winter, the days will be longer in winter than in ſummer. Fig. 72.

It is plain (by Cor. 5. Prop. II.) that the mean motion about S is equal to the true motion; when SN = ſquare root of the product of the ſemiaxes; and that happens in two points of the orbit.

Let AEPQ be the prime vertical, EQ the equinoctial, BL the ecliptic. Suppoſe the ſun to move uniformly along the ecliptic CL, ſuppoſe it to be at D; thro' D, and the poles P, A, draw the meridian PDFA. It is plain, when D is near C, the motion CD, along the ecliptic is greater than that along the equinoctial CF; becauſe the hypothenuſe CD is greater than the baſe CF. But when D is near L, the motion along the equinoctial FQ, is greater than that in the ecliptic DL. For the part DL almoſt coinciding with a parallel circle, muſt be leſs than the correſponding part FQ of the great circle. Therefore, there is a point between C and L, where the motion along the ecliptic CL, is equal to the motion in the equator CQ, or where the motion in longitude, is equal to the motion of right aſcenſion; but in all other points theſe two motions are unequal. After a quadrant of the ecliptic is deſcribed, they coincide at L and Q. And 73.

ſo

Fig. ſo they coincide at all the equinoctial and ſolſtitial
73. points.

Hence the equation of time depends upon the ſaid two cauſes, which are independent on one another. Sometimes they increaſe this inequality or equation of time, when they concur together; and ſometimes they diminiſh it, if they happen to be contrary. Therefore to find the equation of time anſwering to theſe two cauſes ſeparately.

1. For the elliptic orbit of the earth. Find the true anomaly from the mean, by a table computed by Prop. I. Then the difference between the true and mean anomalies converted into time, gives the equation of time; and this muſt be done for all the degrees of anomaly. Where obſerve, that reckoning from the aphelion, the mean place precedes the apparent place, from the aphelion to the perihelion, where they coincide; but from the perihelion to the aphelion, the apparent place precedes the mean; as is evident from what is delivered in Prop. I. whence, the mean time is longer or ſhorter accordingly.

2. To find the equation, in regard to the obliquity of the ecliptic. Let CD be the ſun's longitude, in the equinoctial CQ, take the arch CI = CD. Then in the right angled triangle CDF, we have given the hypothenuſe CD, and the angle C, to find the right aſcenſion CF; then CF taken from CD or CI, gives the difference FI, which converted into time, gives the equation of time for the longitude CD. And this is to be done for all the degrees of the ecliptic, and put into a table. Where note, the mean place I, precedes the apparent place F, in the firſt and third quadrants of the ecliptic. But the apparent place precedes the mean, in the ſecond and fourth quadrants.

72. Hence therefore, in the firſt and ſecond quadrants of anomaly, the equation muſt be ſubtracted from

the apparent, to give the mean time. But in the third and fourth quadrants of anomaly, the equation muſt be added to the apparent time, to have the mean. And this from the elliptic orbit of the earth. Fig. 72.

And upon account of the obliquity of the ecliptic, in the firſt and third quadrant of the ecliptic, the equation muſt be ſubtracted from the apparent time to get the mean; and in the ſecond and fourth quadrants, the equation muſt be added to the apparent time, to have the mean. 73.

And when both theſe cauſes are conſidered, the entire equation of time will conſiſt of two parts, and is always the ſum or difference of two arches; one of which is the difference of the ſun's true and mean motion; and the other the difference of the ſun's longitude and right aſcenſion. From theſe ſeparate parts two tables are computed as was ſaid before, which will be perpetual. And from theſe two tables a third is compoſed; which will not be perpetual, but will ſerve without ſenſible error for an age; becauſe the earth's aphelion and vernal equinox, which are the initial points of theſe motions, vary a little in their poſition, which when it becomes ſenſible requires a new table. This third table is uſed to ſave the labour of taking out the equations at twice out of the two tables, which by this is done at once; which ſhortens the work in the calculation of the planet's places.

The conſtruction of this table is thus. Begin at the vernal equinox, and take the equations correſponding to the 0, 1, 2, 3, 4, &c. degrees, from the table of the ſun's mean anomaly. Then in the table of the ſun's longitude, ſeek the place of the earth's aphelion, (which is about 8 degrees of Cancer), begin there, and write out the equations of all the ſucceeding degrees in order; which muſt be reſpectively annext to the former, either by adding or

Fig. 73. or ſubtracting as the two tables direct, prefixing the ſign of the major quantity. Theſe muſt be placed againſt the reſpective degrees 0, 1, 2, 3, &c. in the 3d table of the ſun's longitude, and if this be continued quite thro' all the ſigns of the ecliptic, you'll have the table required. This third table will either be in degrees or in time, according as the two firſt are. But it is eaſily converted from one to the other; for a degree is equal to 4 minutes of time.

In ſhort thus.

Let ♈ be the firſt point of Aries, A the aphelion, ☉ the ſun's place; then put

e = ſun's equation for his true anomaly; which is + in the firſt 6 ſigns, and — in the laſt 6.
l = ſun's longitude.
r = ſun's right aſcenſion.
then $e + l - r$ = equation in degrees.
and let $e + l - r$ reduced to time = t the abſolute equation of time.
then mean time + t = apparent time.
or apparent time — t = mean time.
And thus a general table is conſtructed.

The uſe of the table.

1. To reduce the apparent to the mean time; find the ſun's place, againſt which, you'll find the equation; which add to, or ſubtract from, the apparent time, as the table directs, gives the mean time.

2. To turn the mean into the apparent time; find the equation as before, which add to, or ſubtract from, the mean time (contrary to what the table directs), gives the apparent time.

73. Cor. 1. *If the earth's orbit was a circle, coinciding with the plane of the equinoctial; the apparent and true time would be the ſame.* For

Fig. 73. For in this caſe the earth's motion in its orbit would always be uniform, and conſequently the ſun would ſeem to advance equally every day, and there would be always the ſame conſtant addition to the earth's rotation, in order to overtake the ſun. And the ecliptic and equator coinciding, there could be no difference between the ſun's longitude and his right aſcenſion. So that the mean and ſolar days would always be the ſame.

Cor. 2. *It is evident, if the earth's aphelion and perihelion coincided with the equinoctial points, that in theſe points the equation of time would be nothing.*

For theſe are the two beginnings of theſe ſeveral equations. But in proceſs of time, theſe points being removed from one another, the places where the equation is nothing will alſo be removed from theſe points; becauſe in combining theſe two different tables, they will affect one another, and the reſult compounded of both, can neither agree with one nor the other.

Cor. 3. *The mean aſtronomical day exceeds the time of the earth's rotation by* $3^m\ 56^s$. *And the time of the earth's rotation to the ſame ſtar is* $23^h\ 56^m\ 4^s$.

For $\frac{360}{365\frac{1}{4}}$ or .9856 is the degrees the ſun advances in a day, and which the earth has to overtake, after its revolution to the ſame ſtar; and .985 × 4 minutes = $3^m\ 56^s$, which taken from 24^h, leaves $23^h\ 54^m\ 4^s$, the time of the earth's rotation.

72. Cor. 4. *Hence, in a year the earth makes* $366\frac{1}{4}$ *revolutions about its axis.*

For the earth revolving the ſame way about its axis, as it moves in its orbit; it has to revolve thro' the 365th part of the circumference, to overtake the ſun, after its rotation is performed; and therefore in the whole years or 365 revolutions, it has another

Fig. another whole revolution to make, which in all is
72. 366¼.

Cor. 5. *In a mean ſolar day, 360° 59′ 8″ of the equator paſſes under the meridian.*

For .9856 degrees = 59′ 8″, and this is what the ſun advances in a day by his mean motion.

Cor. 6. *The equations of time are greateſt and leaſt, when the apparent time is equal to the mean, or the ſun's motion in a day, is 59′ 8″. And that happens in this age, about the 10th of February, 14th of May, 26th of July, and 1ſt of November.*

For when the mean day is equal to the ſolar day, the equations are at a ſtand; and are then either greateſt or leaſt. And theſe times are to be had from the tables.

Cor. 7. *When a clock is rightly adjuſted to keep mean time, it muſt go 23h 56m 4s, from the inſtant of any fixed ſtar's paſſing thro' a fixed point, to the time of its returning to the ſame point again.*

SCHOLIUM.

From hence a table may be made of the equation of time for all the days in the year. And this is done by conſidering in what points of the ecliptic the ſun is, on every day at noon, and taking the correſpondent equations, from the table of the ſun's longitude. And it is not ſufficient to make one table; but four muſt be made; for leap year, and for the firſt, ſecond, and third years after leap year. For there are 6 hours difference between one year and another; and in 4 years it returns again nearly to the ſame. But this table will only ſerve for an age.

PROP.

PROP. IX.

Fig. 74.

The coſine of the ſun's declination is a mean proportional between the radius and the coſine of the obliquity of the ecliptic, at the point where the ſun's motion in the ecliptic is equal to his motion in right aſcenſion.

Let EQ be the equinoctial, CL the ecliptic, P the pole, S the place of the ſun, PSd a meridian paſſing thro' the ſun, and ASB a parallel of declination. Draw the meridian Prf infinitely near PSd, cutting AB in r, and CL in t.

The triangle Srt, right angled at r, being very ſmall, may be taken for a plain triangle. Therefore $\text{S.S}tr : \text{S}r :: \text{rad} : \text{S}t = \frac{\text{rad} \times \text{S}r}{\text{S.S}tr} = \frac{\text{rad} \times \text{S}r}{\text{S.CS}d}$. But (Trig. B. III. Prop. IV. Cor. 5.) $\text{coſ. } d\text{S} : \text{rad} :: \text{S}r : df = \frac{\text{rad} \times \text{S}r}{\text{coſ. } d\text{S}}$. Then ſince the motion in longitude is equal to the motion in right aſcenſion; therefore $\text{S}t = df$; that is, $\frac{\text{rad} \times \text{S}r}{\text{S.CS}d} = \frac{\text{rad} \times \text{S}r}{\text{coſ. } ds}$; therefore $\text{S.CS}d = \text{coſ. } ds$. But in the right angled ſpherical triangle CdS (by Caſe 11.) $\text{rad} : \text{S.CS}d$ or $\text{coſ. } ds :: \text{coſ. } ds : \text{coſ. SC}d$.

Cor. 1. *Hence dS or the ſun's declination is 16° 44′, when the ſun's motion in the ecliptic is equal to the mean motion.*

Cor. 2. *And his longitude at that time is, in the firſt quadrant,* 46° 14′, *or in* 16° 14′ *of Taurus.*

Cor. 3. *Hence there are four points in the ecliptic, each at* 46° 14′ *diſtance from the equinoctial points, or* 43° 46′ *from the tropics; where the mean and true motion of the ſun are equal.*

This is regarding only the obliquity of the ecliptic, ſetting aſide the excentricity of the earth's orbit.

Cor.

Fig. 74. Cor. 4. *Alſo in theſe four points, the equations are the greateſt and leaſt.*

SCHOLIUM.

It was ſhewn before (Cor. 5. Prop. II.) that the earth's motion about the ſun is equal to the mean motion, when its diſtance from the ſun is a mean proportion between the ſemiaxes of its orbit. And this is, ſetting aſide the conſideration of the obliquity of the ecliptic.

PROP. X.

The ſolar days are longeſt when the earth is in its perihelion, and ſhorteſt in the aphelion; in regard to its elliptic orbit. But the days are longeſt when the ſun is in the tropics, and ſhorteſt in the equinoxes; regarding only the obliquity of the ecliptic.

72. If ANP be the earth's orbit, then ſince the velocity is greateſt at P and leaſt at A; the diurnal arch PD, deſcribed by the earth, will be greateſt at P; and AB the leaſt, deſcribed at A. Therefore the angle PSD is the greateſt, and ASB the leaſt. But theſe angles, are what the earth wants to revolve thro' to overtake the ſun, and are the additions to the ſydereal day to make the ſolar day. But this addition is greateſt at P and leaſt at A. Therefore the ſolar days are greateſt at P and leaſt at A; and in other places are of a mean quantity.

73. Then in regard to the obliquity of the ecliptic, the motion of right aſcenſion will be greateſt at L, for two reaſons; firſt, becauſe it is then parallel to the equinoctial, and without any obliquity. And ſecondly, becauſe it then moves equably in a parallel at L, and therefore the motion referred to Q, in the equinoctial will be greater than that, and at L is the greateſt it can be, as the point L is the

furtheſt from the equinoctial, that it can be. But at the equinox C, the ſun moving uniformly thro' CL, will make a leſs motion along the equinoctial CF, than along the ecliptic CD, as the baſe of a triangle is leſs than the hypothenuſe. And the leaſt motion will be at C, where the obliquity is greateſt. Therefore the addition to the days will be leaſt at C; that is, the ſolar days will be greateſt at L and leaſt at C. And at other places are of a mean quantity. Fig. 72.

Cor. 1. *Hence the days are unequal thro' the whole year, being ſometimes longer and ſometimes ſhorter.*

Here an hour, being a twenty-fourth part of a day, will alſo be unequal at different times of the year.

Cor. 2. *Having regard both to the elliptic orbit of the earth, and the obliquity of the ecliptic; the longeſt and ſhorteſt days will fall upon ſome intermediate points.* 72. 73.

This will be when the equations of time in the general table differ the moſt; which is eaſieſt found by looking into ſuch a table. Hence,

Cor. 3. *About Mar. 27, the natural ſolar days are* $19''$ *leſs than the mean days; and about June 24,* $14''$ *greater; and Sept. 19,* $22''$ *leſs; and December 22,* $30''$ *greater. So that a day at the end of December is* $52''$ *longer than a day at the end of September.*

PROP. XI.

The velocity of a planet in its orbit at any place N *: is to its velocity at the mean diſtance* SD *: :* $\sqrt{FN}$, *the ſquare root of its diſtance from the other focus :* $\sqrt{SN}$, *the ſquare root of its diſtance from the ſun.* 76.

For if PC or SD $= r$, SN $= d$, then FN $= 2r - d$. And (by Cor. 3. Prop. XVI. centr. forces)

Fig. 76. the velocity in N is as $\sqrt{\frac{2r-d}{d}}$. But in D, the diſtance SD or $d = r$, therefore the velocity in D is $\sqrt{\frac{2r-r}{r}} = \sqrt{\frac{r}{r}} = 1$. Whence, velocity in N : velocity in D :: $\sqrt{\frac{2r-d}{d}} : 1 :: \sqrt{2r-d} : \sqrt{d} :: \sqrt{FN} : \sqrt{SN}$.

Cor. 1. *The velocity in the aphelion* A : *is to the velocity at the mean diſtance* SD :: AF : CD *the ſemiconjugate.*

For vel. at A : vel. at D :: $\sqrt{FA} : \sqrt{SA}$ or $\sqrt{FP} :: \sqrt{FA^2} : \sqrt{FA \times FP} :: FA : CD$, (by Prop. III. Ellipſis).

Cor. 2. *The velocity in the perihelion* P : *velocity at the mean diſtance* SD :: FP : CD, *the ſemiconjugate.*

For vel. at P : vel. at D :: $\sqrt{FP} : \sqrt{SP} :: \sqrt{FP^2} : \sqrt{SP \times FP} :: FP : CD :: SA : CD$.

Cor. 3. *The velocity in any place* N *is as* $\sqrt{\frac{FN}{SN}}$. By the demonſtration.

PROP. XII. *Prob.*

77. *To find the ſun's place and diſtance at a given time.*

Let L = arch ♈A☉, the ſun's mean longitude or mean place.
P = ♈A, the place of the apogee.
A = ſun's mean anomaly, AD☉.
E = equation belonging to the anomaly A, which is + or —.
G = arch belonging to the equation of time, which is + or —.

D =

D = log. diſtance for the ſame anomaly. Fig. 77.
Then L + E + G = ſun's true place, or P + A + E + G = ſun's place.
and numb. of the log. D = his diſtance.

The Rule at length.

1. From the tables of the mean motion of the ſun, write on it the numbers for the year, month, day, hour, minute and ſecond; for the ſun's anomaly, and place of the apogee, in two ſeparate columns. And add them up.

2. To the ſun's mean anomaly add the place of the apogee; and the ſum is the ſun's mean longitude.

3. Seek the mean anomaly in the table of the ſun's equations, and take the equation and proportional part, anſwering thereto; which ſubtracted from, or added to, the mean longitude, as the table directs, gives the ſun's true longitude, to the mean time; rejecting 360 deg. when it exceeds that.

4. Find the equation of time in the general table, anſwering to the ſun's true longitude. Find that equation in the table of the ſun's mean anomaly, for hours, &c. and againſt it is the arch, which (according as the abſolute equation of time was + or —) is to be added to, or ſubtracted from, the ſun's longitude, gives his correct longitude for the apparent time given.

5. In the table of the ſun's mean anomaly, find the log. diſtance anſwering, and the proportional parts; the number thereof is the true diſtance.

Note, if you make uſe of tables, printed before 1752; you muſt take out for 11 days leſs, after that time.

Fig. 77.

Ex. I.

To find the ſun's place, Jan. 31, 1766, *at noon.*

By HALLEY's *Tables.*

	☉ mean anom.				Long. apogee.			
	s	°	′	″	s	°	′	″
An. 1766	6	12	13	28	3	8	44	53
Jan. 31.		19	42	43				3
☉ m. anom.	7	1	56	11	3	8	44	56
ap.	3	8	44	56				
☉ m. long.	10	10	41	7	eq. time			
equa.	+	1	2	40	+ 14′ 8″			
☉ true long.	10	11	43	47	arch + 35″			
eq. time			+	35				
	10	11	44	22	his correct place.			

Log. diſtance = 4.993755.

Explanation.

In the table of the mean motions of the ſun, againſt 1766, you have the ſun's mean anomaly 6^s 12° $13'$ $28''$, and the place of the apogee 3^s 8° $44'$ $53''$; to theſe add the motions for Jan. 31, which are 19° $42'$ $43''$, and $3''$. And you have the ſun's mean anomaly 7^s 1° $56'$ $11''$, and place of the apogee 3^s 8° $44'$ $56''$. Whoſe ſum 10^s 10° $41'$ $7''$ is the ſun's mean longitude for that time. Then the equation for the mean anomaly 7^s 1° being 1° $1'$ $2''$, the equation for 7^s 1° $56'$ $11''$ is 1° $2'$ $40''$; which (by the table) added to the mean longitude, gives 10^s 11° $43'$ $47''$ for the ſun's true longitude. Then the abſolute equation of time, anſwering to the ſun's longitude, is + $14'$ $8''$, correſponding to which, in the table of the ſun's mean motion for hours, &c. we have + 35, which added to the ſun's longitude before found, gives $10^s\ 11^\circ\ 44'\ 22''$ for his correct longitude. And finding the mean anomaly in the table of the ſun's diſtances, you'll find (by proportional parts) the log. of his diſtance 4.993755. Fig. 77.

Ex. 2.

Find the ſun's place Oct. 25, 1768, at half an hour paſt nine in the morning.

Here becauſe 'tis in the morning you muſt write Oct. $24^d\ 21^h\ 30^m$; but becauſe it is leap year, and after February, you muſt add a day; ſo the time is Oct. $25^d\ 21^h\ 30^m$.

By HALLEY'*s Tables.*

	☉ mean anom.				Long. apog.			
	s	°	′	″	s	°	′	″
An. 1768	6	11	42	48	3	8	46	55
Biſ. Oct. 25	9	12	52	3				48
21^h			51	45	3	8	47	43
30^m			1	14				
☉ m. anom.	3	25	27	50	equat. time			
+	3	8	47	43	— $15^m\ 50^s$			
☉ m. long.	7	4	15	33	arch			
equat.	—	1	45	57	— 39″			
☉ true long.	7	2	29	36				
			—	39				
	7	2	28	57	☉ correct place.			

Log. diſtance = 4.996930.

Some tables give the ſun's mean longitude at once; and then the place of the apogee is to be ſubtracted from the longitude, to find the mean anomaly. For $P + A = L$, and $A = L - P$. And this way is as ſhort as any.

Fig.

PROP. XIII.

65. *Let* E *be the earth in its orbit* AE, P *a planet in its orbit* NPC, PB *perpendicular to the ecliptic,* S *the ſun; then the ſine of the angle of commutation reduced to the ecliptic,* ESB : *is to the ſine of the elongation* BES : : *as tangent of the heliocentric latitude* PSB : *to tangent of the geocentric latitude* PEB.

For let SAN be the line of the nodes, and draw SB and EB, then in the plain triangle ESB, S.ESB : S.SEB : : EB : SB. And in the right angled triangles EBP, SBP; EB : BP : : rad : tan. BEP $= \frac{BP}{EB} \times$ rad. And SB : BP : : rad : tan. BSP $= \frac{BP}{SB}$ × rad. Therefore *t.*BEP : tan. BSP : : $\frac{1}{EB} : \frac{1}{SB}$: : SB : EB : : S.SEB : S.ESB.

Cor. *As ſine of the angle of commutation of the planet in its proper orbit,* ESP : *ſine of the angle of elongation,* PE*s* : : *ſine of the heliocentric latitude* BSP : *ſine of the geocentric latitude* BEP.

For SP $= \frac{PB \times rad}{S.BSP}$, and EP $= \frac{PB \times rad}{S.BEP}$;

therefore SP : EP : : $\frac{1}{S.BSP} : \frac{1}{S.BEP}$: : S.BEP : S.BSP : : SP : EP : : S.SEP : S.ESP.

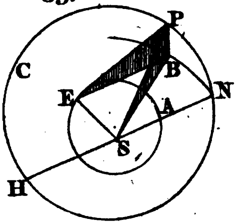
PROP. XIV. *Prob.*

78. *To find the place of a planet, at a given time.*

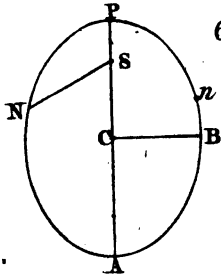
Let L = planet's mean longitude or place, ♈N*p*.
F = longitude of the aphelion, ♈NA.
L — F = A = mean anomaly, A*n*♈*p*.
E = equation for A, + or —.

N =

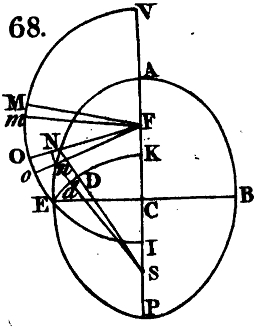
65.



66.



68.



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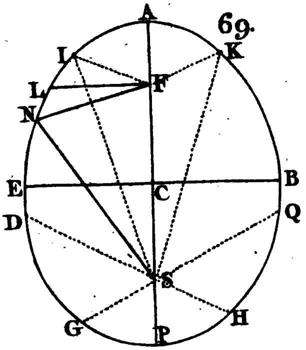
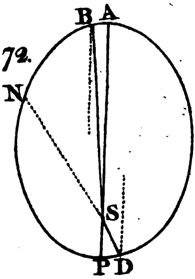


72.



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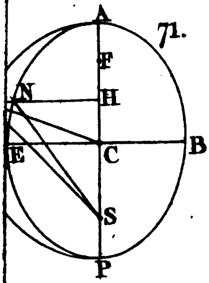
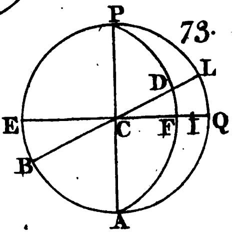


73.



UNIV.

N = place of the aſcending node, ♈N. Fig.
L + E = H = heliocentric longitude.
H — N = argument of latitude, NP.
R = reduction belonging to H — N, + or —.
Then L + E + R = true heliocentric longitude.

Again,

Let B = diſtance, for A, SP.
X = heliocentric latitude PSp, and c = curtation, for H — N.

then B — C = diſtance in the plane of the ecliptic, Sp. and the geocentric latitude is found by Prop. XIII.

Then in the triangle formed by the earth, ſun, and planet; there are two ſides, and the included angle at the ſun, given; to find the angle at the earth, which added to, or ſubtracted from, the planet's true heliocentric longitude, according to their poſition, gives the geocentric longitude.

It is the eaſieſt way to draw a figure, as it directs the whole work, and makes the proceſs intelligible.

If you calculate by tables printed before 1752; you muſt take out for 11 days leſs, after that year.

The Rule at length.

1. Find the ſun's place and curtate diſtance by Prop. XII. to the given time equated. The oppoſite to that is the earth's place.

2. From the tables of mean motion, take out the planet's mean longitude, the places of the aphelion, and node, for the mean time, which put in ſeparate columns; and add them up.

3. From the planet's mean longitude, ſubtract that of the aphelion, the remainder is the mean anomaly; which in ſome tables is given at once.

4. Find the equation for the mean anomaly, and add it to, or ſubtract it from, the mean longitude, as the table directs, gives the planet's heliocentric longitude.

Fig. 5. From the heliocentric longitude of the planet, ſubtract that of the node; the remainder is the argument of latitude.

6. For this argument of latitude, in its proper table, find the reduction, heliocentric latitude, and curtation.

7. The reduction added to, or ſubtracted from, the heliocentric longitude, as the table directs; gives the true heliocentric longitude of the planet in the ecliptic.

8. Find the diſtance for the mean anomaly, from which ſubtract the curtation; the remainder is the curtate diſtance of the planet.

9. Take the difference of the longitudes of the earth and planet, which is the angle of commutation, or angle at the ſun.

10. In the triangle made by the ſun, earth and planet; find the angle of elongation; which add to, or ſubtract from, the ſun's place; according as the planet's apparent place or the ſun's place is foremoſt; which will appear by the figure; and you have the geocentric place of the planet.

11. The latitude is found by the proportion in Prop. XIII.

Examp. 1.

To find the geocentric place of Mars, Nov. 12, 1761, at noon, mean time.

By Prop. XII. The ſun's place is $7^s\ 21^d\ 11'\ 45''$, and the earth's $1^s\ 21^d\ 11'\ 45''$. And the log. curtate diſtance = 4.995016, and diſtance = 99859. Then for the planet's place.

By HALLEY's *Tables.*

year	m. long.	aph.
1761	11 3 27 42	5 1 44 30
Nov. 12	5 9 50 31	58
m. lon.	4 13 18 13	5 1 45 28
equa.	+ 3 2 9	11 11 32 45
hel. lon.	4 16 20 22	mean anom.
reduc.	— 2	node
tr. h. lon.	4 16 20 20	1 17 3 20
		32
log. diſt.	5.219915	1 17 3 52
curt.	226	2 29 16 28
l. curt. diſt.	5.219689	arg. lat.
diſtance	165840.	hel. lat. 1 50 59 north.

♂ place 4 16 20 20 at *p*
☉ place 1 21 11 45 at T
< com. 2 25 8 35 = < TS*p*.

Then in the triangle TS*p*, the ſides TS, S*p*, and angle TS*p* are given, to find the angle of elongation ST*p* = 2 2 33 18, by Caſe 4, of oblique plain triangles; then,

from ☉ place 7 21 11 45
take the < 2 2 33 18
remains ♂ geocent. place } 5 18 38 27

Laſtly, S. com. TS*p*, 85 8 35 — 9.9984377 78.
S. elong. ST*p*, 62 33 18 — 9.9481457
tan. hel. lat. PS*p*, 1 50 59 — 8.5091348
tan. geoc. lat. PT*p*, 18.4572805
1 38 58 Nor. 8.4588428

Examp. 2.

To find the place of Venus Sept. 9, 1768, at 5 o'clock in the morning; that is Sept. 8^{d} 17^{h}.

The

Fig. 78. The ſun's place by Prob. XII. is 5 17 35 10; the equation of time (3′ 8″) deduct; then the mean time is 29^d 16^h 56^m 52^s. And log. ſun's diſtance 5.002607, and the diſtance 100600. Then for the place of Venus.

By Halley's *Tables*.

year	mean lon.	aphel.
1768	5 6 36 42	10 7 35 29
leap year		37
Sept. 9.	0 26 7 20	10 7 36 6
16^h	1 4 5	7 26 15 48
56^m	3 44	mean. anom.
52^s	3	node
m. lon.	6 3 51 54	2 14 33 1
equa.	+ 40 6	21
hel. lon.	6 4 32 0	2 14 33 22
reduc.	+ 1 56	3 19 58 38
tr. hel. lon.	6 4 33 56	argum. lat.
log. diſt.	= 4.857664	hel. lat. 3 11 1
curt.	20	
l. cur. diſt.	4.857644	
	diſtance 72052.	

♀ lon.	6 4 33 56
☉ lon.	11 17 35 10
< com.	5 13 1 14

79. Then in the triangle ♀S☉, we have two ſides given, and the angle at S; to find the angle at ☉, 7° 4′ 34″.

Then to the ſun's place	5 17 35 10
add the angle at ☉	7 4 34
♀'s hel. place	5 24 39 44

Laſtly,

Laſtly, S. com.	(163 1 14)	9.4654254	Fig.
S. elong.	(7 4 34)	9.0905675	79.
T. hel. lat.	(3 11 1)	8.7452446	
T. geo. lat. north		17.8358121	
	1 20 38	8.3703867	

In ſome tables the mean anomaly is had directly from the given year, month, day, &c. And by the mean anomaly, the planet's heliocentric place is had, with its diſtance and heliocentric latitude. And that method is ſhorter than the foregoing one.

PROP. XV.

If AR, CB *be the orbits of two planets, both circular, and in the ſame plane;* S *the ſun in their common center. Put the radius* SA *or* SR $= r$, *radius* SB $=$ R. *Then ſuppoſing one of them* A *or* B *to be the earth; the other, any planet; then that planet will appear ſtationary to a ſpectator at the earth, when the coſine of the angle* ASB $= \frac{\sqrt{Rr}}{R + r - \sqrt{Rr}}$; *radius being* 1. 80.

Let the two planets be at A and B. Let A move thro' the ſpace *a*A, whilſt B moves thro' the ſpace *b*B. Therefore *a*A and *b*B are the velocities of the two planets; and theſe (by Cor. 5. Prop. V. cent. forces) are reciprocally as the ſquare roots of the radii *r* and R. Now that one of the planets may appear ſtationary at the other, the lines connecting them, *ab* and AB, muſt be parallel; otherwiſe they cannot be directed to the ſame ſtar or point in the heavens. Produce *a*A and *b*B to interſect in F; or which is the ſame thing, draw AF, BF perpendicular to the radii SA, SB, to meet in F. Then ſince the angles at A and B are right, the figure FBAS will be inſcribed in a ſemicircle whoſe diameter

—B

Fig. 80. meter is FS; therefore the angle $BFA = BSA$, $AFS = ABS$, $FSB = FAB$, and $DFS =$ ſupplement of BAS.

Let $FB = x$, $BD = y$, then $DS = R - y$, and ſince ab, AB are parallel, $FB\,(x) : FA :: Bb : Aa :: \surd r : \surd R :: 1 : \sqrt{\frac{R}{r}}$; therefore $FA = x\sqrt{\frac{R}{r}}$, and $FA^2 = \frac{Rxx}{r}$. Alſo in the right angled triangles FBS and FAS, $FA^2 + rr = FS^2 = xx + RR$, and $FA^2 = RR - rr + xx$; therefore $\frac{Rxx}{r} = RR - rr + xx$, and $Rxx - rxx = \overline{RR - rr} \times r$, whence $xx = \frac{RR - rr}{R - r} \times r = \overline{R + r} \times r = Rr + rr$. Alſo $xx - rr = Rr$. And $FA = \sqrt{Rr + rr} \times \sqrt{\frac{R}{r}} = \sqrt{RR + Rr}$.

Again, by the ſimilar triangles SAD, FBD; $FB\,(x) : BD\,(y) :: SA\,(r) : AD = \frac{ry}{x}$. Alſo, $SA\,(r) : SD\,(R - y) :: FB\,(x) : FD = \frac{Rx - yx}{r}$. And $FA = FD + DA = \frac{Rx - yx}{r} + \frac{ry}{x} = \frac{Rxx - yxx + rry}{rx} = x\sqrt{\frac{R}{r}}$, by what went before. Therefore $Rxx - yxx + rry = rxx\sqrt{\frac{R}{r}} = xx\surd Rr$. And $Rxx - xx\surd Rr = xxy - rry$, and $y = \frac{R - \surd Rr}{xx - rr}\,xx = \frac{R - \surd Rr}{Rr} \times \overline{Rr + rr} = \frac{R - \surd Rr}{R} \times \overline{R + r}$. Or multiply both numerator and denominator by $R + \surd Rr$, then $y =$

RR

Fig. 80.

$$\frac{RR-Rr}{R\times\overline{R+\sqrt{Rr}}}\times\overline{R+r}=\frac{\overline{R-r}\times\overline{R+r}}{R+\sqrt{Rr}}=\frac{RR-rr}{R+\sqrt{Rr}}.$$

Hence SD $= R-y = R-\frac{R-\sqrt{Rr}}{R}\times\overline{R+r}$ $=\frac{RR-RR-Rr+R\sqrt{Rr}+r\sqrt{Rr}}{R}=$ $\frac{\overline{R+r}\sqrt{Rr}-Rr}{R}$; and SD : rad (1) : : SA (r) : S.SDA $=\frac{Rr}{\overline{R+r}\sqrt{Rr}-Rr}=\frac{\sqrt{Rr}}{R+r-\sqrt{Rr}}$ = cos. angle ASB.

Cor. 1. *The ſine of the angle* ASB $=\frac{R-r}{R\sqrt{R}+r\sqrt{r}}$ $\sqrt{\overline{R+r}}$.

For in the triangle FBD, FB (x) : S.D $\left(\frac{\sqrt{Rr}}{R+r-\sqrt{Rr}}\right)$: : BD (y) : S.F $=\frac{y}{x}\times$ S.D $=$ $\frac{\overline{R-\sqrt{Rr}}\times\overline{R+r}}{R\sqrt{\overline{Rr+rr}}}\times\frac{\sqrt{Rr}}{R+r-\sqrt{Rr}}=$ $\frac{\overline{\sqrt{R}-\sqrt{r}}\times\sqrt{\overline{R+r}}}{R+r-\sqrt{Rr}}$; which may be otherwiſe expreſſed by multiplying both numerator and denominator by $\sqrt{R}+\sqrt{r}$; then S.F $=$ $\frac{\overline{R-r}\times\sqrt{\overline{R+r}}}{R\sqrt{R}+r\sqrt{R}-R\sqrt{r}+R\sqrt{r}+r\sqrt{r}-r\sqrt{R}}$ $=\frac{\overline{R-r}-\sqrt{\overline{R+r}}}{R\sqrt{R}+r\sqrt{r}}=$ S.ASD.

Cor. 2. *The tangent of the angle* ASB $=$ $\frac{R\sqrt{r}-r\sqrt{R}}{Rr}\sqrt{\overline{R+r}}$.

For

Fig. 80. For in the triangle FBD, FB (x) : rad (1) : : BD (y) : tan. BFD or ASB $= \frac{y}{x} = \frac{\overline{R-\sqrt{Rr}}\times\overline{R+r}}{R\sqrt{Rr+rr}}$ $= \frac{R-\sqrt{Rr}}{R\sqrt{r}}\sqrt{R+r} = \frac{R\sqrt{r}-r\sqrt{R}}{Rr}\sqrt{R+r}$. This alſo may be reduced to another form by multiplying both numerator and denominator by $R\sqrt{r} + r\sqrt{R}$, and then it becomes $\frac{\overline{R-r}\sqrt{R+r}}{R\sqrt{r}+r\sqrt{R}}$.

Cor. 3. *The tangent of the angle* ABS, *at the upper orbit, is* $= \frac{r}{\sqrt{RR+Rr}}$.

For in the triangle AFS, FA $(\sqrt{RR+Rr})$: rad (1) : : SA (r) : tan. AFS $= \frac{r}{\sqrt{RR+Rr}} =$ tan. ABS.

Cor. 4. *The tangent of the angle* BAS, *at the lower orbit, is* $= \frac{R}{\sqrt{Rr+rr}}$.

For in the triangle SBF, FB $(\sqrt{Rr+rr})$: rad (1) : : BS (R) : $\frac{R}{\sqrt{Rr+rr}} =$ tan. angle BFS or its ſupplement BAS, which is greater than a right angle.

Cor. 5. *When any planet is ſtationary, the tangent of the angle at the earth, is to the tan. of the angle at the planet; as the periodic time of the planet, to the periodic time of the earth.*

For tan. A : tan. B : : $\frac{R}{\sqrt{\overline{R+r}\times r}} : \frac{r}{\sqrt{\overline{R+r}\times R}}$: $\frac{R}{\sqrt{r}} : \frac{r}{\sqrt{R}}$: : $R\sqrt{R} : r\sqrt{r}$: : $R^{\frac{3}{2}} : r^{\frac{3}{2}}$: : per. time of B : per. time of A.

Cor.

Fig. 80. Cor. 6. *There is another point* G, *as far beyond* R; *where the planets at* G, B, *will appear ſtationary from one another.*

For the poſitions of all the lines remain the ſame at G and B; as at A and B.

Cor. 7. *Let the two curves* AR, CR *be ellipſes or any other curves as well as circles; the planets* A, B *moving in them, will appear ſtationary, when the tangents* AF, BF *are proportional to the velocities at* A *and* B.

For if AF, BF be as the velocities in A and B, they will be as aA and bB, and therefore ab AB will be parallel; and conſequently the planets will appear ſtationary to one another.

Cor. 8. *Hence in circular orbits, and nearly in elliptical ones; the place of the earth may be determined, from which a planet, in a given point of its orbit, will appear ſtationary. For there is no more to do, than to find the angle at the ſun, between the planet and earth, which will determine the two points of the earth's orbit, where it muſt be at the time of ſtation.*

PROP. XVI. *Prob.*

To find the time when any planet will appear ſtationary.

If the orbits of the planets were exact circles, the time might be exactly found. But as they are all elliptical, there is no direct method to find the time. Yet as their orbits do not differ a great deal from circles, one may approach near hand it, and the reſt muſt be done by trials. To proceed then,

1. Having the mean diſtances of the earth and planet, find the angle at the ſun between the earth and planet when the planet is ſtationary by the laſt Prop.

2. For

Fig. 2. For the time you enquire, or any other fixt time, find the heliocentric places of the earth and planet, and conſequently the angle between them, made at the ſun, (by Prop. XII. and XIV). If that angle be the ſame with the angle of ſtation found by the firſt article; then this will be the time required nearly. But if not, then

3. Find out the number of days, that the earth and planet have to move in their orbits, till they make that angle; which may be done thus. Let P be the periodic time of the planet, p that of the earth, $x =$ number of days required. Then $\mathrm{P} : 360° :: x : \frac{360x}{\mathrm{P}}$, the arch run thro' by the planet. And $p : 360° :: x : \frac{360x}{p}$, the arch run thro' by the earth. Then the difference of theſe arches or angles muſt be put equal to the ſum or difference of the angles, found by art 1 and 2, as the caſe requires; then by reduction x is found, which reckoned forward from the time you compute, gives the time of ſtation nearly.

4. But becauſe of the elliptic figures of the planetary orbits, this may not happen to be the preciſe day; therefore by Prop. XIV, find the geocentric longitude of the planet for that day, and alſo for the next. If they happen to be the ſame, then the planet is ſtationary, in one of theſe days. If not, obſerve whether it is direct or retrograde. If it be direct, it is either ſhort of the firſt ſtation or paſt the ſecond. If it is retrograde; it is either paſt the firſt or ſhort of the ſecond ſtation. Accordingly you muſt take 3, 4 or 5 days ſooner or later than this time, as the caſe requires, and compute anew the geocentric longitude of the planet for that time and the day after. If theſe do not ſtill agree, you muſt take a new time, and compute again, till at laſt you find them to agree.

5. When you are within a day of the true time, Fig. compute the planet's geocentric longitude for three ſucceſſive days, and by having the progreſs or regreſs of the planet for theſe times; the method of interpolation will give the exact time of the ſtation.

Example.

To find the time of Jupiter's ſtation next after 81. April 1, 1768.

Let AG be the earth's orbit, BC Jupiter's, SA $= 10000 = r$, SB $= 52010 = R$, $p = 365\frac{1}{4}$ days, $P = 4332\frac{1}{2}$. Then,

1. $\frac{\sqrt{Rr}}{R + r - \sqrt{Rr}} = .58187 = \text{coſ.}\ 54.25 =$ angle DSG, the angle of ſtation.

2. On the firſt of April the earth's heliocentric longitude is — — — $6^s\ 12^\circ\ 16'$
Jupiter's heliocentric longitude — $6\ 17\ 36$
the difference is the angle ASB — — $5\ 20$

So that Jupiter is paſt the firſt ſtation, and is conſequently retrograde; but not come to the oppoſition with the ſun.

3. Here $\frac{360x}{p} = .9856x =$ ASG, and $\frac{360x}{P} = .0831x =$ BSD; and $.9856x - .0831x$ or $.9025x$ $=$ ASG $-$ BSD $=$ ASB $+$ DSG $= 5^\circ\ 20' + 54^\circ\ 25' = 59^\circ\ 45' = 59^\circ\ 75'$; therefore $x = \frac{59.75}{.9025} =$ 66.2 days after the firſt of April, which falls on the 6th or 7th of June, which is nearly the time of the ſecond ſtation, or that where the planet ceaſes to be retrograde, and begins to be direct, its place being at D, and the earth's at G.

4. But Jupiter's geocentric place on June 6th is $6^s\ 13^\circ\ 1'\ 5''$; and on June 7th, $6^s\ 13^\circ\ 0'\ 45''$; therefore he is ſtill retrograde, and not come to his

Fig. 81. ſecond ſtation. Therefore advance three days more, which brings it to June 9th and 10th; then Jupiter's geocentric place computed for theſe two days is $6^s\ 13^\circ\ 0'\ 12''$ for both; ſo that the ſtation is between theſe times. And computing alſo for the 8th or elſe for the 11th; the differences will find the exact time by interpolation.

SCHOLIUM.

It may be obſerved, that if A = ſum or difference of the angles found by Art. 1. and 2. that $.9025x = A$, is a general formula for finding the time; ſerving in all caſes for Jupiter.

They that calculate the geocentric places of the planets for every day in the year, may ſee at once, by inſpection, when the ſtations happen; and whether the planets are retrograde or direct at any time. This is done in ſome ephemerides.

PROP. XVII. *Prob.*

Having given the time of any one tranſit of Mercury or Venus over the ſun's diſk; to find all the times wherein we may expect to ſee the tranſit of the ſame planet, when it is in the ſame node.

It is matter of obſervation, that the inferior planets ſometimes paſs over the ſun's diſk, and appear therein like black ſpots. This can only happen when theſe planets are retrograde, and the earth and the planet in or near the line of the nodes. Conſequently if the planet enter into the ſun's diſk, the ſun muſt be ſo near the node, that the geocentric latitude of the planet exceed not the ſun's apparent ſemidiameter. For if it does, it is evident, the planet will not appear in his diſk.

Since the line of the nodes of each planet is nearly fixt in one place, it follows, that the earth will be in the line of the nodes, always in the ſame point of its orbit; and therefore the planet being retrograde in the aſcending node, and at the ſame time the earth in the line of the nodes, will always be about the ſame time of the year. And in the deſcending node at another certain time of the year.

Now the time being given, when the earth, ſun and planet, are in this poſition once; it is required to find when this will happen again. Therefore let P = periodic time of the earth, p = the periodic time of the planet. Then if we find two whole numbers x and y, ſuch that x revolutions of P may be exactly, or very nearly, equal to y revolutions, of p; then the planet and earth will come into the ſame poſition in the line of the nodes as before. Therefore $Px = py$, and $\frac{p}{P} = \frac{x}{y}$. Therefore (by Prob. IX. Ch. IV. B. II. Arithmetic) to find two numbers x, y, in the ſame ratio as P and P, and they will ſatisfy the demand.

1. *For Mercury.*

Here $P = 365^d\ 6^h\ 9^m = 365.256$; and $p = 87^d\ 23^h\ 15^m = 87.968$. Then we ſhall find $\frac{p}{P} = \frac{x}{y} = \frac{1}{4} = \frac{6}{25} = \frac{7}{29} = \frac{13}{54} = \frac{33}{137} = \frac{46}{191}$, &c. each ratio approaching nearer and nearer, x is the number of ſydereal years wherein this event may be expected again; at the end of 1 year it cannot happen, at the end of 6 years it rarely happens, but often at the end of 7. Alſo at the end of 13, 33, 46, or any additions made by them, it will probably appear.

Dr. *Halley*, in the iſland of St. *Helena*, obſerved Mercury in the ſun, Oct. 28, 1677, old ſtile. To this

Fig. this year add 13, 33, 46, or their doubles, &c. and that will give the year, this event will probably happen again, which will always be about the end of October old ſtile, or the beginning of November new ſtile. The planet being then in its aſcending node. Thus adding 92 or twice 46 to 1677, you have 1769 for the year, when Mercury will appear again in the ſun, in the month of November new ſtile. But to know more nearly the time of the tranſit, theſe ſydereal years ſhould be reduced to Julian years, by adding ſo many times 9 minutes, as is the number of years elapſed; but without this it is exact enough for our uſe. Hence in the year 1769, about Oct. 28, old ſtile, or Nov. 8, new ſtile, we may expect this tranſit will happen again at London.

The number 92 is diviſible by 4, but if it had not, you muſt compare the odd years with the common and leap years, to find how much the time overruns or falls ſhort, in theſe odd years.

Again for the deſcending node; *Hevelius* obſerved Mercury in the ſun 1661, Apr. 23, old ſtile; reduced to London, it happened Apr. 23^d 4^h 52^m, old ſtile. By the nearneſs of the earth and Mercury in this node, it happens that Mercury and the ſun come not in conjunction, in either 6 or 7 years; therefore to the year 1661, add continually 13, 33 and 46 years, or their doubles, &c. and you will have all the years wherein the tranſit is to be expected, when the planet is in its deſcending node; which will always be towards the end of April old ſtile, or the beginning of May, by the new ſtile. Thus add 13 and 46, or 59 to 1661, and the ſum is 1720 for the year when a tranſit will happen. Alſo to 1720 add twice 33 or 66, and the ſum is 1786 for the year of another tranſit, to be looked for Apr. 23^d 5^h. The laſt odd year is a common year of 365 days only. Therefore add 6 hours, and the time is Apr. 23^d 11^h, old ſtile, or May 5^d Fig. 11^h new ſtile, 1786.

Dr. Halley has calculated the times of the tranſits of Mercury as follows; for the old ſtile.

Years	Month	d	h	m	Years	April d	h	m
1677	Octob.	28	0	28	1661	23	4	52
1690		30	18	2	1674	26	12	29
1697		23	18	11	1707	24	12	6
1710		26	11	45	1720	26	19	43
1723		29	5	19	1740	21	11	43
1730		22	5	28	1753	24	19	20
1736		30	22	53	1786	22	18	57
1743		24	23	2	1799	26	2	34
1756		26	16	36				
1769		29	10	10				
1776		22	10	19				
1782	Nov.	1	3	44				
1789	Oct.	25	3	53				

2. *For Venus.*

Here $P = 365.256$, and $p = 224{,}700$. Whence $\frac{p}{P} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. omitting ſome ſmall fractions of no conſequence. Therefore if a tranſit appears at any time, it will probably appear again after 8 years, or 235 years, or 713 years, or their compounds; but not often after 8 years, their periods not being near enough in that ſmall time. In the year 1639, Nov. 24^d 6^h 37^m, old ſtile, *Horrox* obſerved Venus to tranſit the ſun, being in the aſcending node. Add 8 and 235, and it is probable it will appear again in 1747, and 1874, towards the end of November, old ſtile.

Again, for the deſcending node, Dr. Halley computes that Venus will appear in the ſun in 1761, Fig. about May 25^d 18^h, old ſtile, or June 5^d 18^h, new ſtile. Therefore ſhe may be expected to be ſeen again in the ſun, in 1769, June 5^d 18^h. And in 1796, June 7^d 8^h, and in 2482, June 10^d 5^h, &c.

Dr. Halley has calculated the tranſits of Venus as follows, for the old ſtile.

Years	Novem.			Years	May		
1639	24	6	37	1761	25	17	55
1874	26	16	46	1769	23	11	0
2109	29	2	56	1996	28	2	13
2117	26	16	3	2004	25	19	18

SCHOLIUM.

It may happen that our computations may miſs the true time, ſeveral hours, or even a day or two, or perhaps the tranſit may not happen at all; occaſioned by the periods x, y, not being exact. For if the error that happens to be in one of them, be often multiplied, it will amount to a conſiderable difference in the end. But theſe errors may be rectified, by finding the heliocentric place of the planet at the time expected, and comparing it with the place of the earth at that time, as in the next Prop.

PROP. XVIII. *Prob.*

To find the time of the ingreſs and egreſs of an inferior planet, paſſing over the ſun's diſk; and its neareſt diſtance from the ſun's center.

Find the time (by the laſt Prop.) when the tranſit will nearly happen. For this time find the place of the earth, and the planets heliocentric longitude. If theſe agree, they are then in conjunction; but if not, the time of their conjunction muſt be found thus. Put P, p for the periodical times of the earth and planet; then $\frac{360}{p} - \frac{360}{P}$ = degr. which the planet gains at the ſun in a day, put it = D. Then ſay as D° : 1^{day} :: difference between the earth's and planet's heliocentric places in degrees : to the time in days, to the conjunction, or paſt it; according as the earth or planet is foremoſt. But if that time does not ſtill hit the time of conjunction, the ſame operation muſt be repeated till it does; or elſe the exact time will be found by the rule of falſe, or the method of trial and error. When you have found the heliocentric places of the earth and planet to agree, you have their places, being then in conjunction; and at the ſame time you have the place of the node, and the diſtance between them; and by your calculation you have the daily or horary motions of both the ſun and planet; and having the inclination of the orbit, which is the planet's path upon a fixt plane; from theſe data, the path of the planet on the moveable plane of the ſun's diſk will be found, its diſtance from the ſun's center, when neareſt; and the time of the beginning and ending of the tranſit. Fig.

Example.

To find the time of the tranſit of Venus in 1769. By the laſt Prop. the time will be about June $5^d\ 18^h$. Then by Prop. XII. and XIV. on June $5^d\ 18^h$,

☉ place is	—	$8^s\ 15^\circ\ 40'\ 23''$
♀ place	—	— 8 17 8 8
♀ paſt the conjunction		1 27 45 = 1°.46251

Here $p = 224^d.66$, and $\frac{360}{p} = 1.60214$, and $\frac{360}{P} = .98561$, whence D = 1.60214 − .98561 =

Fig. $= .61653^d$. Then $\frac{1.4625}{.6165} = 2.371^d = 2^d\ 8^h\ 54^m\ 14^s$, the time paſt the conjunction; therefore the conjunction was about June $3^d\ 9^h\ 5^m\ 46^s$. Again, for June $3^d\ 9^h\ 5^m\ 46^s$,

	s	°	′	″
☉ place is —	8	13	24	13
♀ place — —	8	13	22	33
♀ ſhort of the conjunction			1	40 $= .0278^\circ$.

Then by theſe two errors the true time will be found June $3^d\ 10^h\ 9^m\ 27^s$. At that time,

	s	°	′	″
☉ place is — —	8	13	26	46
♀ place — — —	8	13	26	45
place of the deſcend. node —	8	14	33	46
And Venus ſhort of the node —		1	7	0

The time from June $3^d\ 9^h\ 5^m\ 46^s$, to the conjunction, is $1^h\ 3^m\ 41^s = 63.68^m$; and the ſpace gone thro' in that time by Venus, is $4'\ 12'' = 4'.2$, and by the earth $2'\ 32'' = 2.533$.

82. Let S be the ſun, ETB the earth's orbit, *an* a portion of the orbit of Venus, VIC its projection on the plane of the ecliptic, N the node. Let the earth be at E, Venus at *a* in conjunction with the ſun at S; then AS, *a*V are perpendicular to the plane of the ecliptic. Thro' S draw a plane perp. to SE, cutting the plane of the ecliptic in the line SN; then the earth being at E, a ſpectator will ſee Venus at *a*, in the plane ASN at A, where AS is perp. to SN. Now the way of a body or planet upon a fixt plane, placed at a diſtance from that body; ſeen from a moveable point, is different from the path ſeen from a fixt point. Let the earth move from E to T, whilſt Venus moves from V, or *a* to *n*; draw S*n* and SIT. Now if the earth had ſtaid at E, Venus at *n* would appear at N, and would ſeem to deſcribe the path or way AN. But the

Fig. 82.

the earth being really at T; Venus at *n* will appear at D, in the line T*n*D. And therefore ſhe will ſeem to have deſcribed the path AD, to a ſpectator moving along with the earth.

To find the time of Venus moving thro' V*n*, and the arch ET or VI moved through by the earth in the mean time. The diſtance from the node is 1° 7′ = 67′; therefore ſay, as 4.2 (ſpace) : 63.68^m (time) : : 67′ : 1016^m the time of Venus paſſing to the node. Alſo as 63.68^m (time) : 2′.533 (the earth's motion) : : 1016^m (time) : 40′.4, the earth's motion in that time = arch ET or VI. Alſo at V the heliocentric latitude of Venus is 3′ 58″ = 3′.966 north = angle *a*SV. Hence we have *a*V = 3′.966, V*n* = 67′, VI = 40′.4, and I*n* = 26′.6; but theſe are the heliocentric motions; and to reduce them to the geocentric motions, as they appear in the figure ASDN; we muſt find the diſtance of Venus from the earth EV, and from the ſun; at the time of the conjunction.

Log. diſt. of ♀ is 4.861095, and the diſt. = 72626.5
Log. diſt. of ☉ is 5.006566, and the diſt. = 101523.
the difference 28896.5

this is the diſtance of Venus from the earth, or EV = 28896, and SV = 72626; therefore EV : SV : : 28896 : 72626 : : 1 : 2.513; then we ſhall have,

EV (1) : VS (2.513) : : *a*V or < *a*SV (3′.966) : < *a*EV = 9′.968 the geocentric latitude AS.

Alſo ſince SN ſubtends the angle SEN or VE*n*, and SD ſubtends the angle STD or IT*n*; therefore

EV (1) : SV (2.513) : : VS*n* or V*n* (67′) : < VE*n* or SN = 168′.4; and

EV (1) : SV (2.513) : : IS*n* or I*n* (26′.6) : IT*n* or SD = 66′.84; and DN = 101′.6. Then in the right angled triangle ASD, we have the ſides SD, SA given; to find the angle ADS = 8° 29′, the angle

Fig. angle of the planet's apparent way with the ecliptic; and the hypothenuſe AD = 67′.57.

83. In the triangle ASD, draw SP perp. to DA; then by ſimilar triangles, DA (67′.57) : SA (9′.968) :: DS (66′.84) : SP = 9′.86 :: SA (9′.968) : AP = 1′.47; and SP is the planet's neareſt approach to the ſun's center. Then AD : AP :: 1016^m (time of deſcribing AD) : 22.14^m = 22^m 8^s, this added to June 3^d 10^h 9^m 27^s the time of the conjunction, gives June 3^d 10^h 31^m 35^s for the middle of the tranſit.

The ſun's apparent ſemidiameter, at the time of the conjunction is 15′ 50″. Therefore if FGH be the ſun's diſk, FAG the way of Venus thereon; draw SF, SG. Then in the right angled triangle SPG; there are given SG (15′.83), and SP (9′.86), to find PG or PF = 12′.36.

Laſtly AD (67′.57) : PG (12′.36) :: 1016^m (time in AD) : time in AG, or half the duration = $185^m.85$ = 3^h 5^m 51^s; therefore the beginning of the tranſit is June 3^d 7^h 25^m 44^s; and the end is at 1^h 37^m 26^s next morning; and the whole duration, 6^h 11^m 42^s. Theſe things will be ſo if Dr. Halley's tables are true. And therefore in England we may ſee the beginning of the tranſit, but neither the middle nor the end of it.

SCHOLIUM.

It may be obſerved that .61653 is a conſtant number for Venus in either node; and in all tranſits of Venus in the deſcending node, the ſun's apparent ſemidiameter will be conſtantly 15′ 50″; the apparent way of Venus with the ecliptic, 8 29; the diſtance of Venus from the ſun and from the earth, 72626, and 28896. But in the aſcending node, they will be different.

PROP.

PROP. XIX. *Prob.*

To find the time of the heliocentric conjunction of any two planets.

Let p, q be the periodic times of the two planets, p being the greater. Then $\frac{pq}{p-q}$ will be the mean time between one conjunction and another, when ſeen from the ſun.

For $p : 360 :: 1 : \frac{360}{p}$, the degrees gone thro' in a day by the ſlower planet. And $q : 360 :: 1 : \frac{360}{q}$, the degrees gone through in a day by the ſwifter planet. Therefore $\frac{360}{q} - \frac{360}{p}$ = the degrees gained in a day by the ſwifter. Whence $\frac{360}{q} - \frac{360}{p} : 1^d :: 360$: time of gaining the whole circumference $= \frac{360}{\frac{360}{q} - \frac{360}{p}} = \frac{1}{\frac{1}{q} - \frac{1}{p}} = \frac{pq}{p-q}$, in days.

Then if the time of one heliocentric conjunction be known, the mean time of the next will be known, and the mean times of all ſucceeding ones. Therefore (by Prop. 14) calculate their heliocentric places to that time, and note the error; and it will eaſily appear whether the true time is paſt or to come, by conſidering whether the ſwifter or ſlower planet is foremoſt. Then aſſume a new time accordingly, and calculate their places again, and note the error. Laſtly, from theſe two errors the true time will be found by the common methods. And this time may be proved by a new calculation of their heliocentric places.

Fig. Cor. *Hence the mean time between one heliocentric conjunction and another, will be for Saturn and Jupiter, about* 20 *years, Saturn and Mars* $2\frac{1}{7}$ *years, for Jupiter and Mars* $2\frac{2}{5}$ *years. And for any ſuperior with an inferior planet, ſomething more than the periodic time of that inferior. And laſtly, for Venus and Mercury about five months.*

Scholium.

The geocentric conjunctions of the planets, can be had no otherwiſe than by repeated trials, by reaſon they do not revolve regularly about the earth, as they do about the ſun. As the two higheſt planets come neareſt to a regular motion round the earth, their geocentric conjunctions will differ but little from their heliocentric conjunctions; and will happen a little ſooner or a little later, according as the earth is on one ſide or the other of its orbit. Therefore ſince there was a conjunction of Saturn and Jupiter, March 14, 1762, there will be no more till the year 1782. The geocentric conjunction of Mars with either Saturn or Jupiter, may happen a good while before or after the heliocentric conjunction, becauſe the orbit of Mars is near the earth's orbit; and that difference of time will depend upon the place of the earth in its orbit. The time of conjunction of any inferior with a ſuperior planet, is more uncertain ſtill. And it may happen, that two conjunctions may be within a little time of one another, and that is, if the Sun be not far from a conjunction with the two planets. And in general the interval of time between the geocentric conjunctions of an inferior with a ſuperior planet, will be a year and ſomething more; for the earth has to make a revolution, and ſomething more before the inferior can come again between it and the ſuperior; and eſpecially in Mars, where the time will be prolonged, by reaſon of his ſwifter motion. Venus is ſometimes in conjunction

252.

UNIV OF

with Mercury, when ſhe is retrograde; but very Fig.
ſeldom. And when ſhe is direct, there is but about $\frac{1}{3}$ of her orbit, wherein ſhe can be in conjunction with Mercury; and when ſhe is there, ſhe will be in conjunction with him generally once, and ſometimes twice. So that in about 20 months (the time of Venus revolving to the Sun) ſhe will be once or twice and ſometimes thrice in conjunction with Mercury; all which are irregular, and the times uncertain without particular calculations.

PROP. XX.

The times, in which different comets move thro' ſimilar parts of their orbits, are as the ſquare roots of the cubes of their perihelion diſtances.

Let PN, Pn, be the parabolic orbits of two co- 92.
mets; S, s, the foci; P, p, the perihelion; N, n, 93.
the places of the comets. Then (Prop. II. parabola) 4SP and $4sp$ are the parameters of the orbits. And ſince PN, pn, are by ſuppoſition, ſimilar parts of the orbits; the areas SPN and spn, will be as SP^2 and sp^2. Let T and t be the times of deſcribing PNS, and pns; alſo let pbs be deſcribed in the time T. Then (by Cor. 2. Prop. 16. centripetal forces), area PNS : area pbs : : $\surd 4PS$: $\surd 4ps$: : $\surd PS$: $\surd ps$. And (by Prop. 11. ib.), area pbs : area pns : : T : t. Therefore area PNS : area pns : : $T\surd PS$: $t\surd ps$; that is, $PS^2 : ps^2 :: T\surd PS : t\surd ps$; whence $PS^2 \times t\surd ps = ps^2 \times T\surd PS$, and $PS^{\frac{3}{2}} \times t = ps^{\frac{3}{2}} \times T$, and $T : t :: PS^{\frac{3}{2}} : ps^{\frac{3}{2}}$.

Cor. 1. *Two comets will have the ſame true anomaly in moving from the perihelion, in times, which are in the ſeſquiplicate ratio of their perihelion diſtances. And their diſtances from the ſun will be as the perihelion diſtances.*

For

Fig. 92. For PNS and *pns* being ſimilar figures, the angles PSN and *psn* are equal; and SN to *sn*, as SP to *sp*.

Cor. 2. *If a comet's peribelion diſtance be equal to the mean diſtance of the ſun and earth; it will deſcribe a quadrant of its orbit* POS *in* 109 *days*, 14 *hours*, 46 *minutes*, 12 *ſeconds*.

For let SP (p) = mean diſtance of the earth = 1, the time of deſcribing the area POS = x, and a = area the earth deſcribes in that time, t = periodical time of the earth; then (by Cor. 3. Prop. XIV. centrip. forces) $a\sqrt{2}$ = area POS = $\frac{4}{3}$; and 3.1416 = area of the earth's orbit; therefore $t : x :: 3.1416 : a$; and $ta = 3.1416x$, whence $x =$

$$\frac{ta}{3.1416} = \frac{4}{3\sqrt{2}} \times \frac{t}{3.1416} = 109.615 \text{ days.}$$

Cor. 3. *If* p *be the peribelion diſtance of any comet; the time of its moving from the peribelion thro' 90 degrees of its orbit, is* $109.615 \times p\sqrt{p}$ *days*.

For $1 : 109.6 :: p\sqrt{p} : 109.6 \times p\sqrt{p}$ days.

PROP. XXI.

92. *Having given either the comet's true anomaly, or its diſtance from the ſun; to find the other; and the time it is there; and alſo the area, or mean anomaly proportional thereto.*

The comet's orbit, and the time of the perihelion are ſuppoſed to be known. Let PNO be the orbit, S the ſun, P the perihelion, N the place of the comet. Put the perihelion diſtance SP = p; comet's diſtance SN = y, coſ. true anomaly PSN = z, and v = verſ. ſupplement thereof; alſo t = 109.615 days, the time of a comet's deſcribing a quadrant, whoſe perihelion diſtance is 1, equal to the mean diſtance of the earth; D = time from the perihelion. Fig. 92.

Then (by Cor. 1. Prob. XXII. Sect. IV.) $z = \frac{2p}{y} - 1$, whence $zy = 2p - y$, and $zy + y = 2p$; therefore $y = \frac{2p}{1+z} = \frac{2p}{v}$, becauſe $v = 1 + z$, by trigonometry.

Alſo (by Cor. 2. ib.) the area PSN $= \frac{2p+y}{3}\sqrt{py - pp}$, and the quadrantal area PSO $= \frac{4}{3}pp$. Then (by Cor. 3. Prop. XX.) $t\sqrt{p^3}$ = time of the comet's deſcribing PSO or $\frac{4}{3}pp$; and area PSO $(\frac{4}{3}pp) : t\sqrt{p^3}$ :: area PSN $\left(\frac{2p+y}{3} \times \sqrt{py - pp}\right)$: D; therefore $\frac{\overline{2p+y} \times t}{4}\sqrt{y-p} = D$, the number of days from the perihelion.

Therefore if z or v be given y is known, and the contrary, by the equation $vy = 2p$. And y being known, D is known, by the equation $\frac{\overline{2p+y} \times t}{4}\sqrt{y-p} = D$, and the area PSN alſo.

Example.

In the comet of 1748, whoſe perihelion diſtance is ,84067, and time of the perihelion Apr. 17^{d} 19^{h} 26^{m}; let the true anomaly be 40 degrees. To find the diſtance, and time it will be there.

Here $z = .76604$, and v or $1 + z = 1.76604$, whence $y = \frac{2p}{v} = \frac{1.68134}{1.76604} = .95204$, and $2p + y = 2.63338$, and $y - p = .11137$; whence $\frac{\overline{2p+y} \times t}{4}\sqrt{y-p} = \frac{2.63338 \times 109.615}{4}\sqrt{.11137}$ $= 24.083$ days $= 24^{d}\ 1^{h}\ 59^{m}\frac{1}{2}$, which is beſt found by

Fig. 92. by logarithms. So the time will be May $11^d\ 21^h\ 25^m\frac{1}{2}$.

Cor. 1. *If c be the coſine of half the true anomaly; then the comet's diſtance,* $y = \frac{p}{cc}$, *radius being* 1.

For by the principles of trigonometry, $v = 2cc$.

Cor. 2. *Having the time from the perihelion given; the diſtance, and true anomaly, may be found; by reducing the equation* $\frac{\overline{2p+y}\times t}{4}\sqrt{y-p} = D$; *which will be an adfected cubic equation.*

Cor. 3. *Hence a table may eaſily be made, for finding the time when a comet will be in any given place of its orbit; or on the contrary, to find in what point of its orbit it is, at any time given; and is done as follows.*

Since a comet, at the diſtance of the Sun, deſcribes a quadrantal area, in 109.615 days; therefore if that area be divided into any number of equal parts, as ſuppoſe 100; one of theſe parts will be .912286, which is the part of the area which the comet daily deſcribes, or rather divide the quadrantal area into 109.615 parts, that is, as many as there are days; and the comet will deſcribe one part every day, and any number of parts in the ſame number of days; and therefore theſe parts or days will ſtand for the mean motion, ſimilar to degrees.

Therefore take the angle PSN ſucceſſively equal to 1, 2, 3, 4, 5, &c. degrees, as far as convenient, for the true anomaly. Then y is had, the correſpondent diſtance; which diſtances are to be ſet againſt the true anomalies in the table. Then y being had, D will be had or the number of days from the aphelion; and theſe days are alſo to be ſet

Fig. 92.

ſet againſt the true anomalies in the table. The table being thus conſtructed, will ſhew the time, and the comet's diſtance from the perihelion, for any given true anomaly; if it is even degrees; if not, by taking a proportional part. And on the contrary, having the time given, one may, by the ſame table, find the diſtance, and the true anomaly correſponding thereto, in their ſeveral columns, by taking a proportional part, when neceſſary. And that with the ſame eaſe, as finding the number from a given logarithm, by the common tables.

But if any one had rather have a table where the time or proportional parts of the area, increaſe uniformly; the former is eaſily transformed into ſuch a one, by placing the time, or parts of the area, which repreſent the mean motion, in the firſt column, increaſing by 1, 2, 3, 4, &c; and finding the correſpondent angles PSN, and diſtances SN, from the ſaid table by taking proportional parts. And putting theſe ſeveral angles and diſtances in the ſecond and their columns; tho' the diſtances are not ſo material, being eaſily found from the true anomaly. And thus a table will be conſtructed eaſier than by continually extracting the root of an adfected cubic equation, with three terms, for every number, to conſtruct it by. Such a table is the following one, by the help of which, the place of any other comet may be found for any time. For if p be the perihelion diſtance of any comet, then if you take $\sqrt{p^3}$ to 1 (or $\sqrt{1^3}$); as the time given, is to a fourth quantity; it is plain (by Prop. XX. Cor. 1.) this quantity is the time, in which the comet p has the ſame true anomaly, as the comet 1 has in the given time. Therefore, looking for that time ſo found, in the table; you will have the true anomaly of the comet p. And the diſtance againſt it being multiplied by the perihelion diſtance, gives the true diſtance.

Fig. 92. In the following table, the firſt column is days, and repreſents the mean motion; the ſecond is the true anomaly, or angle from the perihelion reckoned in the comet's orbit; the third is the logarithms of the diſtances, inſtead of the diſtances themſelves.

A general TABLE *for calculating the motion of Comets in parabolical Orbits.*

Days.	Angle from the perihel. °	′	″	Logar. Diſtance.	Days.	Angle from the perihel. °	′	″	Logar. Diſtance.
1	1	23	37	0.000064	31	39	44	4	0.053298
2	2	47	12	0.000257	32	40	49	3	0.056261
3	4	10	40	0.000577	33	41	53	9	0.059298
4	5	34	0	0.001026	34	42	56	19	0.062410
5	6	57	8	0.001602	35	43	58	35	0.065596
6	8	20	2	0.002298	36	44	59	58	0.068768
7	9	42	38	0.003117	37	46	0	27	0.071970
8	11	4	54	0.004060	38	47	0	2	0.075202
9	12	26	46	0.005126	39	47	58	44	0.078464
10	13	48	14	0.006317	40	48	56	34	0.081757
11	15	9	14	0.007616	41	49	53	32	0.085065
12	16	29	43	0.009026	42	50	49	38	0.088397
13	17	49	39	0.010547	43	51	44	52	0.091745
14	19	9	1	0.012179	44	52	39	15	0.095111
15	20	27	47	0.013923	45	53	32	48	0.098496
16	21	45	53	0.015809	46	54	25	31	0.101888
17	23	3	19	0.017766	47	55	17	26	0.105289
18	24	20	3	0.019795	48	56	8	31	0.108699
19	25	36	3	0.021897	49	56	58	48	0.112118
20	26	51	17	0.024073	50	57	48	18	0.115544
21	28	5	45	0.026373	51	58	37	2	0.118971
22	29	19	25	0.028751	52	59	25	1	0.122399
23	30	32	16	0.031297	53	60	12	15	0.125830
24	31	44	17	0.033742	54	60	58	44	0.129261
25	32	55	27	0.036356	55	61	44	30	0.132695
26	34	5	45	0.039030	56	62	29	32	0.136122
27	35	15	11	0.041772	57	63	13	52	0.139544
28	36	23	45	0.041583	58	63	57	30	0.142961
29	37	31	25	0.047462	59	64	40	28	0.146372
30	38	38	11	0.050409	60	65	22	45	0.149779

Fig. 92.

Days	Angle from the perihel. °	′	″	Logar. Diſtance.	Days	Angle from the perihel. °	′	″	Logar Diſtance.
61	66	4	23	0.153177	112	90	49	8	0.307283
62	66	45	22	0.156565	114	91	29	17	0.312458
63	67	25	42	0.159945	116	92	8	30	0.317570
64	68	5	27	0.163315	118	92	46	48	0.322621
65	68	44	34	0.166676	120	93	24	13	0.327611
66	69	23	5	0.170024	122	94	0	48	0.332542
67	79	1	1	0.173360	124	94	36	34	0.337416
68	79	38	22	0.176684	126	95	11	32	0.342234
69	71	15	9	0.179996	128	95	45	45	0.346985
70	71	51	23	0.183295	130	96	19	14	0.351684
71	72	27	4	0.186579	132	96	52	0	0.356330
72	73	2	13	0.189848	134	97	24	4	0.360924
73	73	36	51	0.193104	136	97	55	28	0.365463
74	74	10	58	0.196345	138	98	26	14	0.369945
75	74	44	34	0.199574	140	98	56	22	0.374369
76	75	17	40	0.202784	142	99	25	54	0.378757
77	75	50	18	0.205978	144	99	54	50	0 383093
78	76	22	28	0.209156	146	100	23	13	0.387377
79	76	54	9	0.212321	148	100	51	2	0.391609
80	77	25	23	0.215472	150	101	18	20	0.395790
81	77	56	10	0.218603	152	101	45	6	0.399939
82	78	26	30	0.221717	154	102	11	22	0.404040
83	78	56	25	0.224815	156	102	37	9	0.408093
84	79	25	55	0.227896	158	103	2	27	0.412098
85	79	54	59	0.230961	160	103	27	18	0.416055
86	80	23	37	0.234004	162	103	51	42	0.419976
87	80	51	55	0.237034	164	104	15	40	0.423854
88	81	19	48	0.240048	166	104	39	13	0.427693
89	81	47	18	0.243046	168	105	1	20	0.431490
90	82	14	25	0.246027	170	105	25	4	0.435248
91	82	41	10	0.248988	172	105	47	25	0.438969
92	83	7	33	0.251932	174	106	9	23	0.442654
93	83	33	36	0.254860	176	106	30	59	0.446299
94	83	59	17	0.257771	178	106	52	13	0.449904
95	84	24	38	0.260665	180	107	13	7	0.453469
96	84	49	39	0.263542	182	107	33	40	0.457002
97	85	14	20	0.266400	184	107	53	54	0.460515
98	85	38	42	0.269243	186	108	13	48	0.463975
99	86	2	44	0.272066	188	108	33	23	0.467402
100	86	26	28	0.274875	190	108	52	41	0.470798
102	87	13	2	0.280441	192	109	11	40	0.474165
104	87	58	26	0.285941	194	109	30	22	0.477499
106	88	42	43	0.291375	196	109	48	47	0.480802
108	89	25	53	0.296743	198	110	6	55	0.484070
110	90	8	1	0.302044	200	110	24	47	0.487306

Fig. 92.

Days	angle from the perihel.			Logar. Diſtance.
	°	′	″	
204	110	59	43	0.493692
208	111	33	40	0.499959
212	112	6	40	0.506119
216	112	38	44	0.512173
220	113	9	56	0.518120
224	113	40	17	0.523982
228	114	9	51	0.529738
232	114	38	40	0.535386
236	115	6	43	0.540928
240	115	34	5	0.546363
244	116	0	46	0.551736
248	116	26	49	0.557021
252	116	52	14	0.562227
256	117	17	3	0.567354
260	117	41	18	0.572402
264	118	5	0	0.577387
268	118	28	10	0.582293
272	118	50	49	0.587121
276	119	12	58	0.591869
280	119	34	39	0.596537
284	119	55	52	0.601112
288	120	16	38	0.605646
292	120	36	59	0.610140
296	120	56	55	0.614593
300	121	16	27	0.619006
310	122	3	12	0.629587
320	122	48	34	0.639893
330	123	31	24	0.649924
340	124	12	26	0.659680
350	124	51	38	0.669161
360	125	29	13	0.678367
370	126	5	14	0.687298
380	126	39	53	0.695954
390	127	13	10	0.704335
400	127	45	12	0.712442
410	128	16	2	0.720481
420	128	45	49	0.728285
430	129	14	31	0.735887
440	129	42	16	0.743315
450	130	9	4	0.750564
460	130	35	3	0.757663
470	131	0	9	0.764588
480	131	24	31	0.771374
490	131	48	6	0.778005
500	132	11	1	0.784506
510	132	33	15	0.790879
520	132	54	51	0.797106
530	133	15	50	0.803215
540	133	36	15	0.809209
550	133	56	7	0.815089
560	134	15	27	0.820857
570	134	34	17	0.826518
580	134	52	39	0.832082
590	135	10	33	0.837545
600	135	28	0	0.842910
620	136	1	40	0.853370
640	136	33	45	0.863477
660	137	4	25	0.873267
680	137	33	44	0.882746
700	138	1	51	0.891951
720	138	28	48	0.900800
740	138	54	40	0.909549
760	139	19	33	0.917984
780	139	43	30	0.926191
800	140	6	35	0.934185
820	140	28	50	0.941970
840	140	50	18	0.949557
860	141	11	2	0.956956
880	141	31	5	0.964179
900	141	50	30	0.971238
920	142	9	17	0.978130
940	142	27	30	0.984972
960	142	45	10	0.991466
980	143	2	19	0.997922
1000	143	18	57	1.004235
1100	144	35	31	1.033967
1200	145	42	42	1.061013
1300	146	42	20	1.085817
1400	147	35	45	1.108711
1500	148	24	1	1.129975
1600	149	7	55	1.149817
1700	149	48	6	1.168416
1800	150	25	5	1.185921
1900	150	59	17	1 202451
2000	151	31	2	1.218103
2500	153	41	48	1.285925
3000	155	20	24	1.341028
4000	157	42	28	1.427478
5000	159	22	34	1.494251
10000	163	45	13	1.699705
100000	172	32	6	2.372859

PROP. XXII. *Prob.*

To find the place of a comet in its orbit, at a given time.

1. Find the ſun's place and diſtance for the time given, by Prop. XII.

2. Find the difference between the time of the perihelion, and the time given; and reduce it to days and decimal parts of a day. Find its logarithm, from which ſubtract $\frac{3}{2}$ the log. of the perihelion diſtance; find the number of the remaining logarithm, and that is the mean motion of the comet.

3. Seek the mean motion in the firſt column under the title of days; againſt which, in the ſecond column, you have the comet's true anomaly, or angle from the perihelion; and in the 3d column the log. of the diſtance.

4. To this log. diſtance, add the log. perihelion diſtance of the comet; and the ſum is the log. of the comet's true diſtance from the ſun.

5. Find the angular diſtance of the perihelion, from the node (by column 2 and 4, of the table of comets); and having before, the angular diſtance of the comet from the perihelion; by adding, or ſubtracting, as the caſe requires, you'll find the angular diſtance of the comet from the node, or the argument of latitude. A ſketch of a figure will be uſeful here.

6. Then having the argument of latitude, and inclination of the orbit; the comet's place reduced to the ecliptic, the heliocentric latitude, and log. curtate diſtance from the ſun, will be found by the common rules of trigonometry. And thence the comet's heliocentric longitude.

7. Theſe things being had; by the very ſame rules that the geocentric place of a planet is found, the geocentric place of the comet will be found.

Example.

94. Let us take for an example, the glorious comet of 1744. And let it be required to find its place January 5, at 9 o'clock at night.

Sun's long. at that time	297° 21′ 7″
and the log. diſtance —	1.992935
difference of times	44ᵈ 23 4
that is —	44.961
Log. 44.961 —	1.652836
$\frac{3}{2}$× —1.347325 ſubtr. —	1.020987
log. mean motion	2.631849
mean motion —	428.40
true anomaly —	129° 9′ 58″ = CP.
log. diſtance —	0.734671
log. perihel. diſtance —	1.347325
log. comet's diſtance	0.081996 = SC.
ang. diſtance of the perihelion and node	151 23 49 = arch NCP
anom. ſubtract —	129 9 58 = CP
argument of latitude	22 13 51 = NC
reduced to the ecliptic	15 33 4 = ND
com. heliocent. lat.	16 5 13 = CD
lon. of the aſcend. node	45 46 11
add the reduction	15 33 4
com. heliocen. longit.	61 19 15
the earth's longitude	117 21 7
95. angle of commutation	56 1 52 = ESC
log. curtate diſt. of the comet from the ſun	0.064648 = SC
the angle of elongation add to the ſun's longit.	70 47 7 = SEC
the comet's geoc. long.	8 8 14
and com. geocent. lat.	18 10 41 N.

Let

Fig. 94. Let NCP*n* be the comet's orbit, S the ſun, N*n* the ſection of its plane with the plane of the ecliptic, N the aſcending node, P the perihelion, C the comet's place, moving towards P; let CD be perpendicular to N*n*.

Then the angle PSC (129 9 58) is the anomaly; SC the diſtance, angle NSP the diſtance of the perihelion and node (151 23 49); the angle NSC or arch NC (22 13 51) the argument of latitude, being what the comet is paſt the node, in its orbit; ND (15 33 4) its reduction to the ecliptic; CD (16 5 13) the heliocentric latitude. Then ſince the comet is gone paſt the node, and direct; the arch ND muſt be added to the longitude of the node N; and then the heliocentric longitude of the comet at D will be 61 19 15. The difference between the heliocentric longitudes of the earth and comet, or the angle of commutation, is 56 1 52.

95. Then in the triangle SEC (made by the ſun, earth, and comet;) two ſides, ES, SC are given, and the included angle ESC; to find the angle of elongation SEC (70 47 7;) and the comet at C, being before the ſun at S, to a ſpectator at E; the angle of elongation muſt be added to the ſun's place (297 21 7,) and the ſum (368 8 14, or 8 8 14) is the comet's geocentric longitude. And the geocentric latitude is found by Prop. XIII. to be 18 10 41 north; which are very near what I found them to be, by my own obſervations; by taking the diſtance of the comet from ſeveral fixt ſtars, at that very time.

PROP. XXIII. *Prob.*

The periodic time, and perihelion diſtance of a comet being given; to find the time when the comet will be in any given point of its elliptical orbit.

96. This is to be done by the very ſame rule, that was laid down for the planets. Let the ſemitranſverſe CP $= a$, ſemiconjugate CB $= c$, perihelion diſtance SP $= p$, excentricity CS $= n$, comet's diſtance SN $= y$, coſine of the anomaly PSN $= z$, periodic time $= t$, and $d =$ number of degrees in the arch whoſe coſine is $\frac{a-y}{n}$. Then by Prop. I. $y = \frac{cc}{a+zn}$, and the time of deſcribing PN is $= \frac{dt}{360} - \frac{t\sqrt{-cc+2ay-yy}}{2a \times 3.1416}$.

But by the nature of the ellipſis, $2ap - pp = cc$, and $n + p = a$, and a is known from the periodic time by Prop. VII; therefore

$$\frac{\sqrt{-cc+2ay-yy}}{2a} = \sqrt{\frac{-2ap+pp+2ay-yy}{4aa}}$$

$$= \sqrt{\frac{y-p}{2a} - \frac{yy-pp}{4aa}} = \sqrt{\frac{y-p}{2a} - \frac{y+p}{2a}\text{A}};$$

where $\text{A} = \frac{d-p}{2a}$. Hence, the time of deſcribing PN is $= \frac{dt}{360} - \frac{t}{3.1416} \times \sqrt{\frac{y-p}{2a} - \frac{y+p}{2a}\text{A}}$.

Cor. *Hence a table may be made for any given comet, to find its place in its orbit at any time; in like manner as was done for the planet's.*

For if the anomaly be taken ſucceſſively 1, 2, 3, 4, &c. degrees, z will be known for each of them, and conſequently y; and thence the correſponding time in moving from the perihelion. All which being put into a table, it will be ready for uſe. And this table need be continued no further than till the diſtance y be ſo great, that the comet becomes inviſible. Then by this table the comet's place may be found at any given time; by finding the time from the perihelion in the table, and taking the anomaly and diſtance correſponding thereto; uſing proportional parts when neceſſary. Fig.

PROP. XXIV. *Prob.*

To find the quantity of matter in any body that has another revolving about it.

The quantity of matter in any central body, is as the cube of the diſtance of the revolving body directly; and the ſquare of the periodical time inverſly (by Prop. VII. centripetal forces).

Let S be the Sun, E the earth, P a planet, T or Q a ſatellite, and let radius PT $= r$, periodical time of T or Q $= p$. Then the quantity of matter in the body P is as $\frac{r^3}{p^2}$ or $\frac{PT^3}{pp}$. 97.

Now p or the periodic time is known for all the planets and ſatellites; and to find PT, let ST, EQ be tangents to the orbit at T and Q; then PT is as SP × angle PST; or PQ, as PE × angle PEQ: becauſe theſe angles are very ſmall. Therefore SP × angle PST = PE × PEQ. Now the greateſt elongation, or the angle PEQ is found by obſervation; and at the ſame time the heliocentric places of the planet P, and the earth E are known, by the theory of the planets. Therefore in the triangle ESP, two ſides ES, SP, and the included angle ESP are given to find EP. And EP being

Fig. 97. known, as alſo PS, the angle PST will be known; and from hence QT or TP. And therefore the quantity $\frac{PT^3}{pp}$ will be known, which is proportional to the quantity of matter in P.

Therefore it will be as rad : PE : : S.QEP : PQ or PF.

Or elſe PS : PE : : angle PEQ : angle PST the heliocentric elongation; and then as rad : PS : : S.PST : PT. Thus the diſtance of Venus from the Sun is known to be 723.4. And PT for Jupiter's 4th ſatellite, is found to be 12.508; for Saturn's 4th ſatellite 8.5084; for the Moon 3.0546. And *p* or the periodic time for Venus, is $224^d.7$; for Jupiter's 4th ſatellite, 16.687^d; for Saturn's 4th ſatellite, 15.944; and for the Moon, 27.322. Therefore $\frac{PT^3}{pp}$ for the Sun, Jupiter, Saturn, and the Earth, comes out 7497.8; 7.0255; 2.423; and .03817. Therefore the quantities of matter in the Sun, Jupiter, Saturn, and the Earth, are as 196420, 184.05; 63.48; and 1, reſpectively. This is ſuppoſing the ſun's parallax to be 10½ ſeconds, and conſequently the angle, the radius of the moon's orbit, ſubtends at the ſun, 10′ 30″. If the ſun's parallax be more or leſs than $10'\frac{1}{2}$; the quantity of matter in the earth muſt be increaſed or diminiſhed in the triplicate of that ratio; the reſt remaining the ſame.

Cor. 1. *The real diameters of the ſun and planets, are; for the Sun* 10000, *Saturn* 790, *Jupiter* 996, *Mars* 57, *the Earth* 109, *Venus* 112, *Mercury* 40, *the Moon* 30.

For we found before, that PQ is as PE × angle PEQ. Therefore ſuppoſing QT to be any planet, the radius QP (and conſequently the diameter) is as the diſtance EP of the earth × by the apparent

radius

Fig. 97.

radius PEQ, or by the apparent diameter of the planet. But the apparent diameter of the planet is had by obſervation, and the diſtance EP is known from the aſtronomical tables; by finding thereby the heliocentric places of the earth and planet. The diameter of the earth here given, ſuppoſes the ſun's parallax to be $10\frac{1}{2}$ ſeconds. But if the ſun's parallax be greater or leſs, the earth's diameter muſt be increaſed or decreaſed in that ratio; the others remaining as before. The moon's diameter is found the ſame way as the reſt.

Cor. 2. *Hence the denſity of the ſun, and ſuch of the planets as have ſatellites, are; the Sun* 100, *Saturn* $65\frac{1}{2}$, *Jupiter* 95, *the Earth* 393.

For the quantity of matter in any of theſe bodies, is as the denſity and cube of the diameter. Therefore the denſity is as the quantity of matter divided by the cube of the diameter. But the quantities of matter are known by this Prop. and their diameters by Cor 1. and therefore you will have the denſities, in the proportion above laid down.

Cor 3. *The weight of a body at the ſurface of any planet, is as the denſity and diameter of the planet. And therefore is at the Sun* 23.34, *Saturn* 1.208, *Jupiter* 2.209, *and the earth* 1.

For let d = denſity of the planet, r = its radius. Then the weight at the given diſtance 1, is as the quantity of matter, that is, as dr^3; conſequently the weight at the diſtance r, is as $\frac{dr^3}{rr}$ or dr. But at the diſtance r the body is upon its ſurface; therefore at the ſurface, the weight of the body is as dr, or as $d \times 2r$. But d and $2r$ are both known by Cor. 2. and Cor 1. And therefore the weights at the ſurfaces, are found as above.

PROP.

Fig.

PROP. XXV. *Prob.*

To find the proportion between the axis of the ſun or a planet, and its equinoctial diameter.

Let r = radius of the ſun or planet, f = force of gravity on its ſurface, v = verſed ſine of the arch deſcribed in 1 ſecond by its rotation, to the radius r. H = equinoctial or greateſt radius. Then if the force of gravity f be denoted by the ſpace a heavy body deſcends in a ſecond at its ſurface, v will denote the centrifugal force ariſing from its revolving; and this is equal to the diminution of its gravity. But ſo much as the gravity of a column of the planet is diminiſhed, ſo much is its height increaſed, to make an equilibrium. And this is the ſame within the planet as at the ſurface; as both theſe forces decreaſe in the ſame ratio. Therefore we ſhall have $f : f - v :: H : r$. And $f : f - v :: 2H : 2r$, that is $f : f - v ::$ as the equinoctial diameter : to the polar diameter nearly.

Therefore if the diameter and time of revolution be known; the arch deſcribed in a ſecond will be known, and v the verſed ſine of that arch. Alſo f will be known, by having the gravitating force upon the ſurface. And hence the ratio of the diameters will be found.

Cor. *In the earth, the ratio of the axis to the equatorial diameter, will be found as 285 to 286. And in Jupiter as* $11'$ *to 12. In the ſun the difference is quite inſenſible. In the reſt of the planets, there wants the proper data.*

SCHOLIUM.

The times of the revolutions of the planets round their axes have been found out by obſerving certain ſpots upon their ſurfaces. Theſe ſpots ad-

hering to the ſurfaces of the planets, revolve along Fig.
with them. Therefore if the time be meaſured between the appearance of any ſpot, and its coming again into the ſame poſition; the time of the planet's revolution is had. Hence the time of rotation of any planet may be found, if there be any dark ſpots upon its ſurface, that can be ſeen and obſerved by a teleſcope. But ſome of them afford no ſuch thing; as Saturn and Mercury. The periodic times of theſe that have been obſerved are as follows.

The Sun	in	25^d	15^h	16^m.
Jupiter	—		9	56.
Mars	—	1	0	40.
Earth	—		23	56.
Venus	—		23	20.

PROP. XXVI. *Prob.*

To determine the diſturbing forces wherewith the planets act upon one another; and their effects.

Let A, B be two planets, as Saturn and Jupiter. 98.
Then ſince all bodies attract one another, with forces which are as the quantities of matter; the planet B will attract A, and retard it in its orbit; and at the ſame time A will attract B, and accelerate it in its orbit; but when B has got to C, it will attract A, and accelerate it in its orbit; and likewiſe A will attract C, and retard it in its orbit. All this follows from the mutual actions of theſe two bodies on one another. And the forces of B and A upon one another will be as the quantities of matter in B and A reſpectively; and the acceleration or retardation, will be in the ſame ratio. Alſo theſe forces and their effects are the greater, the nearer the bodies are; and that in the reciprocal duplicate ratio of the diſtance. Whence, after the bodies

are

Fig. 98. are removed to a great diſtance from one another, their effects upon one another become inſenſible.

And to determine the quantity of force, each body exerts on the other. Let D = diſtance of A from the ſun. S = quantity of matter in the ſun, B and A = quantities of matter in B and A. Then the force of the ſun upon A : is to the force of B upon A : : as $\frac{S}{DD}$: to $\frac{B}{BA^2}$. And the like for the force of A upon B. Therefore the quantities of matter in the bodies, S and A; and alſo their diſtances being known; the diſturbing force of B upon A, or of A upon B will be known.

Cor. 1. *Hence the linear error, cauſed by one body* B *acting upon another* A, *for any time, will be known. And that is done by finding the verſed ſine* v, *of the arch deſcribed in that time by* A *in its orbit; and making as* $\frac{S}{DD}$: *to* $\frac{B}{BA^2}$: : v : *to a fourth, which will be the linear error required.*

For the line v is that, which the force of the ſun, would cauſe the body A to deſcribe, in the given time; and therefore v is the genuine effect of that force. Therefore the effect of the force $\frac{B}{BA^2}$ upon A, will be found by the ſaid proportion, as all ſimilar effects are proportional to their cauſes.

It is found by experience, that Saturn's periodic time is ſometimes prolonged 13 days, by the action of Jupiter upon him. For when Saturn and Jupiter are neareſt together, their diſtance will be to Saturn's diſtance from the ſun, about as 4 to 9. And therefore the force of Jupiter is to the force of the Sun, upon Saturn, as $\frac{184}{16}$ to $\frac{196420}{81}$ or as 1 to 211. Therefore it is no wonder, if the periodic

riodic time of Saturn be conſiderably ſhortened, Fig. 98.
when its centripetal force is increaſed $\frac{1}{211}$ part.

The diſturbing force of Saturn upon Jupiter is much leſs, for two reaſons, 1. becauſe Saturn is about 3 times leſs than Jupiter; and 2. becauſe Jupiter is nearer the Sun than Saturn, and ſo the Sun's force upon him is more than 3 times as great. So that Jupiter is about 10 times leſs diſturbed by Saturn than Saturn is by Jupiter; which therefore is ſcarce ſenſible.

Cor. 2. *And hence it is evident, that when a comet comes near Saturn or Jupiter, it muſt needs have the poſition and figure of its orbit changed, as well as its periodical time, from the diſturbing force of ſo great a body.*

For the orbits of comets being very eccentric ellipſes; a ſmall change of the centripetal force, will make a great change in theſe orbits; and even be ſufficient to change them into parabolic or hyperbolic ones.

Cor. 3. *Hence, when any two planets (as Saturn and Jupiter) are in heliocentric conjunction. The periodical time of the outermoſt will be ſhortened, and of the innermoſt lengthened.*

This ariſes from the action of the two planets, upon one another, when ſo near together. For the centripetal force of the ſun upon the outermoſt will be increaſed by the action of the inner one, which ſhortens the periodic time of the outer one. And the ſun's centripetal force upon the inner one will be diminiſhed, by the action of the outer one; and that lengthens its periodic time; by Prop. III. Centripetal forces.

SECT. VI.

The Theory of the Moon, and the Satellites; the Sun's diſturbing Force upon them. The errors in the Moon's and Satellite's Motions. Conſtruction of Tables, and their Uſe in finding the Moon's place, and time of the Syziges. Moon's Libration, her Quantity of Matter, Denſity, &c. To find her diſtance from a Star. The Errors of the Satellites.

AS the primary planets revolve about the ſun as their center of motion, ſo does the moon about the earth, and the ſatellites about their reſpective primary planets. And if the moon was acted upon by no force but the centripetal force of the earth, ſhe would deſcribe a perfect ellipſis about the earth in the focus thereof; the principal axis whereof would remain at reſt, and always of the ſame quantity; her periodic times would all be equal; and the inclination of her orbit invariable, as alſo the place of her nodes. And then it would be eaſier to calculate her place, than it is to calculate the place of any of the planets.

But ſince her motions are diſturbed by the action of the ſun, they become ſubject to ſo many inequalities, that it is a matter of the greateſt difficulty to calculate her place exactly. And no ſet of tables have ever been made, by which her place can be truly found at all times; but in an age they will become erroneous and almoſt uſeleſs; tho' they were at firſt perfectly agreeable to the heavens. And perhaps no ſet of tables will ever be framed, which will give her place true and exact for all ages. And what is here ſaid of the irregular motions of the moon, is likewiſe true of all the ſatellites in ſome degree or other; tho' by reaſon of their great diſtance, they cannot be exactly obſerved here.

One would be ſurprized that ſo ſmall a matter, as a little variation in the ſun's force, ſhould cauſe ſo many irregular motions in the moon, as ſhe is found to have; and much more, that they can be brought to no certain rules, but ſuch as are in the higheſt degree intricate and laborious. So that this ſingle ſecondary planet gives the aſtronomers more trouble, than all the celeſtial bodies beſides. Mention has been made of theſe irregular motions in the Schol. of Prob. XVIII. Section IV; and the methods of finding the mean quantities of theſe motions by obſervation, is ſhewn in that Prop. But to treat this matter at large, in all particulars relating to the moon's motion, would require a large volume of itſelf, which would not at all ſuit with the intended brevity, in this courſe of mathematics. But that I may give the reader as much as I can in ſo little compaſs; I ſhall, in what follows, demonſtrate the principal irregularities that attend her motion, and the calculation of her place, by ſuch tables as we have already; leaving the conſtruction of new ones to thoſe that have more time and leiſure, and better convenience; which is properly a work for thoſe that have the uſe and advantage of obſervatories, where they can prove their truth, and rectify them at pleaſure. But cannot be truly executed by private perſons, that are deſtitute of ſuch helps.

PROP. I. *Prob.*

To find the forces by which the ſun diſturbs the motions of the moon.

99. Let CADB be the moon's orbit, T the earth, S the ſun, P the moon; make SK = ST; and let SK : SL : : $SP^2 : SK^2$. Then if SK or ST repreſent the ſun's force at T, SL will repreſent his force at P. Draw LM parallel to PT; divide the force LS into the two forces LM acting parallel to PT, and MS acting parallel to TS. But the force LM, and the part TM diſturb the moon's motion. The force LM in its mean quantity is equal to PT, and by ſo much the force of the earth upon the moon is increaſed. Alſo TM in its mean quantity is equal to 3PK, acting in direction PN parallel and equal to TS; and this force MT draws the moon out of her orbit. Let P, *p* be the periodical times of the earth and moon; then (by Prop. V. Cent. Forces), the ſun's centripetal force at T (ST) : the earth's centripetal force at P : : $\frac{ST}{PP} : \frac{PT}{pp}$; therefore the earth's centripetal force at P = $\frac{PT \times PP}{pp}$. And this is to the additional force PT : : $\frac{PT \times PP}{pp}$: PT : : PP : *pp*. That is, the force by which the moon is retained in her orbit : is to the increaſe of centripetal force by the ſun's action : : PP : *pp* : : 178.725 : 1. Therefore the increaſe of the moon's centripetal force is $\frac{1}{178{,}725}$ of that force.

Alſo force PT : force 3PK or PL : : PT : 3PK, Therefore, ex æquo, the force by which the moon is retained in her orbit : diſturbing force PL or TM : : PT × 178.725 : 3PK. Therefore the diſturbing force TM = $\frac{3PK}{PT \times 178.725}$ × earth's centrip. force on the moon = $3a$ the ſine of the moon's diſtance from the quadratures × earth's centripetal force, divided by 178.725 × radius. Fig. 99.

Cor. 1. *Let* C, c *be the centripetal forces of the ſun and earth,* S = *ſine of the moon's diſtance from the quadrature, radius* = r. *Then the additional force* (PT) = $\frac{c}{178.725}$. *And the diſturbing force* (TM) = $\frac{3sc}{178.725r}$.

Cor. 2. *Produce* TP, *and make* PR = PL *or* TM, *and draw* RQ *perp. to* TQ. *Then* QR *is the force that accelerates the moon, and* PQ *is the diminution of its centripetal force.*

For the force PR is divided into the two forces, PQ and QR, of which PQ acting towards Q diminiſhes the moon's centripetal force; and QR being parallel to the tangent at P, accelerates the moon at P.

Cor. 3. *There are four points in the moon's orbit, each* 35° 16′ *from the quadratures, where the moon's diſturbing force makes no alteration in the earth's central force.*

For the triangles PKT and PQR are ſimilar, whence PK : PT : : PQ (PT) : PR (3PK); therefore $3PK^2 = PT^2$, or $3ss = rr$, whence $s = \sqrt{\frac{rr}{3}}$ = S. 35° 16′. And when PQ = PT, the diminution of the force is equal to the addition, which makes no alteration.

Fig. 99.

Cor. 4. *The mean force* PT *is* $= \frac{1}{643410}$ *of the force of gravity at the earth.*

For the force of gravity is 3600 times greater than the force at P.

Cor. 5. *The whole increaſe of the centripetal force at* P *is* $\frac{c}{178.725} \times \overline{1 - \frac{3ss}{rr}}$.

For rad $(r) : s : :$ PT $\left(\frac{c}{178.725}\right)$: PK $= \frac{cs}{178.725r}$. And PR = 3PK $= \frac{3cs}{178.725r}$. And rad (r) : PR $\left(\frac{3cs}{178.725r}\right)$: : S.R (s) : PQ $= \frac{3css}{178.725rr}$. And PT — PQ = whole additional force $= \frac{1 - \frac{3ss}{rr}}{178.725} c$.

Cor. 6. *If* A = *ſine of twice the moon's diſtance from the quadratures; then the force* QR, *accelerating or retarding the moon's motion in its orbit, is* $\frac{3c}{178.725} \times \frac{A}{2r}$.

Let z = S.QPR or KPT = coſ. PTK; then rad (r) : RP $\left(\frac{3cs}{178.725r}\right)$: : S.QPR (z) : QR $= \frac{3c}{178.725rr} \times sz =$ (by trigonometry) $\frac{3c}{178.725} \times \frac{A}{2r}$.

Cor. 7. *Hence the moon is accelerated in the quadrants* CA, DB; *and retarded in the quadrants* AD, BC.

Cor. 8. *The force which accelerates or retards the moon's motion, is greateſt in the octants.*

For it is greateſt when A is greateſt, that is when 2CP is 90 degrees, or CP = 45°. Fig. 99.

Cor. 9. *The diſturbing force* TM, *in the ſyziges* A *and* B, *is* 2PT. *And therefore the earth's force upon the moon in the ſyziges, is twice as much diminiſhed; as it is increaſed in the quadratures.*

Cor. 10. *The moon's orbit is more flat in the ſyziges, and more curve in the quadratures; and therefore ſhe goes farther from the earth in the quadratures.*
For the orbit will be more curve where the central force is greater, that is in the quadratures.

PROP. II. *Prob.*

To find the moment of the area which the moon deſcribes in any place P; *her orbit being nearly circular.* 100.

Let c = earth's centripetal force on the moon, s and z the ſine and coſine of PTK. Let RPL be parallel to AB, and ſuppoſe the arch Co and alſo Pp to be deſcribed by the moon in a ſecond. Draw no, Pt parallel to CT. Then (by Cor. 6. Prop. I.) the force QR that accelerates the moon at P, is $= \frac{3c}{178.725} \times \frac{A}{2r} = \frac{3sz}{178.725rr} \times c$; and no = ſpace deſcended at C in a ſecond, by the force c; and $2no$ or $\frac{Co^2}{CT}$ = velocity generated by c in a ſecond. And the velocity generated (in an equal time) being as the force, $\frac{3sz}{178.725rr} \times \frac{Co^2}{CT}$ = velocity generated in a ſecond at P, by the force $\frac{3sz}{178.725rr} \times c$. And the whole velocity generated by all the forces $\frac{3sz}{178.725rr} \times c$, is = ſum of all the

Fig. 100. the $\frac{3sz}{178.725rr} \times \frac{Co^2}{CT} = \frac{3}{178.725rr} \times \frac{Co}{CT} \times$ ſum of all the $Pp \times sz$ in the arch CP.

Therefore if $TC = r$, $PK = s$, $TK = z$, arch $CP = v$. Then by ſimilar triangles, $TP \times Pt = PK \times Pp$; that is, $-r\dot{z} = s\dot{v}$. Then the ſum of all the $Pp \times sz =$ fluent of $sz\dot{v}$ or $-rz\dot{z} = -\frac{rzz}{2}$. But at C, $z = r$, therefore the correct fluent is $\frac{r}{2} \times \overline{rr - zz} = \frac{rss}{2} =$ ſum of all the $Pp \times sz$. Therefore the whole velocity generated at P, by the force QR, is $= \frac{3}{178.725rr} \times \frac{Co}{CT} \times \frac{rss}{2} = \frac{3}{178.725} \times \frac{Co}{2rr}ss$. Whence, moon's velocity (Co): this increment of velocity $\left(\frac{3}{178.725} \times \frac{Co}{2rr}ss\right) :: \frac{2}{3} \times 178.723rr : ss :: 119.15rr : ss$.

Therefore the increment of velocity at any place P, above the velocity at C, is $= \frac{ss}{119.15rr} \times Pp = \frac{PK^2}{119.15PT^2} \times$ moon's velocity. All this ſuppoſes the ſynodic revolution to be $27^d\ 7^h\ 43^m$. But ſince it is $29^d\ 12^h\ 44^m$; the increment of velocity muſt be increaſed in the ſame proportion; that is, in the ratio of 10808 to 10000; and then the whole increment of velocity at any place P will be $= \frac{PK^2}{112.4PT^2} \times$ by the moon's velocity. Therefore the velocity at C : to the velocity at any place P :: $Co : CO + \frac{ss}{112.4rr} \times Co :: 112.4rr : 112.4rr + ss$. That is, the velocity at C : velocity at P :

$112.4PT^2 : 112.4PT^2 + PK^2$. And the moments of the area at C and P (being as the velocities) are in the ſame proportion. Fig. 100.

Cor. 1. *Hence the whole increment of the moon's velocity at any place* P, *in paſſing thro'* CP; *is as the ſquare of* PK. *But the momentary increment (being as* PK × KT) *increaſes to the octants where it is greateſt, and from thence decreaſes to* A *where it is nothing.*

Cor. 2. *The moment of the area in the quadrature* C, *is to the moment of the area in the ſyzigy* : : *as* 112.4 : 113.4; *and the ſame is true of the velocities.*

For if the velocity at C be 112.4, the increment of velocity at A is 1 where $s = r$; and the whole is 113.4. And the ſame for the areas, deſcribed in equal times.

Cor. 3. *In the octants, the moment of the area, is to the moment of the area in the quadratures* : : *as* 225.8 *to* 224.8.

For then $ss = \frac{1}{2}rr$. And the velocity or increment of the area at C, to the velocity or increment at P, is as $112.4rr$ to $112.4rr + \frac{1}{2}rr$, or as 224.8 to 225.8.

Cor. 4. *The ſpace deſcribed by all the additional velocities, in paſſing thro' the quadrant* CA, *is* = $\frac{1}{224.8} \times CA$. *And in paſſing thro' any arch* CP, *is as the area of the ſegment* CPK.

The ſpaces being as the velocities, whilſt the moon deſcribes the ſpace Pp, with the velocity at C; ſhe will (by Prop. II.) deſcribe the ſpace $\frac{ss}{112.4rr} \times Pp$, by her additional velocity. And whilſt ſhe deſcribes (the ſum of all the Pp) or CP, ſhe will deſcribe an additional ſpace = ſum of

Fig. 100. all the $\frac{ss}{112.4rr}$ Pp $= \frac{1}{112.4rr} \times$ ſum of all the Pp $\times ss$. But the ſum of all the P$p \times ss =$ ſegment CPK $\times r$. Therefore whilſt ſhe deſcribes CP, ſhe will deſcribe (by her additional velocity) an arch $= \frac{1}{112.4rr} \times \mathrm{CPK} \times r = \frac{1}{112.4} \times \frac{\mathrm{CPK}}{r}$. And when ſhe comes at A, ſhe will deſcribe the additional arch $\frac{1}{112.4} \times \frac{\mathrm{CAT}}{r} = \frac{1}{112.4} \times \frac{\mathrm{CA} \times \mathrm{TA}}{2r} = \frac{1}{224.8}\mathrm{CA}$.

PROP. III. *Prob.*

To find nearly the proportion of the diameters of the moon's orbit; ſuppoſing there is little or no excentricity.

101. As this orbit differs not much from an ellipſis, we will ſuppoſe it to be the ellipſis CADB, C and D the quadratures, A and B the ſyziges, T the earth in the center. Let the ſemiconjugate AT $= x$, the ſemitranſverſe TC $= y$.

By Ex. I. Prop. XIX. Sect. II. B. II. Curve Lines, the radius of curvature at the vertex of an ellipſis is $\frac{1}{2}$ latus rectum; therefore the radius of curvature at A $= \frac{yy}{x}$, and at C $= \frac{xx}{y}$. Alſo (by Cor. 5. Prop. I.) the increaſe of the centripetal force at C (where $s = 0$) is $\frac{1}{178.725}c$. And the decreaſe at A (where $s = r$) is $\frac{2}{178.725}c$. Therefore the force at C is $\frac{179.725}{178.725}c$, and the force at A is $\frac{176.725}{178.725}c$. Therefore the force at A to the force at C, is as 176.725 to 179.725. Alſo (by Cor. 2. Prop. II.) the velocity at A is to the velocity at C, as 113.4 to

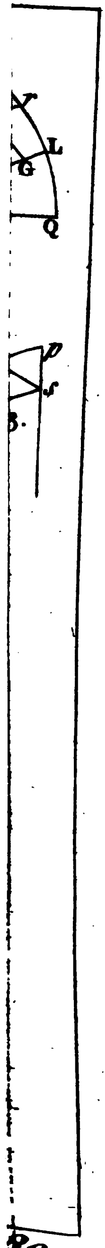


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Fig. to 112.4, and ſo are the ſmall arches Aa, Cc deſcribed in equal times. But the verſed ſines of theſe arches are as the forces 176.725 and 179.725. And the ſquares of the arches (Aa, Cc) divided by their verſed ſines, are as the radii of curvature in A and C. Whence it follows (putting the velocities for the arches, and the forces for the verſed ſines) that $\frac{\overline{113.4}^2}{176.725} : \frac{\overline{112.4}^2}{179.725} :: \frac{yy}{x} : \frac{xx}{y}$; therefore $\frac{\overline{113.4}^2}{176.725} \times \frac{xx}{y} = \frac{\overline{112.4}^2}{179.725} \times \frac{yy}{x}$, and $\overline{113.4}^2 \times 179.725 \times x^3 = \overline{112.4}^2 \times 176.725 \times y^3$; whence $x^3 : y^3 :: \overline{112.4}^2 \times 176.725 : \overline{113.4}^2 \times 179.725$. And $x : y :: \sqrt[3]{\overline{112.4}^2 \times 176.725} : \sqrt{\overline{113.4}^2 \times 179.725} :: 130.7 : 132.22 :: 86.2 : 87.2$. Therefore the moon's diſtance from the earth in the ſyziges, is to her diſtance in the quadratures, as 86 to 87. It is to be noted, that Sir Iſaac Newton makes them to be as 69 to 70, which doubtleſs are more exact. But it depends upon a very intricate calculation.

PROP. IV. *Prob.*

To find the variation of the moon.

The *variation* is the difference between the mean and true place of the moon; and is owing partly to the elliptic figure of the moon's orbit, and partly to the inequality of the areas deſcribed in different places thereof.

102. About the orbit CAD deſcribe the ſemicircle CFD. Thro' P draw BPK perp. to DC, and draw TP, TB. Then the angle CTB will be the mean motion, and CTP the true motion. For the area

TPC

Fig. TPC is to the area TBC : : as TFC to TAC : : or as TF to TA. Therefore if BTC = 45 deg. and BK to PK as FT or TC to AT, that is, as 87 to 86. Then PTK will be 44° 40′ 8″, and BTP = 19′ 52″, which is one part of the variation, or that ariſing from the elliptic figure of the orbit. Here if we had made uſe of Sir J. Newton's numbers 69 and 70, we ſhould have had the angle BTP = 24′ 42″.

103. Let CA be a quadrant of the orbit, take C*m* = 45 degrees, and let *m* be the mean place, *a* the true place, and *f*, F her places which ſhe would have without any acceleration. Since ſhe moves ſloweſt at firſt, her mean place *m* will be before her true place *a*; and both of them before her place *f*. By Cor. 4. Prop. II. $AF = \frac{1}{224.8} \times CA = 24'.02$, and ſince both *f* and *m* move uniformly, therefore C*f* is as C*m*, and *fm* as C*m*; and ſince C*m* = ½CA, *fm* = ½FA = 12′.01. But (by Cor. 4. Prop. II.) *fa* : FA (24.02) : : ſegment C*m*K : area CAT : : 1427 : 7854; therefore *fa* = 4′.36. Conſequently *ma* (*fm* — *fa*) = 7′.65 = 7′ 39″, which is the other part ariſing from the acceleration. This added to the angle BTP before found makes 27′ 31″ for the greateſt variation. But if Sir J. Newton's numbers were made uſe of, where the angle PTP is 24′ 42″, the geateſt variation in the octants would be 32′ 21″. But this is true ſuppoſing the moon to deſcribe only 90 degrees from the quadratures to the ſyziges. But as ſhe deſcribes a greater angle in proportion of the ſynodic revolution to the periodic, or as 10808 to 10000; therefore all the angles are dilated in that proportion; and therefore the greateſt equation will be 29′ 44″. Or from Sir J. Newton's numbers, 34′ 58″.

PROP.

PROP. V. *Prob.*

To find the motion of the moon's nodes, her orbit being nearly circular.

Let $AqBQ$ be the moon's orbit, T the earth, P the moon, SAB the line of the apſides, Q, *q* the quadratures, *m*N*n* the line of the nodes. PK, PH, AZ perpendiculars upon TQ, and N*n*. Then (by Cor. 1. Prop. I.) the force by which the moon is drawn out of her orbit is $\frac{3s}{178.725r}c = \frac{s}{59.575r}c$. Let PM be the arch which the moon deſcribes in any ſmall time, and ML a ſmall line, which the moon deſcribes in the ſame time by the force $\frac{s}{59.575r}c$, as this force is directed to the ſun, the line ML will be parallel to TS. As ML is the diſtance that the moon is drawn from the arch PM, by the ſaid accelerative force; 2ML will be the uniform motion it has acquired in that time, by the ſaid force. Let MP be continued back to *m*, in the moon's orbit, to cut the line of the nodes TN in *m*. Now ſince ML is parallel to the ecliptic; a plane drawn thro' ML and MP*m*, will cut the ecliptic in a line *ml*, which will be parallel to ML (by Cor. Prop. XI. B. V. Geometry); therefore draw LP*l* to cut *ml* in *l*, and the triangles PML and P*ml* are ſimilar, and therefore $ml = \frac{mP \times ML}{MP}$, but ſince MP is given, and ML is as the force 3PK, therefore *ml* is as *m*P × PK. Now when the moon was at P, the line of the nodes was at T*m*, where the plane of the moon's orbit TMP cuts the ecliptic. But when the moon comes to L (inſtead of M), the plane of her orbit will then be in the plane TLP; and the line of the nodes at T*l*, where the plane TLP*l* cuts the ecliptic. Therefore 104.

Fig. fore the angular motion of the nodes generated in 104. that time will be = angle mTl. But the angle mTl is as $\frac{ml}{mT} \times$ ſine of Tml or STN; that is, as $\frac{mP \times PK}{mT} \times AZ$; that is, (becauſe by ſimilar triangles $\frac{mP}{mT} = \frac{PH}{PT}$) as $\frac{PH \times PK}{PT} \times AZ$, that is, (becauſe PT is given) as $PH \times PK \times AZ$.

The line ML is to the verſed ſine of the arch PM, as the forces that produce them; that is, as $\frac{s}{59.575r}c$ to c, or as s to $59.575r$. That is $ML : \frac{PM^2}{2MT} :: s : 59.575r$; therefore $ML = \frac{PM^2}{2MT} \times \frac{s}{59.575r}$. When P falls upon A, or the moon is in the ſyzigy; then $s = r$, and the angle PML is a right angle; therefore in the triangle PML, $PM : ML \left(\frac{PM^2}{2MT \times 59.575}\right) ::$ radius : S.LPM or $mPl = \frac{PM}{2MT \times 59.575}$. And if N be at Q, or the nodes in the quadratures, and P at A; then PM and PL being parallel to the ecliptic, m and l will be at an infinite diſtance, and then the angle mTl will be equal to mPl, whoſe ſine is $\frac{PM}{2MT \times 59.575}$. And the angle mTl anſwering to 2ML is the motion of the node, whoſe ſine is $\frac{PM}{MT \times 59.575}$. Suppoſe PM to be deſcribed in an hour, then PM or the mean horary motion is 32′ 56″½, whoſe ſine, as it differs inſenſibly from the arch, we ſhall have the arch $= \frac{32' \, 56''\frac{1}{2}}{59.575}$ (putting

♈MT

PM = 1) = $33''.18$, where all the angles PTK, PTN and STN are right angles. And in other caſes, the horary motion of the node will be to $33''.18$, as the product of the ſines of the three angles PTK, PTN and STN, to the radius cube; and the nodes are regreſſive when all the ſines are poſitive. But if any ſine changes to be negative, the nodes will be progreſſive. Fig. 104.

Cor. 1. *The nodes are progreſſive, when the moon is between either quadrature, and the node neareſt that quadrature; otherwiſe they are regreſſive. And by the exceſs of the regreſs above the progreſs, they are in the whole moved forward.*

For in the arches QA*n* and *q*BN; PK and PH are both affirmative or both negative. And in the arches NQ, *nq*; only one is negative, the other being affirmative.

Cor. 2. *In any given poſition of the nodes in reſpect of the ſun, their mean horary motion is half their horary motion in the moon's ſyziges; and is to* $16''.59$, *as* AZ^2 *to* AT^2.

For let $QP = z$, $QK = x$, $PK = y$, tan. $QN = t$, $AZ = s$; $PT = r$, $h = 33'.18$. Then if $PM = \dot z$, by the nature of the circle $y\dot z = r\dot x$. But by this Prop. the horary motion of the nodes at P $= \dfrac{AZ \times PH \times PK}{PT^3} \times 33''.18$. Produce PK to D, and by ſimilar triangles $TQ\,(r) : TK\,(r - x) :: t : KD = \dfrac{r - x}{r}t$, and $PD = y + \dfrac{r - x}{r}t$; again by ſimilar triangles, $AT\,(r) : AZ\,(s) :: PD \left(y + \dfrac{r - x}{r}t\right) : PH = \dfrac{sy}{r} + \dfrac{r - x}{rr}ts$. Therefore the horary motion of the nodes at P $= \dfrac{syh}{r^3} \times$

Fig. 104. $\overline{\frac{sy}{r}+\frac{r-x}{rr}ts}$. And the ſum of all the horary motions in the ſemicircle = ſum of all the $\frac{ssyb}{r^4}\times\overline{y+\frac{r-x}{r}t}\times PM$ = flu. of $\frac{ssyb}{r^4}\times\overline{y+\frac{r-x}{r}}:$ $\times\dot{z}$ = flu. of $\frac{ssrb}{r^4}\times\overline{y+\frac{r-x}{r}t}\times\dot{x}$ = flu. of $\frac{ssb}{r^3}\times\overline{y\dot{x}+\frac{r\dot{x}-x\dot{x}}{r}t}=\frac{ssb}{r^3}\times\overline{\text{area QPK}+\frac{rx-\frac{1}{2}xx}{r}t}$. And when $x=2r$, that ſum is $=\frac{ssb}{r^3}\times\overline{\text{ſemicircle QA}q+\frac{2r^2-2r^2}{r}t}=\text{QA}q\times\frac{ssb}{r^3}=\frac{ssb}{2rr}\times$ arch QAq.

Again, when P is in A, PH = AZ, and PK = AT. Then the ſum of all the $\frac{AZ\times PH\times PK}{PT^3}b$ = ſum of the $\frac{AZ^2}{PT^2}b$ = fl. $\frac{ss}{rr}b\dot{z}=\frac{ssbz}{rr}=\frac{ssb}{rr}\times$ arch QAq. And the former ſum is to the latter, as $\frac{ssb}{2rr}$ to $\frac{ssb}{rr}$, or as 1 to 2. Therefore if the nodes went on with the velocity they have in the moon's ſyziges, they would deſcribe twice the ſpace they now deſcribe; and therefore the mean motion is but half their motion in the ſyzigy.

Laſtly, ſince $\frac{AZ^2}{PT^2}b$ = horary motion at A, the mean horary motion is $\frac{AZ^2}{2PT^2}b$, and this is to $\frac{1}{2}b$ (16″.59), as $\frac{AZ^2}{PT^2}$ to 1 or as AZ² to PT².

Cor. 3. *If the moon's orbit be an ellipſis inſtead of the circle* QAqB; *the mean horary motion will be contracted in the ratio of the tranſverſe to the conjugate.* Fig. 104.

For the force 3PK being leſſened in the ratio of TQ to TA; the line LM, and conſequently the angle LPM will be diminiſhed in the ſame ratio. Therefore the horary motion, in any place, and the ſum of all the horary motions will be diminiſhed in the ſame ratio, or as the area of the circle to that of the ellipſis. That is, the mean motion of the nodes in the circle will be to the mean motion of the nodes in the ellipſis, as the circle to the ellipſis, or as the tranſverſe to the conjugate.

Cor. 4. *But in the ellipſis, if* V *and* v *be the moon's velocities in the ſyziges and quadratures; the motion of the nodes in the quadratures, to the motion in the ſyziges, in deſcribing equal parts of its orbit, will be as* $\frac{1}{vv}$ *to* $\frac{1}{VV}$.

For the motion of the nodes is as ML, which is as the ſquare of the time in MP, and therefore reciprocally as the ſquare of the velocity.

Cor. 5. *Hence the mean horary motion of the node throughout the year will be about* $8''.17$; *and in the whole year* 19.9 *deg.*

The mean horary motion throughout the year is the mean of all the mean horary motions, for all poſitions of the node. Let $t = \text{TQ}$, $c = \text{TA}$, then (by Cor. 2. and 3.) the mean horary motion $= \frac{ssb}{2rr} \times \frac{c}{t} = \frac{cb}{2rrt} \times ss$. And the ſum of all the horary motions in the circle is $= \frac{cb}{2rrt} \times$ ſum of all the $ss =$ (becauſe the ſum of all the $ss = r \times$ area of the circle) $\frac{cbr}{2rrt} \times$ area

Fig. 104. QAqB $= \frac{cbr}{4rrt} \times r \times$ circumference QAqB $= \frac{cb}{4t} \times$ circumference QAqB.

But when $s = r$, the mean horary motion is $\frac{cb}{2t}$, and the ſum of all the $\frac{cb}{2t}$ is $= \frac{cb}{2t} \times$ circumference QAqB; and the former ſum to the latter, is as $\frac{cb}{4t}$ to $\frac{cb}{2t}$ or as 1 to 2. Therefore the mean annual horary motion of the nodes is $= \frac{cb}{4t} =$ 8″.176; and in the whole year is 19° 54′.

PROP. VI.

The horary variation of the inclination of the moon's orbit, is to the horary motion of the nodes : : as the ſine of the inclination of the orbit, to the tangent of the moon's diſtance from the node.

105. Let NPn be the moon's orbit, Nn the line of the nodes, P the moon, Pp a perpendicular upon the plane of the ecliptic, NTm the horary motion of the nodes. Draw PH perp. to TN, and draw pHg to cut Tm in g, and draw Pg. Then the angle PHp will be the inclination of the moon's orbit, to the plane of the ecliptic when the moon is at P, and Pgp is the inclination after an hour; therefore HPg is the horary variation of inclination. Let $p = s.$inclination, $q =$ tan. NP, and let Ho be perp. to PH, then rad (r) : Hg : : ſine of Hgo or PHp (p) : Go $= \frac{p}{r} \times$ Hg. Then the motion of the nodes or angle HTg : variation of inclination or angle HPg or HPo : : $\frac{\text{H}g}{\text{HT}} : \frac{\text{H}o}{\text{HP}}$ or

$p \times$

$\frac{p \times Hg}{r \times HP} :: \frac{r \times HP}{HT} : p ::$ tan. PN $(q) : p$; and Fig. 105.
$HPg = \frac{p}{q} \times HTg = \frac{p}{q} \times NTm$.

Cor. 1. *If* Q, *q are the quadratures*; NR, *nr quadrants. Then the variation of inclination is nothing at the four points* Q, R, *q*, *r*. *And the inclination decreaſes from* Q *to* R, *and from q to r, where it is leaſt; and increaſes from* R *to q, and from r to* Q, *where it is greateſt.*

For at Q, q the nodes are ſtationary, and NTm is $= 0$, therefore $HPg = 0$; and at R and r, q is infinite, therefore $\frac{p}{q} \times NTm$ or $HPg = 0$. And the inclination decreaſes in the arches QR, and qr. And increaſes in the arches Rq, and rQ. For Pgp is leſs than PHp, &c.

Cor. 2. *If* TL *be perp. to* TN, *and* KP *produced* 106. *to* D, *and let* n = *number of hours in deſcribing* QP, f = *ſine of* 2ATN *the double diſtance of the ſun from the nodes. Then the decreaſe of inclination from* Q *to* P *will be* $= \frac{pnf \times \text{area QLDP}}{2r^3 \times QP} b$.

Putting the ſymbols as before; then the horary variation $HPg = \frac{p}{q} \times NTm = \frac{p}{r} \times \frac{HT}{HP} \times NTm$ $=$ (Cor. 2. Prop. V.) $\frac{p \times HT}{r \times PH} \times \frac{AZ \times PH \times PK}{AT^3} b$ $= \frac{pb}{r} \times \frac{HT \times AZ \times PK}{AT^3} = \frac{psyb}{r^4} \times HT$. But $PK \times \dot{z} = AT \times \dot{x}$; and drawing HI parallel to KD, we have, by ſimilar triangles, TH : IH or DP :: TZ : AT, whence $TH = \frac{DP \times TZ}{AT}$; and there-

Fig. 106. fore HP$g = \frac{psyb}{r^4} \times \frac{DP \times TZ}{r}$ = (becauſe AZ × TZ $= \frac{rf}{2}$) $\frac{pyfb}{2r^4} \times DP = \frac{pfb}{2r^4} \times DP \times PK$. And the ſum of all the variations in the arch QP = ſum of all the $\frac{pfb}{2r^4} \times DP \times PK$ = fluent of $\frac{pfb}{2r^4} \times DP \times PK \times \dot{z} = \frac{pfb}{2r^4} \times$ fl. DP × AT × $\dot{x}$ = fl. $\frac{pfb}{2r^3} \times DP \times \dot{x} = \frac{pfb}{2r^3} \times$ area QPDL. But as b the horary motion of the nodes is given, we ſhall have QP : n (the number of hours) : : $\frac{pfb}{2r^3} \times$ QPDL : whole variation in that time $= \frac{pnfb}{2r^3} \times \frac{QPDL}{QP}$.

And at R where the inclination is leaſt, the whole diminution $= \frac{pnfb}{2r^3} \times \frac{\text{area } QRL}{QR}$.

Cor. 3. *If the nodes are in the quadratures, the variation in the arch* QP $= \frac{pnyy}{2rr \times QP} b$. *And at A where it is leaſt, it is* $= \frac{pn}{2QP} \times b$.

In this caſe the area QPDL is infinite, and $f = 0$, from which no ſolution can be had. Therefore we muſt have recourſe to the original proceſs, where HG$p = \frac{pb}{r} \times \frac{HT \times AZ \times PK}{AT^3}$ = (in this caſe) $\frac{pb}{r} \times \frac{KT \times TA \times PK}{AT^3} = \frac{pb}{r^3} \times TK \times PK$. Therefore the ſum of all the HG$p = \frac{pb}{r^3} \times$ ſum of all the TK × PK. But the ſum of all the TK × PK =

$\frac{ryy}{2}$. Whence the ſum of all the variations in QP = Fig. 106. $\frac{pbyy}{2rr}$. But this muſt be diminiſhed according to the number of hours, whence QP : n :: $\frac{pbyy}{2rr}$: $\frac{pnyy}{2rr \times \mathrm{QP}}b$. And when $y = r$, the whole diminution in paſſing from the quadratures to the ſyziges is $= \frac{pn}{2\mathrm{QP}}b$ $= 162''$. And by ſo much it is increaſed again at the quadrature q. For here $p = .087$ (the inclination being 5 deg.), and $n = 177$, and QA = 1.57. And in the octants, where the horary variation is greateſt, the variation comes out $\frac{pn}{4\mathrm{QP}} = 81''$, in the moon's paſſing from Q to P, the node remaining at Q.

PROP. VII.

The excentricity of the moon's orbit is continually varying; and is greateſt when the line of the apſides is in the ſyziges; and leaſt, when it is in the quadratures.

If it was not for the ſun's perturbating force the moon would always deſcribe the ſame ellipſis, and the tranſverſe axis and excentricity of the orbit would remain unaltered. But ſince the perturbating force of the ſun always acts upon her more or leſs, and cauſes all the irregularities of her motion; it is evident that all theſe effects will be the greater as that force is the greater. But when the tranſverſe axis or line of the apſides is in the ſyziges, then the perturbating force TM or 3PK (fig. 99.) is the greateſt poſſible, by which means the moon is removed farther from the earth, and con-

Fig. conſequently the tranſverſe axis is lengthened, and the excentricity becomes greater. And the contrary happens when the tranſverſe is in the quadratures; for by its being in the quadratures, the force LM is greater; and the whole centripetal force towards T being greater, the body will be drawn nearer the earth and deſcribe a leſs orbit, or one leſs excentric than before. Therefore when the apſides are in the quadratures, the excentricity is leſs; and when they are in the ſyziges, it is greater. But how much it is greater or leſs depends upon the mean excentricity; and that depends upon obſervation.

Cor. 1. *Hence the excentricity continually increaſes, as the apſides move from the quadratures to the ſyziges; and decreaſes from the ſyziges to the quadratures.*

Cor. 2. *Likewiſe the excentricity of the orbit continually increaſes, as the moon paſſes from the quadratures to the ſyziges; and decreaſes, in paſſing from the ſyziges to the quadratures.*

For the perturbating force increaſes from the quadratures to the ſyziges; and decreaſes from the ſyziges to the quadratures.

PROP. VIII.

107. *The apſides of the moon's orbit move forward, when the moon is in the ſyziges; but backward when the moon is in the quadratures.*

Let ACBQ be the moon's orbit; A, B the ſyziges; Q and *q* the quadratures, T the earth, CG the line of the apſides. Then the velocity of the moon at the ſyziges A, and the area deſcribed there, will (by Prop. II.) be greater for the time, than the velocity and area at the quadrature Q;

and therefore the velocity and area will be greater at A, and leſs at Q, than it ought to be when the orbit is deſcribed regularly by the moon, without any acceleration or retardation from the diſturbing force. Hence it follows, that the moon being too ſwift at A will leave the orbit behind; that is, the moon, inſtead of being at A will be got to *a*; and therefore to keep it ſtill in the orbit, the point A of the orbit muſt be moved to *a*, thro' the angle AT*a*, and therefore the whole orbit muſt be moved thro' the angle AT*a*; that is, the apſides C, G muſt be moved forward thro' the angle AT*a* or CT*c* equal to it. But near the quadrature Q, where the moon is too ſlow, ſhe will be left behind at *p*; and therefore to keep her in the ſame point of her orbit, the orbit muſt be moved back to the moon, or the point Q muſt be moved back to *p*; and therefore the whole orbit muſt move backwards thro' the angle QT*p*. Therefore when the moon is in the ſyziges, where her motion is too faſt, the apſides move forward; but when ſhe is in the quadratures, where her motion is too ſlow, the apſides move backward. Fig. 107.

Cor. 1. *When the apſides are in the ſyziges, they move faſteſt forward; and in the quadratures they move faſteſt backward.*

For in the ſyziges the perturbating force is greater; and the effects greater; and the contrary in the quadratures.

Cor. 2. *In a whole revolution of the moon the apſides are carried forward.*

For (by Cor. 9. Prop. I.) the centripetal force of the earth is twice as much diminiſhed in the ſyziges as it is increaſed in the quadratures; whence the effects will be greater in one caſe than the other; and therefore the progreſs of the apſides will be greater than their regreſs.

Cor. 3. *There are certain points of the orbit, where*

Fig. *the apſides are at reſt; and theſe are at about* 35°¼
107. *degrees from the quadratures.*

For (by Cor. 3. Prop. I.) in theſe points, the diſturbing force ceaſes, and if the force was always to continue proportional to that, the moon would move in an immoveable orbit; therefore in theſe points, the orbit is at reſt.

SCHOLIUM.

In conſequence of the greater and leſſer perturbating force of the ſun in different parts, and in different ſituations of the moon's orbit, the apſides are carried with an unequal motion, increaſing to the ſyziges, and decreaſing towards the quadratures till they ſtand; then moving backwards faſter and faſter to the quadratures, where the motion begins to decreaſe again. And all theſe motions are greater or leſſer according as the forces that produce them are greater or leſſer; and the ſame is true of all irregular motions of the moon. And ſince theſe forces are a little greater at the conjunction than at the oppoſition of the luminaries; all theſe effects will be ſomething greater at the conjunction than at the oppoſition.

Theſe diſturbing forces of the ſun, and their effects, are alſo different at different times of the year. For (by Prop. XXVIII. Cor. 4. Centripetal forces) theſe forces or their effects are reciprocally as the cube of the ſun's diſtance at that time. And therefore the motion of the apogee and nodes, and the excentricity of the orbit, will be greater in winter than in ſummer. In winter the ſun's greater force dilates the moon's orbit, and then ſhe moves ſlower, and her periodic time is greater. In ſummer his leſſer force contracts the orbit, and ſhe moves faſter; and her periodic time is leſs.

From the foregoing principles, and proper obſervations made upon the moon's motions, tables are

Fig. 107.

are conſtructed for readily finding the moon's place; without which it would be an immenſe labour to do it. The elements of the moon's motion that have been gained by obſervation, beſides theſe mentioned in Prob. XVIII. Sect. IV. are the following.

	°	′	″
The greateſt equation of the apogee	12	18	00
The greateſt equation of the node	1	29	40
The mean inclination of the orbit	5	8	30
The greateſt inclination —	5	17	30
The leaſt inclination —	4	59	30
The mean variation in the octants		35	10
The greateſt variation —		37	25
The leaſt variation — — —		33	4
The mean horizontal parallax —		57	5

The mean excentricity 55 (of 1000 parts), the greateſt $66\frac{7}{10}$, the leaſt $43\frac{3}{10}$.

The greateſt annual equation of the moon's place, $11' \, 50''$; of the apogee, $20' \, 0''$; of the node, $9' \, 30''$.

The ratio of the moon's parallax to her diameter, as 167 to 90, or as 11 to 6.

PROP. IX. *Prob.*

To explain the nature, conſtruction, and uſe of the lunar tables, for finding the moon's place.

For this purpoſe different authors have different methods, and different ſets of tables. Some having more, ſome fewer. Some throw ſeveral equations together in one table, whilſt others give them in ſeparate tables. However the principal of them are theſe that follow.

1. A table of the epocha's or radix's of the moon's mean place, her apogee and aſcending node, made for the beginning of ſeveral years; beginning

Fig. December 31 at noon, and for ſome particular place you pitch on. Theſe are matters of obſervation; and this table is generally made to agree with the moon's motion when the earth is in the aphelion. In this table is inſerted a table of the moon's mean motion from the ſun, for theſe times; or elſe a ſeparate table is to be made for that purpoſe.

2. A table of the mean motions of the moon, of its apogee and node, is made for years and centuries of years; and for months, days, hours, &c. And likewiſe the mean motions of the moon from the ſun. This table is calculated from the moon's periodical time, which is known; and ſerves to find the mean place of the moon, apogee, and node for a given time.

3. A table of the annual equations of the moon, apogee, and node, correſponding to the equation of the ſun (called the equation of the center), theſe equations, in different parts of the earth's orbit, are reciprocally as the cube of the ſun's diſtance. And theſe four annual equations are proportional to one another; and when one is greateſt, the reſt are greateſt; and increaſe and diminiſh in the ſame ratio. Theſe are to correct the mean place of the moon, her apogee and node, found before when the equation of the ſun is to be added, that of the apogee is to be added; and thoſe of the moon, and of the node, ſubtracted; and the contrary.

4. A table of the correction of the moon's motion, depending on the angle between the ſun and the apogee; and is greateſt when the apogee is in the octants, and amounts to 3′ 56″ when the earth is in the perihelion; and to 3′ 34″, in the aphelion. But this greateſt equation varies in the reciprocal proportion of the ſun's diſtance. But when the apogee is out of the octants, the equation is leſs, in proportion of the ſine of twice the diſtance of the

295.

UNIV

he apogee (from the next ſyzigy or quadrature) to adius. This equation is to be added to the moon's motion, when the apogee paſſes from the quadrature to the ſyzigy; but ſubtracted, from the ſyzigy to the quadrature.

5. A table of corrections of the mean place of the apogee, and contains two equations. The 1ſt. depends on the angle between the ſun and apogee; for the apogee goes forward in the ſyziges and backward in the quadratures. The 2d is a menſtrual equation, and depends on the angle between the ſun and moon.

6. A table of equations of the apogee and excentricity of the orbit. This depends on the angle between the apogee and ſun; here the excentricity is put down over againſt the equation of the apogee which depends on the ſaid angle. This equation is to be added (if that angle be leſs than 90 or between 180 and 270, otherwiſe ſubtracted) to the place of the apogee equated; which gives a more correct place of the apogee; and having the moon's place, the moon's mean anomaly will be had. But theſe excentricities ought to be corrected, according to the diſtance of the moon from the ſyziges, and of the ſun's diſtance.

7. Another table of the correction of the moon's motion, depending on the angle between the ſun and the node. This equation is greateſt when the nodes are in the octants, and is 0 at the ſyziges and quadratures. It is calculated by Prop. V. When greateſt it is 47″, and is as the ſign of twice the diſtance of the node from the ſyzigy or quadrature. This is to be added to the moon's place if the nodes move from the ſyziges to the quadratures; or elſe ſubtracted.

8. A table of the equation of the orbit; this is calculated like the equations in a primary planet, and by it is found the moon's place in its orbit,

the ſame way that a planet's place is found. This is called the *equation of the moon's center*, and anſwers to the degrees of the moon's mean anomaly. Here, as the excentricity varies, there muſt be two tables of this ſort, anſwering thereto. And for a mean excentricity, a proportional equation muſt be taken.

9. A table of the moon's variation or reflection. This is calculated by Prop. IV. It is greater when the earth is in the perihelion, and leſs in the aphelion, reciprocally as the cube of the ſun's diſtance. This table is to be corrected according to the ſeveral degrees of the ſun's mean anomaly. By this the moon's place is corrected by adding this correction in the 1ſt and 3d quadrants; and ſubtracting it in the 2d and 4th.

10. A table of three equations of the nodes. The 1ſt depends on the angle between the ſun and the nodes; for the nodes are at reſt in the ſyziges, and move backward at the quadratures; the 2d is a menſtrual equation of the nodes depending on the angle between the ſun and moon. The third is annual; for the nodes are ſwifter in the perihelion. The calculation of theſe depend on Prop. V.

11. A table ſhewing the correction of the inclination of the orbit. For the inclination is leaſt when the nodes are in the quadratures with the ſun; and greateſt in the ſyziges. And in a revolution of the moon; leaſt when ſhe is in the ſyziges, and greateſt in the quadratures. Theſe are like the equations of the nodes; and may therefore be put into the 10th table.

12. A table ſhewing how much theſe equations and corrections are to be increaſed in the conjunction; and decreaſed in the oppoſition to the ſun. For the forces being greater in the conjunction, their effects will be greater.

13. A table of the moon's latitude for every degree of diſtance from the node. This is made the ſame way as the tables of the primary planets. But becauſe the inclination varies, the table is made for the leaſt; and there is added ſuch parts as make up the greateſt. And in the mean inclinations a proportional part thereof is to be added.

14. A table ſhewing the moon's place reduced to the ecliptic, for all degrees of the moon's diſtance from the node; and is juſt the ſame as in the primary planets. The difference between this diſtance from the node, and the reduced arch (or the hypothenuſe and baſe) is called the *reduction*, which is put into the table in a proper column. This is alſo for the leaſt variation, and muſt be increaſed upon occaſion like the other.

15. A table of the moon's parallax. This muſt be made for the greateſt and leaſt excentricity; and a mean muſt be taken as there is occaſion. This table is alſo to contain particles to be ſubtracted, as the moon approaches the quadratures, where it is leaſt. The parallax is alſo leſs when the earth is in the perihelion, and greater in the aphelion.

Some of theſe tables may be branched into two or three, or perhaps ſometimes two may be put into one, as may be thought proper; different people having different methods. There are alſo other tables of the moon which may be uſeful, ſuch as a table of epochas of the mean conjunctions of the moon with the ſun. A table of lunar periods. A table of the horary motions of the ſun and moon, their diameters, and parallaxes, in eclipſes. Tables of the plinian periods of ſolar and lunar eclipſes, &c.

PROP.

PROP. X. *Prob.*

To find the moon's place at a given time.

As different authors have different tables and different methods of working; therefore no general rules or precepts can be laid down, that will ſuit all authors and tables. For thoſe that have conſtructed tables have put them into various forms, which require a different proceſs or method of calculation. Yet there are ſome things which are common to them all. I ſhall give the method according to our countryman Dr. Halley, as follows.

The *arguments* in any table for the ſeveral equations, are the leading numbers therein, or the numbers in the firſt column of the table, by which the equations are found.

1. *By* HALLEY's *Tables.*

The precepts for theſe tables are the following.

Let M = ſun's mean anomaly.
☉ = ſun's true place.
A = place of the ſun's apogee.
☽ = moon's mean place.
☽ 1, ☽ 2, ☽ 3, &c. = moon's place once, twice, thrice, &c. equated.
☊, ☊ 1, ☊ 2 = mean place of the moon's aſcending node, and once, twice equated.
G, G1, G2 = mean place of the moon's apogee, and the ſame once, twice equated. Then,

1. Find the true place of the ſun, his apogee, and mean anomaly, to the time given, reduced to mean time. By Prop. XII. Sect. V.

2. From the *tables of the epochas of the moon's mean motions* (H*b*4 — I*i*2, &c.), take the mean longitude of the moon, her apogee and node; for years, months, days, hours, &c. and add them

into ſeparate ſums for the moon and apogee. And for the node, ſubtract the ſum (for the month, day, hour, &c.) from its place in the radical year.

3. From the *table of the annual equations of the moon, &c.* (L*l*3, &c.) take the equations of the moon, its apogee, and node; anſwering to the ſun's mean anomaly for the argument. With theſe correct the places of the moon, its apogee and node; by adding or ſubtracting as the table directs.

4. In the *table of the firſt ſemeſtral equation* (M*m*), find the ſecond equation of the moon's place, correſponding to the argument $☉ - G1 = N$, called *the annual Argument.*

5. In the ſame *table* (M*m*), *with the other ſemeſtral equation*; with the argument $☉ - ☊ 1$, find the third equation of the moon.

6. In the *table of the equations of the apogee and excentricity* (M*m*2, &c.), with the annual argument N, find the log. for the equation of the moon's center; add it to the log. tangent of $\frac{☽\,1 - G1}{2}$, the ſum is the log. tangent of an angle, which doubled and taken from $☽\,1 - G1$ (or its comp. to 12 ſigns, when greater than 6), gives a correction, which muſt be taken from $☽\,3 - \overline{A + N}$, when $☽\,1 - G1$ is leſs than 6 ſigns; or added, if above. With this argument, ſeek in the table of *the fourth equation of the moon* (M*m*), for the equation; which will correct the moon's place a fourth time.

7. In the table of the *equation of the apogee and excentricity* (M*m*2, &c.), with the annual argument N, find the ſecond equation of the apogee, the excentricity of the orbit, and the log. for the equation of the moon's center. The firſt corrects the apogee a ſecond time.

8. In the *table for readily finding the equation of the*

the moon's center (Nn, &c.), with the argument ☽ 4 — G2 (or the moon's mean anomaly), or its ſupplement to 12 ſigns (when it is greater than 6), find the equation in the column having the proper log. for the equation of the moon's center at top. Add (or ſubtract) this equation to half the mean anomaly, and to the log. tangent of this corrected angle, add the log. for the equation of the center (found Art. 7.), gives the log. tangent of half the true anomaly, which doubled is the true anomaly. And the difference of the mean and true anomaly is the equation of the center. This is to be ſubtracted from the moon's place, when the mean anomaly exceeds not 6 ſigns, otherwiſe added. This gives the moon's place equated a fifth time.

9. In the *table of the moon's variation or reflexion* (Nn4), with the argument ☽ 5 — ☉, find the moon's variation; to its logiſtical log. add the log. anſwering to the argument M, in the *table of logarithms for correcting the variation* (Nn4), the ſum is the logiſt. log. of the true variation, by which correcting the moon's place (Art. 8.), gives her true longitude in her proper orbit.

10. In the *table for computing the moon's latitude* (Oo), with the argument ☉ — ☊ 1, find the ſecond equation of the node, the log. ſine of the greateſt inclination, and the greateſt reduction, by this equation of the node, the place of the node is a ſecond time corrected. This place of the node taken from the moon's place, gives the argument of latitude.

To the log. ſine of the greateſt inclination add the log. ſine of the argument of latitude, the ſum (abating radius) is the log. ſine of the moon's latitude; which is north, when the argument of latitude is leſs than 6 ſigns, otherwiſe ſouth.

To the logiſt. log. of the greateſt reduction, add the arith. complement of the log. ſine of twice the argument of latitude; the ſum is the logiſtic log. of the true reduction; which muſt be ſubtracted from the moon's longitude in her own orbit, in the firſt and third quadrants of the argument of latitude, or added in the ſecond and fourth; and there is had the moon's longitude in the ecliptic.

Thus the longitude and latitude of the moon is found, as ſeen from the center of the earth, or where ſhe is vertical.

11. In the *table of the moon's horizontal parallax in the ſyzigy* (Oo2), find the parallax, anſwering to her true anomaly, and alſo to her excentricity; to the logiſtic log. hereof, add the log. anſwering to the moon's diſtance from the neareſt ſyzigy, found in the *table of parallaxes without the ſyziges* (Oo2); the ſum is the logiſtic log. of the moon's true horizontal parallax.

12. Then rad : coſ. altitude : : horizontal parallax : to the parallax at that altitude (by Cor. 3. Prob. XVI. Sect. IV.). Then the parallax in latitude and longitude will be found by Prob. XVII. Sect. IV. and conſequently the apparent latitude and longitude of the moon, or her apparent place.

But after all, if the places thus found do not agree to obſervations; theſe equations ought to be corrected by obſervation, or new ones found out, to make the tables compleat.

Note, If the place you calculate for, is not that for which the tables are made; the time will be different, according to the difference of longitude. Which time muſt be adjuſted, before you begin your work.

Example.

To find the moon's place, Nov. 18, 1768, at 7 o'clock, 21 minutes, mean time.

Sun's mean anomaly = 4 19 31 22 = M.
ſun's true place = 7 27 2 24 = ☉.
his apogee = 3 8 47 47 = A.

This being leap year, a day muſt be added, and the calculation muſt be made for 11 days ſooner, by theſe tables; that is, for Nov. 8.

	Moon.				Apogee.				
	s	°	'	"	s	°	'	"	
1768	6	14	54	14	6	4	36	20	
Nov. 8	5	1	2	7	1	4	45	34	
7 h.		3	50	35			1	57	
21 m.			11	32				6	
☽	11	19	58	28	7	9	23	57	= G
1 equa.		+	7	47		—	13	11	
☽1	11	20	6	15	7	9	10	46	= G1
2 eq.		—	2	10	+	6	3	17	
☽2	11	20	4	5	7	15	14	3	= G2
3 eq.			+	45					
☽3	11	20	4	50	Node.				
4 eq.		—	1	47	9	21	31	23	
☽4	11	20	3	3		16	31	20	
5 eq.	—	6	23	26				56	
☽5	11	13	39	37				3	
6 eq.	var. —		20	7	—	16	32	19	
☽6	11	13	19	30	9	4	59	4	= ☊
	red. —		4	37		+	6	16	
☽ place	11	13	14	53	9	5	5	20	= ☊1
	moon's longit. in the ecliptic				—	1	26	33	
☽ lat.		4	51	13N.	9	3	38	47	= ☊2

Arguments.

s ° ' "

☉ — G1 = 0 17 51 55 = N.

☉ — ☊1 = 10 21 57 21.

☽1 — G1 = 4 10 55 29.

☽3 — $\overline{A + N}$ = 7 23 25 8.

☽4 — G2 = 4 4 49 0 = mean anom.

☽5

☽ 5 — ☉ = 3ˢ 16° 36′ 56″.
☽ 6 — ☊ 2 = 2 9 40 43 = arg. latitude.

Operations.

The excentricity .0649
equation of the center log. 9.943534
$\frac{\text{☽ 1} - \text{G 1}}{2}$ = 65 27 45 its tan. 10.340542
tang. 62 31 47 10.284076
doubled 4 5 3 34
☽ 1 — G 1 = 4 10 55 29
correction 5 51 55

☽ 3 — $\overline{A + N}$ = 7 23 25 8
ſubtr. — 5 51 55
argum. Art. 6 = 7 17 33 13
equation — 1 47
half the m. anom. 62 24 30
equat. Art. 8. — 1 33
62 22 57

its tang. 10.281352
log. eq. center 9.943534
the tangent 10.224886
of ½ true anomaly 59 12 47
true anomaly 3 28 25 34
mean anomaly 4 4 49 0
equat. center 0 6 23 26

variation — — — 19 16
its logiſtic log. 4933
correction log. — — 9.9814
whole variation 20′ 7″ 4747

S. greateſt inclin. 8.95532
S. arg. latitude 9.97209
ſine of 8.92741
the moon's lat. 4 51 13 N.

greateſt reduction — — —	7′ 1″
its logiſt. log. — — —	9320
comp. Art. 2. arg. lat.	1862
reduction 4′ 37″	11182

parallax 59′ 1″ logiſt. log.	72
log. ☽ diſt. from ſyzigy	57
horizontal parallax 58′ 15″	129

The remaining work being purely ſpherical trigonometry, is done by Prob. XXXIX. Sect. II. and Prob. XVII. Sect. IV. for finding the parallax in latitude and longitude.

But experience ſhews, that theſe tables do not now anſwer to the heavens; ſo that they want ſome amendment, or perhaps ſome ſmall equations are ſtill wanting, in order to compute the moon's place correctly; which future induſtry muſt find out.

PROP. XI. *Prob.*

To find the mean and true time of the ſyzigy, or of the new and full moon, for a given month.

The *mean time* of a ſyzigy, is the time when the mean places of the ſun and moon in the ecliptic, are the ſame for the conjunction, or oppoſite for the oppoſition. And the *true time* is when their true places are exactly the ſame, in the conjunction; or directly oppoſite, for the oppoſition.

1. *For the mean time of a conjunction or oppoſition*

From the *table of the mean motion of the moon from the ſun*, take out the numbers for the given year and month, add them up together; and ſubtract the ſum from 12 ſigns, for the conjunction; or from 6 or 18, for the oppoſition. Then ſeek the remainder in the ſame table for days, if it cannot

not be exactly found, take the next leſſer and ſubtract it from that remainder; and note the time correſponding. In like manner ſeek this ſecond remainder, in the ſame table for hours, and ſubtract the next leſſer number from it, and note the time. Do the like for minutes, &c. till nothing remains. The ſum of all theſe parts of time, gives the day, hour, minute, &c. of the ſyzigy ſought.

Note, In leap year after February, this time muſt be leſſened by a day.

This table of the mean motion of the moon from the ſun, you have in almoſt all books of aſtronomical tables.

2. *For the true time.*

Find the true places of the ſun and moon in the ecliptic, for the mean time of the ſyzigy, now found (by the laſt Prop.) If they agree, or be oppoſite, there is no more to be done. But if they differ, take that difference, and ſeek it in the ſame table, for days, hours, &c; and proceed as before, to find the difference of time. Add this to the mean time, if the moon be ſhort of the ſyzigy; or ſubtract it, if paſt; and you have nearly the true time of the ſyzigy.

But to be exact, you muſt repeat the operation, and find the true places of the ſun and moon, for this laſt found time, and their diſtance. Then proceed as before. Otherwiſe the time will be known, by proportion in which this laſt diſtance is run over by the moon. For as the former diſtance, to the time it is run in; ſo this laſt diſtance, to the time from the ſyzigy; which time being added or ſubtracted, gives the correct time of the ſyziges.

If you have a table of the horary motions of the ſun and moon, for their ſeveral anomalies; the horary motion of the moon from the ſun at that time, will be known thereby. Whence, after their true places are once found, and their diſtance, the time of the ſyziges will be found, by making a proportion, according to that horary motion, for that interval of time, to reach the ſyzigy. But if the time of the ſyzigy thus got, be not exact enough, which ſeldom happens, their places muſt be found again, and the reſt of the operation repeated.

Note, the time thus found only anſwers for the place the tables are made for. In other places the time will be different, according to the difference of meridians, which muſt be taken notice of.

Cor. 1. *The mean time of any ſyziges being known, the mean time of any other will be known, by adding continually* 29^d^ 12^h^ 44^m^, *the moon's ſynodic revolution; or ſubtracting it, for the preceeding ſyziges.*

Cor. 2. *Hence the mean time of the new and full moon, for any month, is very eaſily and readily found. But to find the exact time, requires a great deal of labour and calculation.*

Example.

To find the time of oppoſition or full moon, in December 1768.

1. *For the mean time.*

	s	°	′	″
1768	4	10	18	19
Decem.	3	21	42	36
☽ from ☉	8	2	0	55
full ☽	6			
	9	27	59	5
da. 24	9	22	34	41
		5	24	24
ho. 10.		5	4	46
			19	38
min. 38			19	18
ſec. 38				20

The ſum of the times, $24^d\ 10^h\ 38^m\ 38^s$, but becauſe 'tis leap year; the mean time of full moon is Dec. $23^d\ 10^h\ 38^m\ 38^s$.

2. *For the true time.*

Let the time be Dec. $23^d\ 10^h\ 38^m$. The ſun's longitude to that time

	s	°	′	″
(by Prob. XII. Sect. V.) is	9	2	57	5
and the moon's place (Pr. X.)	3	6	56	28
moon paſt the ſun	6	3	59	23
moon paſt the ſyzigy or oppoſition		3	59	23
7 hours mot.		3	33	20
			26	3
51 min. mot.			25	54
				9

From Dec. $23^d\ 10^h\ 38^m$
ſubtract 7 51
the oppoſition Dec. 23 2 47, which is very near the true time; and may be had exactly by a ſecond operation.

At this corrected time the moon's latitude is only $22''$, and therefore there will be an eclipſe of the moon at that time, which will be a total one.

Note, this is the mean time, which is eaſily reduced to the apparent.

PROP. XII. *Prob.*

To find the periods of the new moons, when they ſhall happen again, at the ſame time of the year.

The moon's revolution to the ſun is $29^d\ 12^h\ 44^m\ 3^s = 29.5306$. And the year $= 365^d.25$. Let a = number of lunations in any time; y = number of years in the ſame time. Then that the conjunctions may fall on the ſame time of the year, we

Fig. muſt find a and y ſuch, that $29.5306a = 365.25y$, whence $\frac{y}{a} = \frac{29.5306}{365.25}$. Then all the periods may be found by Prob. IX. Ch. IV. B. II. Arithmetic, and will be $\frac{y}{a} = \frac{1}{12}, \frac{2}{25}, \frac{3}{37}, \frac{8}{99}, \frac{19}{235}, \frac{483}{5974}, \frac{502}{6209}, \frac{6005}{74273}$, &c.

The revolution of the moon to the apſis, is $27^d.5476$, therefore if x = number of theſe revolutions, in the ſame time, $365.25y = 27.547x$. For the apſides ought to be as near as poſſible, in the ſame ſituation, after x revolutions.

If we take the numbers 1 and 12 the periods will differ by 11 days, 12 being too little. If we take 2 and 25, they will differ by 8 days, 25 being too big. Then to try 3 and 37, $3 \times 365.25 = 1095.75$, and $37 \times 29.53 = 1092.63$, and $1095.75 - 1092.63 = 3^d.12$; here 37 is too little, and the moon will be $3^d.12$ or $3^d\ 3\frac{1}{2}^h$ paſt the conjunction in 3 years. And $\frac{365.25 \times y}{27.547} = x = 3.97$, ſo that the apogee wants only .03 part of a revolution to the ſun, which is .8 days or 20 hours.

Again, for the numbers 8 and 99, $365.25 \times 8 - 29.5306 \times 99 = -1^d.53$, ſo the moon wants $1^d\ 12^h\frac{1}{2}$ of the conjunction. And $x = 106$ rev. + 2.07 days.

Then for the numbers 19 and 235, $365.25 \times 19 - 29.5306 \times 235 = .059^d$ or 1 hour, 25 min. Therefore the moon is 1 hour, 25 minutes paſt the change. And $x = 252$ rev. wanting 2.09 days.

Likewiſe the numbers 483 and 5974 being tried; $365.25 \times 483 - 29.5306 \times 5974 = -.0544$ days = 1 hour, 18 minutes, which the moon is ſhort of the change. And $x = 6404$ rev. + 22 hours, which is very near.

And thus the reſt of the periods may be examined. Fig.

Cor. 1. *The ſame periods ſerve for the full moons in any year.*

Cor. 2. *Hence, if the change of the moon happens on any day of the month; then after the ſeveral periods of years, the change will be as follows.*

In	3 *years*	—	3^d	3^h	30^m	*ſooner.*
	8 *years*	—	1	12	30	*later.*
	19 *years*	—		1	25	*ſooner.*
	483 *years*	—		1	18	*later.*
	502 *years*	—		0	6½	*ſooner.*

SCHOLIUM.

We here ſuppoſe the year to be = 365¼ days; but as the common year is but 365 days; allowance muſt be made for the odd years above 4, 8, 12, &c. and for leap year, if it fall in.

PROP. XIII. *Prob.*

To find the mean time of the moon's changing, in any month; by an old almanack.

The moon's ſyn. revolution is	29	12	44
the lunar year (12 l. months)	354	8	48
common ſolar year	365	0	0
wants	10	15	12

and ſo much the moon changes ſooner than in the foregoing year. Again,

twice the remainder —	21	6	24
ſynod. period — — —	29	12	44
almanack 2 years old, later —	8	6	20
alſo thrice the remainder —	31	21	36
ſynodical period —	29	12	44
an almanack three years old, ſooner	2	8	52

And ſo the time ſooner or later may be found for any year after, by multiplying the yearly difference (10 15 12) by the number of years, and finding how much the product falls ſhort or exceeds the ſynodic revolution, or its double or triple, &c. hence by an almanack of

1 year old, the change is ſooner	10^d	15^h
2 years old, later —	8	6
3 years old, ſooner — —	2	9
4 years old, ſooner — —	13	0.
&c.		

But if a leap year (or an intercalary day) falls within the time, the change will be a day ſooner. And if ſeveral fall in, it will be ſo many days ſooner.

Note, by reaſon of the irregular motion of the moon, the change will often happen ſeveral hours ſooner or later, than by this rule.

Otherwiſe.

Amongſt a number of old almanacks, pick out one that is 19 years older. Then the time of the change in the old almanack is nearly the ſame as in the new one.

For 19 years $= 19 \times 365\frac{1}{4} = 6939\frac{3}{4}$ days.
And 235 lunations $= 235 \times 29.5306 = 6939.72$.
So that the days nearly coincide.

But the poſition of the apogee diſturbs this motion ſo much as to make the time alſo to differ for ſeveral hours. So that theſe are no more than ſome eaſy approximations. See the laſt Prop.

SCHOLIUM.

Many other Problems relating to the ſun and moon may be propoſed; as if it was required to find the time of the moon's ſouthing on any day.

From the table of the moon's mean motion from the ſun, take out the numbers for the year, month, and day; which being added together, gives the moon's diſtance from the ſun. Then ſay as 12 ſigns : to 24 hours, or as 1 to 2 : : ſo the moon's diſtance from the ſun, to the time of ſouthing.

But this not being exact enough, find the places of the ſun and moon to that time; and from thence their right aſcenſions in time, and the difference thereof ſhews how far the moon is before the ſun in hours; and conſequently how much ſhe wants or is paſt the ſouthing. This laſt quantity is to be increaſed by as much as the moon gains of the ſun in that time; and then added or ſubtracted to or from the hour of the day, as the caſe requires.

There is more trouble ſtill in finding her riſing and ſetting. For her change of right aſcenſion and of declination muſt be conſidered; ſo as to agree with the oblique aſcenſion and deſcenſion.

PROP. XIV.

The motions of the ſatellites of any planet, are ſimilar to thoſe of the moon, and may be known thereby.

For the ſatellites do all move round their primary planets, like as the moon revolves about the earth, which is the earth's ſatellite. Each of theſe ſatellites, in the part of their orbits next the ſun, being nearer the ſun, are for that reaſon, diſturbed in their motions, juſt as the moon is by the like cauſe. From hence ariſe the like motions as in the moon, ſuch as the progreſſive motion of the apſides, and the regreſſion of the nodes, of the ſatellites. But the forces that produce theſe motions depend upon the diſtances of the ſun from the planet, and of the planet from the ſatellite; or upon their periodic times. And therefore the motions of the ſatellites, may be deduced from the motions of the moon, which are analogous to them. And therefore as we know the motions of the moon, and all her irregularities, we can compute the quantities of theſe motions and irregularities, in any of the ſatellites. Hence,

Cor. 1. *The mean motion of the nodes or apſides of any ſatellite, is to the mean motion of the nodes or apſides of the moon; as the periodic time of the ſatellite, divided by the ſquare of the periodic time of its planet; to the moon's periodic time, divided by the ſquare of the earth's periodic time.*

This appears by Cor. 3. Prop. XXVIII. Centrip. Forces. For theſe angular errors are as the per. times of the ſatellites, directly, and the ſquares of the period. times of the planets reciprocally. And therefore,

Cor. 2. *In the ſame planet, the mean motions of the nodes and apſides, are as the periodical times of the ſatellites.*

Cor. 3. *The mean motion of the nodes of any ſatellite, is to the motion of its apſides; as the motion of the moon's nodes, to the motion of her apogee.*

Cor. 4. *The variation of any ſatellite, is to the moon's variation; as the ſquare of the periodic time of the ſatellite, divided by the ſquare of the periodic time of the planet; to the ſquare of the moon's periodic time, divided by the ſquare of the earth's periodic time; or as the motion of the ſatellites nodes in its time of revolution, to the motion of the moon's nodes in her time of revolution.*

For (by Cor. 6. ib. Cent. Forces), the angular errors in the whole revolutions, are as the ſquares of the per. times of the ſatellites, divided by the ſquares of the per. times of their primary planets.

Cor. 5. *The greateſt equations in a ſatellite and the moon, are as the motions in the times of their revolutions.*

Cor. 6. *In the inmoſt ſatellite of Jupiter, the node recedes* $53'$ *in* 100 *years; and the variation is inſenſible being only about half a ſecond. And the apogee advances about* $1\frac{1}{2}$ *minute in a hundred years.*

PROP. XV.

The moon's libration ariſes from her equal diurnal motion, and her unequal menſtrual motion.

The libration of the moon is that motion, ſhe ſeems to have, of turning round her axis for a ſmall ſpace, in one part of her revolution; and returning back, in another part. By this means ſhe ſhows ſome part of her ſurface on the eaſtern ſide, which was inviſible before; and afterwards ſome part of her weſtern ſide, appears, with the ſpots therein, and then diſappears; and this by an alternate motion.

Since the moon moves uniformly round her axis in a month, ſhe will always turn the ſame face towards the upper focus of her orbit, ſince the angles deſcribed about the upper focus are nearly as the times, in orbits not very excentric, by Schol. Prop. II. Sect. V. Therefore it is evident, that any perſon placed there would always ſee the ſame face of the moon; and conſequently a perſon placed at the earth, cannot always ſee the ſame face, but will ſometimes ſee more of one ſide, and ſometimes more of the other, according to her different ſituations in her orbit. And that increaſe of ſurface will be equal to the equation of the center: and this is her libration in longitude. Her libration in latitude ariſes from her latitude, and the inclination of her axis of rotation to the plane of the ecliptic.

Cor. *The moon muſt nearly, always turn the ſame face*

Fig. *face towards the earth, and that poſition cannot be altered. Setting aſide, her libration.*

This appears by XXIX Centrip. Forces. For if any cauſe, as her libration ſhould put her into a different poſition; ſhe will be gradually drawn back, by the earth's attraction; till ſhe recovers the ſame again.

PROP. XVI.

In every lunation, when the moon is about the equinoctial, advancing northward; ſhe riſes for ſeveral nights about the ſame time, in great latitudes.

108. Let PHQO be the prime vertical, HO the horizon, EQ the equinoctial, P the pole, DBL a parallel of declination, PBAp a meridian. Suppoſe the moon be at C any night, and at B the following night. Now it is plain, if the moon has gained as much time by her increaſe of latitude AB, as ſhe has loſt in her diurnal motion, CA; then ſhe will riſe preciſely at the ſame hour. But this cannot be except ſhe be then in the horizon at B; and therefore at the time of riſing, B is both in the horizen and ecliptic; therefore the comp. of the lat. is equal to the angle of the equinoctial and ecliptic = $23\frac{1}{2}$. And the lat. = $66\frac{1}{2}$, where the moon, about the equinoctial, riſes 2 ſucceeding nights at the ſame hour. And it will do this with little variation for ſeveral nights.

Cor. 1. *When the moon is at the full, about the equinox in September; ſhe then riſes for ſeveral nights together about ſun ſet. So that her light is of great uſe at this time. And this is the original of the* HARVEST *and* HUNTERS MOONS.

For being at the full ſhe is oppoſite to the ſun, and therefore riſes at ſun ſet. And being near the

equinox, it will happen in September or October. Fig. 108.
And thus ſhe gives the greateſt quantity of light, at the moſt proper time of the night; and at the moſt proper ſeaſon of the year too, for the huſbandman to gather his crops.

Cor. 2. *But at other times of the year, ſhe either riſes after this manner, ſoon in the day, or late in the night; and therefore her light is weaker, and therefore ſhe is now not ſo uſeful.*

For at other times of the year, ſhe has a different aſpect with the ſun, and therefore has leſs light. And conſequently cannot riſe when he ſets, but will riſe either in the day time, or in the night.

Cor. 3. *The greater the latitude of the place, the longer the moon continues to riſe at the ſame time.*

For the greater her latitude, the more ſhe riſes above the horizon, by her increaſe of declination; and leſs in the leſſer latitudes. And under the equinoctial ſhe gains nothing by her increaſe of declination; and ſo the difference of the times of riſing there is always the ſame.

Cor. 4. *If the moon's aſcending node be likewiſe near the ſame equinox; ſhe will continue longer, to riſe at the ſame time of night.*

For then her increaſe of latitude is ſtill greater.

Cor. 5. *There is likewiſe a time in every lunation, when the moon ſets for ſeveral nights together, about the ſame hour; in great latitudes. And that is, when ſhe is about the equinoctial advancing ſouthward.*

For by the ſame reaſoning, this will be when ſhe loſes as much time by her decreaſe of latitude (BA), as ſhe gains by her motion in right aſcenſion (AC).

PROP.

Fig.

PROP. XVII. *Prob.*

109. *Having the apparent diſtance of the moon from a ſtar and their zenith diſtances; to find their true diſtance, as ſeen from the earth's center.*

Let Z be the zenith, M the moon's apparent place, S the ſtars apparent place, and MS their apparent diſtance; let *m* and *s* be the true places of the moon and ſtar, clear of the parallax and refraction. Draw ZI perp. to MS, and from D deſcribe the ſmall arches *mo*, *sp*. Then M*m* and S*s* will be known, made up of the parallax and refraction; and all the ſides of the triangle ZMS are given. Therefore,

By Caſe 11th of oblique ſpherical triangles, find the ſegments of the baſe MI and SI. And take $Mo = \frac{\text{tan. } MI}{\text{tan. } MZ} \times Mm$, and $Sp = \frac{\text{tan. } SI}{\text{tan. } SZ} \times Ss$. Then the true diſtance $mS = MS - Mo + Sp$.

Note, if the angle M or S be obtuſe (which is known by the falling of the perpendicular ZI), take $+ Mo$, or $- Sp$.

For in the right angled ſpherical triangle ZIM, (by Caſe 4.) tan. MZ : rad : : tan. MI : coſ. M $= \frac{\text{tan. } MI}{\text{tan. } MZ} \times \text{rad}$. And in the ſmall right angled triangle M*mo*, which may paſs for a plain triangle, rad : M*m* : : coſ. M $\left(\frac{\text{tan. } MI}{\text{tan. } MZ} \times \text{rad}\right)$: M*o* $= \frac{\text{tan. } MI}{\text{tan. } MZ} \times Mm$. And by the ſame way of reaſoning, $Sp = \frac{\text{tan. } SI}{\text{tan. } SZ} \times Ss$. Now ſince the arches M*m*, S*s* are exceeding ſmall, the arch D*m* will be equal to D*o*, or DM — M*o*; and the arch D*s* equal to Dp or $DS + Sp$. Therefore the ſum $Dm + Ds = DM - Mo + DS + Sp$; that is, the whole $ms = MS - Mo + Sp$. Fig. 109.

Example.

$$\text{Let}\left\{\begin{array}{l} MS = 56^\circ\ 17'\ 44'' \\ ZM = 25\ \ 22\ \ 0 \\ ZS = 36\ \ 47\ \ 0 \end{array}\right.$$

moon's parallax	26′ 18″
refraction	— 27
	25 51
or Mm =	1551″
ſtar's refraction	
Ss =	41

Then,

Tan. $\frac{1}{2}$MS.	28	8	52	9.7283719
tan. $\frac{ZS+ZM}{2}$.	31	4	30	9.7800606
tan. $\frac{ZS-ZM}{2}$.	5	42	30	8.9998270
				18.7798876
tan. Q.	6	25	27,	9.0515157
SI =	34	34	19	
MI =	21	43	25.	

Then,

Tan. MZ.	25 22	9.6758903
tan. MI.	21 43	9.6001943
Mm.	1551	3.1906118
		12.7908061
Mo = 1303″ = 21′ 43″		3.1149158

Again,

Fig. 109.

Again,

Tan. SZ.	36 47	9.8736937
tan. SI.	34 34	9.8382164
S*s*.	41	1.6127838
		11.4510002
S*p* = 38″	— — —	1.5773065

$$\begin{array}{rcr} & & {}^{\circ}\quad{}'\quad{}'' \\ MS & = & 56\ 17\ 44 \\ -\ Mo & = & -\ 21\ 43 \\ \hline & & 55\ 56\ 01 \\ +\ Sp & = & +\ 38 \\ \hline ms & = & 55\ 56\ 39. \end{array}$$

the true diſtance.

SCHOLIUM.

The point *m* is always above M, becauſe the moon's parallax is always greater than the refraction. And the points *s* is always below S, becauſe a ſtar has no parallax.

PROP. XVIII. *Prob.*

To find the quantity of matter in the moon.

110. Since (by Prob. XVIII. Sect. IV.) the moon's mean diſtance TL is 60.3 of the earth's radii, or 30.15 diameters; and her mean apparent diameter 2AL is found to be 31′ 16″. Therefore it will be, as rad : S.31′ 16″ : : 30.15 : moon's real diameter =.2742, the earth's diameter being one. For rad : TL : : S.LTA : LA : : S.2LTA : 2LA.

Then (by Cor. 5. Prop. XXVII. Cent. Forces) the ſun's force is to the moon's force; as 1 to 4.46. And (by Cor. 6. ib.) theſe forces are as DF^3y and df^3y; putting D, *d* for the denſities, and F, *f* for the apparent diameters, where *y* is given, being the

ſame in both. Whence we have $1 : 4.46 :: DF^3 :$ Fig.
$df^3 = 4.46 \times DF^3$, therefore $d = \frac{4.46 \times DF^3}{f^3}$. 109.
Where F, the ſun's apparent diameter, is 32′ 12″; and D his denſity, 100. Whence $d = 487$ the moon's denſity.

The quantities of matter are as the denſities, and cubes of the diameters. And the earth's denſity (by Cor. 2. Prop. XXIV. Sect. V.) is 393, and his diameter 1. Therefore $393 \times 1^3 : 487 \times .274^3 ::$ quantity of matter in the earth : quantity of matter in the moon $= \frac{487 \times .274^3}{393} = .055$ of the earth's matter. Therefore the quantity of matter in the moon, to that in the earth; is as 1 to 39.2.

Cor. 1. *The diameters of the moon and earth, are as 1 to 3.64.*

Cor. 2. *The denſity of the moon, to that of the earth; is as 1.23 to 1.*

Cor. 3. *The weight of bodies at the moon, to their weight at the earth; is as 1 to 3 nearly.*

For (by Cor. 3. Prop. 24. Sect. V.) the weight is as the denſity and diameter; which, in the moon and earth, will be as 1.23×1, to 1×3.64; or as 1 to 2.96.

PROP. XIX.

The perturbating forces of the planets, upon the moon, are exceeding ſmall, and their effects inſenſible.

For (by Cor. 6. Prop. XXVII. Cent. Forces) the perturbating force of any of theſe bodies, is as the denſity and the cube of its apparent diameter. Now when Venus is neareſt the earth, her apparent diameter is 70″; and her denſity being unknown, we may ſuppoſe it ſomething greater than that of the earth which is 393, the ſun's being 100. Suppoſe it is to the earth's, reciprocally as its diſtance from the ſun. Then the denſity of Venus will be 560. And the ſun's mean apparent diameter is 1932. Therefore the perturbating force of the ſun, is to that of Venus; as 100×1932^3 to 560×70^3; that is, as 2754 to 1. Now among the moon's equations take the greateſt of any of them, and it will appear how ſmall a ſhare this has in it. Suppoſe the greateſt equation of the annual argument (or the ſun's diſtance from the apogee) be 5 degrees or 18000 ſeconds, the ſhare of Venus in this, is but the 2754th part of it, which amounts only to 6 ſeconds; ſo ſmall a quantity as not to be diſcovered by any obſervations. And in the moon's variation, it would not amount to a ſingle ſecond.

Then for Jupiter. His denſity is 94, and his apparent diameter when neareſt the earth, $46''$; therefore the ſun's perturbating force, to that of Jupiter, is as 100×1932^3 to 94×46^3; that is, as 78820 to 1, which is 28 times leſs than Venus's perturbating force. And Saturn's is far leſs ſtill. So that their forces produce no ſenſible effect in the moon's motion.

Cor. *Hence all the viſible effects relating to the moon's irregular motions, are owing entirely to the perturbating force of the ſun; and that of the planets muſt be quite left out of the account.*

SCHOLIUM.

Altho' all the errors in the moon's motion are to be attributed ſolely to the action of the ſun; yet the motions produced thereby are ſo various and ſo combined with one another, that it is hardly poſſible to aſſign to them all, their due quantities.

From whence it muſt follow, that any ſet of tables, Fig. made upon any ſuppoſed quantities of theſe motions, tho' they may anſwer for the preſent, will deviate in a certain number of years and be wide of the truth. And it cannot be otherwiſe, ſince the equations are ſo many and ſo variouſly mixt. For ſeveral of them being given together by lump, the aſſigning to them their due proportions depends more upon judgment than theory. For all theſe equations depend upon the original projection of the moon which we do not know. And therefore aſtronomers are forced to do as well as they can, in a caſe where they want proper data; and that is, by trying one ſet of equations after another, by increaſing ſome and leſſening others, or adding new ones, till they agree to obſervations; which cannot hold true long upon ſuch a foundation. From hence I think, one may conclude, that no ſet of lunar tables can ever be made, that will ſhew the moon's place to a minute, nor perhaps to two or three, ſo as to be depended on at all times.

As to the acceleration of the moon's motion, if there be ſuch a thing, it can ariſe from nothing but the waſting of the ſun, by its continual emiſſion of ſo much light. For if light is a body, which has a gravitating property like other bodies (as its refraction ſhews); then the ſun's force will be leſſened by the loſs of ſo much matter; which will cauſe the earth to recede further from the ſun, and by that means ſhorten the periodical time of the moon. Whether the earth does really recede further from the ſun, or not, may perhaps be known by comparing the ſun's greateſt or leaſt apparent diameter, obſerved many ages ſince, with the preſent. For I can ſee no reaſon for the acceſſion of new matter to the earth, to increaſe its centripetal force, and ſo ſhorten the periodic time of the moon. But if the waſting of the ſun be the cauſe, it will likewiſe

Fig. lengthen the periodic time of the earth, as well as leſſen that of the moon; whilſt the quantity of matter in the earth, and its diurnal rotation remains unaltered. And then the effect of both theſe together, will be very viſible in a long ſeries of years.

PROP. XX. *Prob.*

To find the errors in the motion of a ſatellite, particularly in the firſt ſatellite of Jupiter.

Let ☽ per. time $= m =$ 655.73 hours.
1 ſatel. per. time $= s =$ 42.47 hours.
earth's per. time $= y =$ 365.25 days.
♃ period. time $= z =$ 4332.5 days.

Then (by Cor. 6. Prop. XXVIII. Cent. Forces), the mean angular errors in the ſatellite and moon, are as $\frac{s}{zz}$ to $\frac{m}{yy}$. And the periodic errors in the ſatellite, and the moon, are as $\frac{ss}{zz}$ to $\frac{mm}{yy}$. Therefore the mean angular error in the ſatellite, to the mean angular error in the moon; is as 1 to $\frac{mzz}{syy}$.

And the periodic error in the ſatellite, is to the periodic error in the moon; as 1 to $\frac{mmzz}{ssyy}$.

Cor. 1. *In the firſt ſatellite of Jupiter, the mean angular error in the ſatellite, is to the mean angular error in the moon; as* 1 *to* 2172.

And the periodic error in the ſatellite, is to the periodic error in the moon; as 1 *to* 33541.

Cor. 2. *The motion of the nodes of Jupiter's* 1*ſt ſatellite is* 53½ *min. in* 100 *years; and of the apogee* 25 *min. in the ſame time.*

For the motion of the moon's nodes in a year Fig. is, 19°.34, and $\frac{1934}{2172} = .89° = 53'\frac{1}{2}$. Alſo the motion of the apogee in a year is 9°.20, and $\frac{920}{2172} = .423° = 25'$.

Cor. 3. *All the equations of Jupiter's firſt ſatellite, as of the node, apogee, variation, &c. generated in its periodical time, are inſenſible and vaniſh.*

The greateſt equation is that of the annual argument, which ſuppoſe 5° = 18000″. Then $\frac{18000}{33541} = \frac{6}{11}$ nearly, being little more than half a ſecond. And therefore the leſſer ones quite vaniſh.

SECT. VII.

The Eclipſes of the Sun and Moon, and of Jupiter's firſt Satellite.

PROB. I.

Fig. *To find all the requiſites relating to the ſhadow and penumbra in an eclipſe of the moon.*

111. Let AB be the ſun, CD the earth. Draw AC, BD, by the edges of the ſun and earth, which will meet in a point V, becauſe the ſun is bigger than the earth. Thro' the centers of the ſun and earth S and T, draw STV. Alſo draw BCE, ADF, touching the contrary ſides of the ſun and earth, interſecting in P; alſo draw SC and CT. Then the figure ACEFDB contains all the requiſites; and all of them will be known, by ſolving ſo many plain triangles. And if the whole figure be turned round, about the axis SV, the lines AV, BV, APF, BPE, will generate the two cones CVD, EPF; the cone CVD is the dark ſhadow of the earth, and EPF infinitely continued, is the penumbral cone. And beyond V, the ſection of the cone EPF, will be all in the penumbra.

Cor. 1. *Half the angle of the cone of the earth's ſhadow* CVT, *is equal to the ſun's apparent ſemidiameter, leſs his horizontal parallax.*

For in the triangle SCV, the external angle SCA = CVS + CST. And CST is the ſun's parallax. Therefore CVT = SCA − CST.

Cor. 2. *Half the angle of the earth's penumbral cone* CPT, *is equal to the ſun's apparent ſemidiameter and his horizontal parallax.* Fig. 111.

For in the triangle CSP, the external angle $CPT = PCS + CST$.

Cor. 3. *Hence, half the angle of the earth's penumbral cone* CPT, *is equal to half the angle of the dark cone* CVT + *twice the ſun's horizontal parallax* CST.

Cor. 4. *The apparent ſemidiameter of the earth's dark ſhadow* IK, *upon the moon's orbit, is equal to the ſum of the horizontal parallaxes of the ſun and moon, leſs the ſun's apparent ſemidiameter.*

For angle $VCI = CIT - CVI =$ (Cor. 1.) $CIT - SCA + CST$.

Cor. 5. *The apparent ſemidiameter of the earth's penumbra* GI, *upon the moon's orbit, is equal to the ſum of the horizontal parallaxes of the ſun and moon* + *the ſun's apparent ſemidiameter.*

For in the triangle PCI, the external angle $ECI = CIT + CPT =$ (Cor. 2.) $CIT + PCS + CST$.

Cor. 6. *Hence to find the length* TV *of the earth's ſhadow. In the triangle* CTV *there is given the angle* V (*Cor.* 1.) = *ſun's apparent ſemidiameter — his parallax, and* CT *the earth's radius, to find* TV.

PROB. II.

To find all the requiſites belonging to the ſhadow and penumbra, in a ſolar eclipſe.

Let AB be the ſun, KL the moon, CD the 112. earth. Draw the tangents AK, BL, by the edges of the ſun and moon, on the ſame ſide, to meet in V; and BKG, ALH to touch the contrary ſides. Draw SK, IK; and thro' S and I the centers of the

Fig. the ſun and moon, draw the axis SIV. Then if
112. the whole figure AKVLB be turned about the axis SV, the ſides AV, BV, and PH, PG, will generate two cones KVL, GPH. The cone KVL is the dark ſhadow of the moon, and the cone GPH is the moon's penumbral cone. Then in the figure AKGVHLB are contained all the requiſites; which will be found, by ſolving the ſeveral plain triangles therein.

Cor. 1. *The angle of the cone of the moon's ſhadow* KVL, *the angle of the penumbral cone* KPL, *the angles* GKV, *and* HLV, *are each equal to the ſun's apparent diameter* AKB, *very near*; *and half the angle of either cone* P *or* V, *is equal to the ſun's apparent ſemidiameter.*

For by reaſon of the great diſtance of the ſun from T, in reſpect of TP, TV, TI; the apparent diameter of the ſun, ſeen from any of the places V, T, I, P, K, will be the ſame; that is, the angles AVB or KVL, APB or KPL, AKB or GKV, ALB or VLH, are all equal; differing only by the angle KSI, which in the moon is inſenſible.

Cor. 2. *The height of the cone* IP *is equal to the height of the cone* VI. *And* KPL, KVL, *are equal and ſimilar.*

For the angles at P and V are equal; and KL is common.

Cor. 3. *The apparent ſemidiameter of the moon's dark ſhadow* QO, *upon the earth at* O, *ſeen from the moon, is equal to the moon's apparent ſemidiameter — the ſun's apparent ſemidiameter. And if the ſun's ap. ſemidiameter be greater, the ſhadow does not reach the earth.*

For draw KO; then in the triangle KOV, VKO = KOS — KVS = KOI — AVS = KOI — ½ ſun's ap. diameter (by Cor. 1.)

Cor. 4. *The apparent ſemidiameter of the moon's penumbra* GO, *upon the ſurface of the earth, as ſeen from the moon, is equal to the ſum of the apparent ſemidiameters of the ſun and moon.* Fig. 112.

Draw GI and TGR. Then in the triangle GPI, the external angle GIO = GPI + PGI = KPI + KGI = KPI + KOI = (Cor. 1.) AKS + KOI.

Cor. 5. *Hence to find the length* IV *of the moon's ſhadow. In the triangle* KVI, *there is given the angle* KVI = *half the ſun's apparent diameter, and* KI *the earth's radius; whence* VI *will be had.*

Cor. 6. *And to find the arch* QN *of the earth, involved in the moon's dark ſhadow. In the triangle* QVT, *we have given* TV *the difference between the moon's diſtance from the earth, and the height of the ſhadow; and the angle* QVO = *the ſun's ap. ſemidiameter (Cor. 1.), and* TQ *the radius of the earth; to find the angle* TQV, *to which add* QVT, *and the ſum is the angle* QTO *or arch* QO; *and doubled gives the whole arch* QN.

Cor. 7. *And to find the arch of the earth* GH *involved in the penumbra. Say as the earth's radius* GT : *to* S. *ſun's ap. ſemidiameter* : : *ſo is* PT *the ſum of the moon's diſtance and cone's height* : *to* S.TGP *or* RGK. *From this take the ſun's ap. ſemidiameter, and there remains* GTO = GO, *and doubled gives* GH.

For in the triangle GPT, there is given the angle P = the ſun's ap. ſemidiameter, and PT the moon's diſtance and height of the cone, and TG the earth's radius; to find the angle RGK = GPT + PTG. Therefore PTG or OTG = RGK — GPT = RGK — ſun's ap. ſemidiameter.

PROB.

PROB. III.

113. *To find the ecliptic limits, or the diſtance from the node; when an eclipſe of the ſun or moon will happen.*

It is evident an eclipſe of the moon can only happen, when the diſtance of the centers of the moon, and of the earth's penumbra, is leſs than the ſum of their ſemidiameters. For if the diſtance is greater, the moon and penumbra cannot touch one another.

Alſo an eclipſe of the ſun cannot happen, unleſs the diſtance of the centers of the ſun and moon, be leſs than the ſum of their ſemidiameters; when ſeen from a certain place. But that it ſhall appear in no place, the moon's parallax muſt be added to the ſum of the ſemidiameters.

Therefore in lunar eclipſes, the moon's latitude muſt be leſs than the ſum of the ſemidiameters of the moon and of the earth's penumbral ſhadow, taken at the moon's orbit. And in eclipſes of the ſun, the moon's latitude muſt be leſs than the ſum of the ſun's and moon's ſemidiameters, added to the moon's horizontal parallax; that the eclipſe may be viſible ſome way. Or without the parallax, to be viſible in a certain place.

Therefore in the right angled ſpherical triangle ☊SM, having the angle ☊, and the diſtance SM, the diſtance of the ſun from the node ☊S will be known, or the ecliptic limits.

Here SM (the ſum of the ſemidiameters) will be known by the two former Props. But ſince all the quantities here concerned vary at different times, all which muſt be found agreeable to a particular time given. I ſhall therefore calculate the ecliptic limits, correſponding to all their mean quantities.

The

Fig. 113.

		′	″
The ſun's mean ap. ſemidiameter		16	4
his parallax — — —			12
moon's mean ap. ſemidiameter		15	38
her parallax — — —		57	5
Inclination of her orbit — —	5	8	30

Hence will be had,

ſemidiameter of earth's penumbra	1	13	21
ſemid. moon and earth's ſhadows		56	51
ſun and moon's ſemidiam. — —		31	42
the ſame with the parallax — —	1	28	47

In the triangle ☊SM for the moon's eclipſe.

Here SM = 1° 13′ 21″ + 15′ 38″ = 1° 28′ 59″.
S. ☊ = 5 8½ — — 8.952398
S.SM = 1 28 59 — — 8.413067
rad. — — — — 10.
S. ☊S = 16 47 — — 9.460669
the limit for the moon's eclipſe at a medium.

In the triangle ☊SM for the ſun's eclipſe.

Here SM = 1° 28′ 47″.
S. ☊. 5° 8′½ — — 8.952398
S.SM = 1 28 47 — — 8.412009
Rad. — — — — 10
S. ☊S. 16 45 — — 9.459611
the limit for the ſun's eclipſe, in any place; about the ſame as for the moon.

But for a particular place, SM = 31 42; and S☊ comes out only 5° 54′ for the limit.

Cor. 1. *Hence there will at leaſt be 4 eclipſes in a year, taking one year with another; two of the moon, and two of the ſun.*

For 16° 47′ + 16° 45′ = 33 32 or 33½. Therefore the ſun ſtays above a month within the ecliptic limits, twice in the year. During which time the moon

Fig. 113. moon makes two revolutions, and therefore muſt cauſe two eclipſes, either time; one of the moon and another of the ſun.

Cor. 2. *In a general way, half of the eclipſes will be invisible at any given place. And conſequently one year with another there can only be two viſible eclipſes in a year, one of the moon, and one of the ſun.*

For the ſun and moon ſpend as much time below the horizon as above it.

Cor. 3. *By the ſame method the ecliptic limits may be found for total eclipſes, as well as for partial ones.*

That is, by taking SM = the difference of the ſemidiameters of the earth's dark ſhadow and of the moon, in lunar eclipſes; or = the difference of the ſemidiameters of the moon and ſun, in ſolar eclipſes, (by Cor. 4. Prop. I.).

Cor. 4. *Eclipſes do not happen always in the ſame places of the zodiac; but in places more and more weſtward.*

For the eclipſes being about the nodes, and the nodes regreſſive at the rate of 19 deg. in a year; therefore the places of the eclipſes are 19 deg. more weſt every ſucceeding year.

PROB. IV.

Having the inclination of the moon's orbit, to the ecliptic, for a given time; and the horary motions of the ſun and moon; to find the angle that the moon's way from the ſun makes with the ecliptic, at the time of an eclipſe.

114. Let ☊S be the ecliptic, ☊M the moon's orbit, ☊ the node. And let S be the ſun, in the ſun's eclipſe; or the center of the earth's ſhadow, in the moon's eclipſe; and M the moon at the time

of the ſyziges. Take ☊A to ☊S as the ſun's horary motion, to the moon's, at that time; draw MA, then MAS is the angle required; and AM the moon's apparent orbit. Fig. 114.

For by conſtruction, in the time that the moon has been moving from ☊ to M (that is, thro' ☊S reckoned in the ecliptic), the ſun has moved thro' a ſpace DS equal to ☊A. Therefore the ſun was in D, when the moon was in the node at ☊. Draw DB, MB parallel to SM, SD; and draw B☊, which will be parallel to MA. Now ſince the moon makes the ſame latitude DB or SM, in the ſame time, whether the ſun moves or ſtands ſtill. And ſince SM is her latitude, when the ſun is at S, DB (equal to SM) will be her latitude, ſuppoſing the ſun had ſtood at D, without any motion towards S; and conſequently ☊B will be her apparent way, to an eye at D, thro' which ſhe ſeems to move in the ſame time. Or, which is the ſame thing, AM will be her apparent way to an eye fixt at S. For the triangles AMS and ☊BD are equal; and MAS is the angle of her way with the ecliptic. By the theory of relative motions, in bodies moving the ſame way, all the apparent motions are the ſame, as if one body ſtood ſtill and the other moved forward, with the difference of their motions. And here ☊D or A3 is the difference of their motions, ſuppoſing S to be fixt.

Cor. 1. *Hence, as the moon's horary motion* : *to the ſun's horary motion* : : S☊ *the diſtance from the node to* A☊. *Then* SA = S☊ — A☊.

Cor. 2. *As ſine of* SA : *rad* : : *tan. moon's lat.* SM : *tan. angle* A.

Cor. 3. *It is the apparent orbit* AM *that muſt be made uſe of in calculating all the particulars of an eclipſe.*

Fig. [illegible] For an obſerver conſiders not S as moving; and therefore only the relative motions are concerned.

PROB. V.

To calculate an eclipſe of the moon.

The Precepts.

1. Find the true time of the oppoſition, when an eclipſe is to happen, by Prop. XI. Sect. VI; and let that be reduced to apparent time.

2. By Prop. XII. Sect. V, and Prop. X. Sect. VI. find the true places of the ſun and moon, when in oppoſition. As alſo the ſun's mean anomaly, and the place of his apogee. Alſo the place of the moon's aſcending node, and of her apogee, and her latitude.

115. 3. Let ☊S be a part of the ecliptic; ☊M the moon's orbit; S the center of the earth's ſhadow, and M the moon, when in oppoſition. Take ☊A, to ☊S which is known by calculation; as the ſun's horary motion, to the moon's. Which motions are known from the aſtronomical tables. Draw AM, for the way of the moon from the ſun, by Prop. IV. Then in the right angled ſpherical triangle ASM, there is given AS (= ☊S — ☊A); and SM the moon's latitude found by calculation: to find the angle SMA.

4. Let fall SP perpendicular to AM. Then ſince the arches SM, MP, SP, are very ſmall, they may be taken for right lines; and the triangle SMP for a plane triangle. Then having SM and angle SMP; MP and SP will be found. Where P is the place of the moon in the middle of the eclipſe. Likewiſe the time of the moon's moving thro' MP will be known by her horary motion; and from thence the time when ſhe is at P, or the middle of the eclipſe.

UNIV

Fig. 115.

5. From the aſtronomical tables, find the ſun and moon's apparent ſemidiameters, for the time of oppoſition; and their horizontal parallaxes.

6. From any convenient ſcale of equal parts, with the center P and radius PB, equal to the minutes contained in the moon's radius, deſcribe the circle BC*o* for the moon. And with the radius SD (equal to the ſum of the ſun and moon's horizontal parallaxes — the ſun's ſemidiameter, all in minutes), deſcribe the circle DEB, from the center S, then this circle (by Cor. 4. Prop. I.) will repreſent the earth's dark ſhadow. Likewiſe with the ſame center S, and radius SF (equal to the ſum of the ſun and moon's parallaxes + the ſun's ſemidiameter, in minutes), deſcribe the circle FQG; and this (by Cor. 5. ib.) will be the earth's penumbra.

7. Theſe things being laid down, it will be eaſy to find all the requiſites by ſcale and compaſs, by meaſuring them; or rather by calculation, in the ſeveral right angled plain triangles, contained in the ſcheme. Thus, to find when the moon firſt touches the penumbra at L; in the right angled triangle SPK, there is given SP, and SK (the ſum of the radii SL and PB), to find PK. Which being known, the time of the moon's paſſing thro' it will be known, by the moon's horary motion from the ſun. Again, to find when the moon firſt enters the dark ſhadow of the earth in D. In the right angled triangle SPI, there is given SP, and SI (the ſum of the radii SD, PB) to find PI; and conſequently the time of half the duration in the ſhadow. Alſo to find the digits or 12th parts of the moon eclipſed. Here *no* the part eclipſed is = $Sn + Po - SP$; and $\frac{12no}{2Po}$ or $\frac{6no}{Po}$ is the number of digits eclipſed.

In total eclipſes of the moon, the earth's ſhadow often reaches further than the moon. And then more

Fig. 115. more than 12 digits are ſaid to be eclipſed, ſuppoſing the moon's diſk to be produced ſo far.

If the time was required when the moon wholly enters into the dark ſhadow BED, it will be found the ſame way, as when it entered into the penumbra GQF; as will be evident, by ſuppoſing GQL the dark ſhadow. In that caſe SI will be the difference of the ſemidiameters of the moon and dark ſhadow.

Now the times of paſſing thro' PI, PK, &c. being known, and the time of the middle of the eclipſe at P; the beginning and ending will be known.

8. Hence if the moon or circle CB*o*, never touches the circle GQF, there will be no eclipſe, not even by the penumbra. And if the ſame circle never touches the circle BDE, there will be no part of the moon totally eclipſed. And if the whole circle CB*o* enter into the circle BED, the whole moon will be totally eclipſed; and that is when SP is leſs than the difference of the ſemidiameters SD and PB. If the point S be in the node, then P falls upon S, and the eclipſe is central. When only a part of the circle CB*o* goes into the circle BED, the eclipſe is a partial one, as in this figure.

9. The time of the eclipſe being known for any particular place, it is eaſy to know, if it be viſible at that place; by knowing if the moon be riſen. Or the place will be known where the moon is vertical; and therefore it will be viſible to all places, within a quadrant's diſtance from it.

10. It muſt be noted, that if the ſpectator lives in the place (or in the ſame longitude) which the tables are calculated for; he will ſee the eclipſe at the time determined by the calculation. If not, he will ſee it an hour ſooner for every 15 degrees difference of longitude, that he lives weſt from it. And ſo much later, if he lives eaſtward; that is in the way of reckoning time. But in regard to

abſolute time, it is ſeen from all places at the ſame Fig. inſtant. 115.

Example.

To find the time of the moon's eclipſe Dec. 13, 1769; *its duration and digits eclipſed.*

1. The mean time of the ſyziges, by the tables, is found to be Dec. $12^d\ 19^h\ 27^m$, at which time the moon's horary motion from the ſun is $35'\ 33''$. At this time, computing the true places of the ſun and moon, the moon will appear to be 35 10 before the ſun. And therefore the time is paſt the ſyziges, $59^m\ 12^s$. Therefore,

from	1^d	19^h	27^m	0^s
take	0	0	59	12
true time	1	18	27	48

The places being computed again, the moon is only $7''$ before the ſun, which amounts to 12^s of time; therefore the time of oppoſition is $12^d\ 18^h\ 27^m\ 36^s$, which reduced to apparent time is Dec. $12^d\ 18^h\ 32^m\ 51^s$.

2. The ſun's place is	8^s	21°	$37'$	$35''$
the moon's place	2	21	37	35
place of the aſcending node	8	14	46	13
her latitude ſouth — —			37	58
the ſun's horary motion			2	33
moon's horary motion			38	6

3. Hence the moon is $6^\circ\ 51'\ 22''$ paſt the deſcending node; that is, ☋S is $6^\circ\ 51'\ 22''$. Therefore ☋A = $27'\ 32''$, and AS = $6^\circ 23'\ 50''$. Therefore the angle SMA = $84^\circ\ 22'\ 28''$.

4. Hence drawing the ecliptic RS, and SM perp. 116. to it, and equal to $37'\ 58''$ from a ſcale of minutes. And making the angle SMA = $84^\circ\ 22\frac{1}{2}$. You will find the perpendicular SP = $37'\ 47''$, and MP = $3'\ 43''$. And therefore, the horary motion of the moon from the ſun being $35'\ 33''$, PM will be paſſed over in $6'\ 17''$. And ſince this is before the

Fig. 116. oppoſition at M, this time muſt be deducted from the time of oppoſition. And the time of the middle of the eclipſe will be Dec. 12^d 18^h 26^m 34^s.

5. The ſun's apparent ſemidiam.	16′	20″
his horiz. parallax — —		12
moon's ap. ſemidiameter	16	48
her horizont. parallax	61	7

6. Hence the radius BP = 16′ 48″.
radius SD = 44 59.
radius SF = 77 39.

7. Hence alſo PK or P*k* = 86′ 34″.
and PI or P*i* = 48 53.

and therefore the time of paſſing thro' PK is 2^h 26′ 6″, and thro' PI = 1^h 22′ 30″. And the whole duration in the ſhadow from I to *i*, is 2^h 45^m. And the digits eclipſed 8½ on the upper ſide. Whence,

firſt entering the penumbra Dec. 13^d	4^h	0^m	28^s morn.
entering the dark ſhadow at	5	4	4
middle — —	6	26	34
oppoſition — —	6	22	51
leaving the ſhadow — —	7	49	4
leaving the penumbra — —	8	52	40
duration — —	2	45	0

digits eclipſed 8½.

All theſe things may be determined near enough by ſcale and compaſs, in a large draught; making uſe of a ſcale of minutes and ſixtieth parts; or rather by making a ſcale of time anſwering thereto, by help of the horary motion of the moon from the ſun. For by this ſcale, the ſeveral hours and minutes may be marked along the line A*k*, by which it will appear at what time the center of the moon is at any given point. For the time is known when the moon is at M, and from thence the points at each hour and minute are eaſily found.

And this conſtruction, with only right lines and circles, will be exact enough in a large figure; for the beſt lunar tables give the times of the phaſes of an eclipſe no nearer than to 4 or 5 minutes of time; and therefore ſuch a conſtruction is ſufficient to anſwer the purpoſe. Fig. 116.

Cor. 1. *No eclipſe of the moon can laſt above* $5\frac{1}{2}$ *hours, from the moon's firſt touching the earth's penumbra, to its laſt leaving it.*

For SK $= 94' \; 27'' = 94.45$, and the horary motion is $35' \; 33'' = 35.55 \; \frac{94.45}{35.55} = 2.66 = 2^h \; 39^m =$ ſemiduration.

Cor. 2. *No eclipſe of the moon, by the earth's ſhadow, can laſt above* $3\frac{3}{4}$ *hours. Nor when total, above* $1\frac{3}{4}$ *hours.*

For SI $= 61' \; 47'' = 61.78$, and $\frac{61.78}{35.55} = 1.745$ $= 1' \; 45'' =$ the ſemiduration, and SD — SI $=$ $28' \; 11'' = 28.18$, and $\frac{28.18}{35.55} = .79 = 47^m$, the ſemiduration.

SCHOLIUM.

The refraction of the earth's atmoſphere, in lunar eclipſes, makes the ſhadow leſs; by bringing the rays, which terminate the ſhadow, ſooner to a point. And hence comes that red colour of the moon, even in total eclipſes. But that light muſt be very dim, by reaſon a great number of the rays are ſtopt and loſt in the earth's atmoſphere.

The circles terminating the ſhadow and the penumbra BED and GQF, cannot be diſtinguiſhed. For the darkneſs from BED, diminiſhes by inſenſible degrees, to GQF, being darkeſt at E, and lighteſt at Q, where it vaniſhes inſenſibly. And therefore the moon does not appear to be eclipſed, Fig. 116. till ſhe is a good way within the penumbra. For that reaſon, there may happen eclipſes of the moon which cannot be diſcovered as ſuch.

It may be obſerved that all lunar tables, ſhew the moon's place in eclipſes, more truly in the ſyziges than in the quadratures, or any other place. For the times of the ſyziges have been more accurately obſerved; and the moon's place more nicely obſerved in eclipſes, than at any other time; and from thence the moon's theory has been deduced. Beſides many of the inequalities ceaſe in the ſyziges, but have ſenſible effects in other places; becoming greater, as the moon is further from the ſyziges; being greateſt in the quadratures. Whence the lunar tables do not determine the moon's place truly in the quadratures. And her place calculated from theſe tables is not ſo exact in the quadratures as in the ſyziges. Several inequalities depend on the aſpect of the nodes and the ſun; but theſe ceaſe when the nodes are in the ſyziges, When the moon and the nodes are in the ſyziges; the moon's place, then wanting fewer equations, as being ſubject to fewer inequalities, will be more correct; than when ſhe is in other places, where there are more and greater inequalities, and more equations; which equations have never been truly fixed. From hence more errors will happen out of the ſyziges, than in them.

PROB. VI.

To find the way of the moon from the ſun, in a ſolar eclipſe, ſuppoſing the obſerver at reſt.

117. Let HZO be the meridian of the place, HO the horizon, EC the equinoctial, EL the ecliptic, Z the zenith, P the pole, S and M the places of the ſun and moon in conjunction, PSD the ſun's meridian.

Having found the ſun's diſtance from the node, ☊S, and the moon's latitude SM, &c. take ☊A to ☊S, as the ſun's horary motion, to the moon's horary motion; then SA is known. Draw MA; then in the ſpherical triangle ASM, right angled at S, there is given SA, SM; to find the angle SMA; AM being the moon's way from the ſun. Fig. 117.

But becauſe the eye of the obſerver is in motion, by the rotation of the earth; which gives an apparent motion to the moon, contrary to that of the obſerver; we muſt find the quantity and direction of that motion. In the firſt place, ſince the obſerver is carried eaſtward, that is towards the point C, the apparent motion in the moon cauſed thereby will be in the line CS. And to determine the poſition of CS in reſpect of AM or SM, ſeveral ſpherical triangles muſt be reſolved, as follows.

In the right angled triangle EDS, there is given ES and angle E to find DS and angle ESD or ASP; or theſe may be eaſier had from the aſtronomical tables. And in the triangle ZPS, there is given PS (the complement of DS), the angle ZPS (from the time of the day), and ZP the complement of latitude; to find ZS, and angles PZS and ZSP. Then ZSP and ASP being known, ZSA will be known. And MSA being a right angle, ZSM will be known. In the right angled triangle CFS, there is given CF, the meaſure of the angle FZC (the difference between the angle PZS and the right angle CZP), and SF the complement of ZS; to find CS, and the angle CSF or BSZ. Then BSZ and ZSM being known, BSM will be known. And SMA being known, its ſupplement SMB is known, and conſequently the angle SBM.

To find the quantity of the motion. That along AM is already known; and to find the apparent motion along SB. The ſine of 15 degrees (the horary motion of a point in the equinoctial) is .259 to the radius 1. And if *b* be the moon's horizontal parallax, then the radius of the earth appears at the moon under the angle *b*, and therefore 15 degrees of the equinoctial appears under the angle of .259*b*; this then is the horary motion of a point in the equinoctial, viewed directly from the moon. And the moon's apparent motion ſeen from that point in the equinoctial is the very ſame. But this motion is to be diminiſhed upon two accounts. 1. Becauſe it is leſs in a parallel circle, in proportion to the coſine of the latitude. And 2. Upon account of the obliquity of the motion, when not perpendicular to the rays of the ſun; and this will be as the ſine of CS, the ſun's diſtance from the eaſt or weſt point of the horizon. Therefore to find the quantity of this motion. Fig. 117.

To the Log. of .259*b*
add the coſ. latitude
and the ſine of CS.

Then the ſum, abating twice radius, is the log. of this apparent horary motion. Then this motion is to be compounded with the motion along AMB as follows.

118. Let AS be a portion of the ecliptic, SB the way of the apparent motion, MA the moon's way from the ſun. Draw NM parallel to SB; and let MN be the horary motion along SB or MN, and MI the horary motion of the moon from the ſun. Then compleat the parallelogram NMIQ; draw the diagonal MQR, which is the direction of the motion, compounded of the obſervers and the moon's motions, and MQ is the total apparent horary motion, ſuppoſing the obſerver at reſt. Then in the plain triangle QMI, there is given MI, and IQ (or MN), and the angle MIQ = MBS; to find the angle QMI, and ſide MQ or the abſolute horary motion. And the angles QMI and IMS being known, QMS is known.

Here obſerve, if the ſun be in the eaſtern hemiſphere, in which caſe the concave ſide of the eaſtern hemiſphere is here projected (in fig. 117), then the moon's motion from the ſun is from M towards A, and the other apparent motion from S towards B or from M towards N. But if the ſun is in the weſtern hemiſphere, this projection repreſents the convex ſide of the ſphere; and then the moon moves from the ſun, in direction AM, and the other apparent motion is from S towards C, being contrary. Fig.

PROB. VII.

To calculate an eclipſe of the ſun, for a particular place given.

The eclipſes of the ſun are more difficult to calculate, than thoſe of the moon; the latter being clear of parallaxes, which the former are incumbered with, which gives a great deal of trouble.

But a great part of it may be avoided by uſing projections inſtead of calculations. The rules are,

1. Find the true time of the conjunction, and the places of the ſun and moon at that time; by Prop. 12. Sect. V. and Prop 10 and 11, Sect. VI. with all other requiſites by the foregoing Props. of this ſection; all for your particular place.

2. Having found the way of the moon from the ſun as directed in the laſt Prob. by projection or calculation; find, by the aſtronomical tables, the moon's horizontal parallax, her apparent diameter, and horary motion; alſo the ſun's apparent diameter, and horary motion.

And here to avoid a great deal of calculation, if the ſphere be projected by a large ſcale, it will give all the requiſites with ſufficient exactneſs, by meaſuring the ſeveral angles and ſides, without any calculation, or very little. And here it is beſt to

Fig. project the concave ſide, and then every thing appears as it is in nature.

119. 3. Find the moon's parallax of altitude, by making as rad : cos. altitude : : ſo the moon's horizontal parallax : to her parallax of altitude Vt or Mm. Then find her parallax of latitude Mn, and longitude Ss, or mn, by Prob. 17. Sect. IV. and from thence her apparent latitude and longitude is known.

120. 4. Draw the line ASL for the ecliptic, and from a large ſcale of minutes, erect SM perp. to AS, and equal to the apparent latitude; make the angle SMR, as found in the laſt Prob. and draw ☊MR for the moon's apparent path. From S let fall SP perpendicular to MR, and SP will be the leaſt diſtance of the centers of the ſun and moon, or the middle of the eclipſe. From the center S with the radius equal to the minutes contained in the ſun's ſemidiameter, deſcribe the circle ABC for the ſun. And from the center P, with the radius equal to the moon's ſemidiameter, deſcribe the circle AOCD for the moon. If theſe circles do not interſect, there will be no eclipſe. But if they interſect, an eclipſe muſt neceſſarily happen.

5. Then P is the place of the moon in the middle of the eclipſe. Make SI and SK equal to the ſum of the ſemidiameters of the ſun and moon; and the moon's center will be at I when the moon firſt touches the ſun, or at the beginning of the eclipſe. And K the place of the moon's center, at the end of it. In the triangle PSI, there is given SI, SP; to find $PI = PK$, which reduced to time, by help of the moon's apparent horary motion, ſhews half the duration of the eclipſe; and conſequently we ſhall have the beginning and end.

6. And to find the quantity no, or the digits eclipſed; we have $no = Sn + Po - SP$, and $\frac{6no}{Po}$ = number of digits.

7. The time found being mean time, it muſt be Fig.
reduced to the common or apparent time, by the
equation of time. And if the given place be not
that for which the tables are made; add ſo much
time, if your place lie eaſtward to the time of
conjunction, as anſwers to the difference of meridians;
or ſubtract it, if you lie weſtward, as is
noted in article 1.

Example.

To find the time of the ſun's eclipſe, June 4, 1769, *its duration and digits eclipſed, at London.*

1. By the tables the mean time of the conjunction is found to be June 3^d 20^h 41^m. And hence the true time of conjunction is June 3^d 20^h 27^m 43^s. And their places are 2^s 13° 51′ 25″. And the moon's lat. 55 32 north. The moon's motion from the ſun 35′ 47″.

2. The angle AMS = 84° 27′. ZSM = 35° 117.
20′. CSF = 5° 18′. SBM = 43° 49′. SF = 118.
42° 16′, CF = 3° 34′. CS = 42° 24′. The angle
QMI = 8° 25′. SMQ = 92° 52′. MN or IQ
= 6° 38′. MQ = 31° 20′. Alſo

The moon's horizontal parallax	60′	58″
her apparent diameter	33	32
her horary motion — —	38	10
the ſun's diameter — —	31	41
his horary motion — —	2	23.

3. The moon's parallax in altitude M*m*, is 45′ 119.
09″; her parallax in latitude M*n*, 38′ 05″; her remaining
latitude S*n*, 17′ 26″; her parallax in longitude
S*s*, 24′ 13″; which is increaſed ſo much.

4. Draw SL for the ecliptic; at any point S, 120.
erect the perp. MS equal to 17′ 26″, the moon's
apparent latitude; thro' M draw the moon's way
☊MR, making the angle SMR = 92° 52′. Draw
SP perp. MR, which here falls very near M. From
the center S, with the radius SA = 15′ 50″ deſcribe

the

Fig. 120. the circle ABC for the ſun. And with the radius MD = 16′ 46″, and center P, deſcribe the circle ADCO for the moon.

5. Hence PI or PK = 27′ 33″. And the time of moving thro' IP or PK, at the rate of 31′ 20″ an hour, is $52^m\ 45^s$, for the ſemiduration. By reaſon of the parallax (24′ 13″), ſhe is paſt the apparent conjunction; the difference being what the parallax cauſes, which comes to $46^m\ 23^s$. Therefore the middle of the eclipſe is ſo much ſooner, being at $3^d\ 19^h\ 41^m\ 20^s$. This reduced to apparent time is $3^d\ 19^h\ 43^m\ 27^s$, for the middle.

6. The digits eclipſed are $5\frac{4}{10}$, nearly.

In this example, I have ſuppoſed that the concave ſide of the ſphere is projected, which ſuits beſt to the appearance of the heavens. And the figures are drawn upon that ſuppoſition.

It appears from the proceſs, that the moon is advancing to her deſcending node, and therefore has north latitude. And by the poſition of that part of the ecliptic, her parallax in longitude, advances her ſo much forward, viz. 24′ 13″. And therefore ſhe is ſo much paſt the apparent conjunction. Hence we gain theſe ſeveral particulars.

The beginning of the eclipſe, June	4^d	6^h	50^m	42^s	morn.
middle — —	4	7	43	27	
end of it — —	4	8	46	12	
total duration — —		1	45	30	

digits eclipſed $5\frac{1}{2}$, on the upper ſide of the ſun, towards the left; as appears by the figure.

Cor. 1. *Hence the poſition of the horns at* C *and* A, *are eaſily found in the middle of the eclipſe.*

For they are in a poſition parallel to RI, the moon's way.

Cor. 2. *The middle of the eclipſe will not be at the ſame time in all places of the ſame longitude.*

For

6.

UNIV.

For the parallax of longitude will be different in different places. Fig. 120.

Cor. 3. *No eclipſe of the ſun can laſt above two hours.*

For SI or SA + MD = 32′ 36″ = 32.6, and the horary motion = 34′ 47″ = 35.78. And $\frac{32.6}{35.78}$ = .91 = 54½ minutes, for the ſemiduration.

SCHOLIUM.

If it was not for the parallax, eclipſes of the ſun were as eaſily calculated, as thoſe of the moon. And in order to get the parallax, the angle ZSM and SF muſt be known, which occaſions the reſolving ſeveral ſpherical triangles before they can be had. Likewiſe it may be obſerved, that the apparent way of the moon is ſtrictly a curve line, concave towards S, which ariſes from the parallel of latitude being a curve, and the moon being out of its plane. Likewiſe the moon's apparent velocity is ſomething greater at the beginning than at the end. But I deſcend not to theſe minute particulars.

PROB. VIII.

To calculate a general eclipſe of the ſun; or an eclipſe of the earth. 121.

1. The elements neceſſary for this are the following. The ſun and moon's place, and likewiſe the time, at the true conjunction. The moon's latitude, horizontal parallax, diameter, and horary motion. The ſun's declination, diameter, and horary motion. And the angle the moon's way makes with a circle of latitude.

2. From a large ſcale of minutes, take the moon's horizontal parallax in your compaſſes, and at any point C, in the right line BD (which repreſents

Fig. 121. ſents the ecliptic), deſcribe the circle ABED, for the earth's diſk, or the earth's flat face as it appears at a diſtance, in a line drawn to the ſun. Draw CM perpendicular to CD, and equal to the latitude of the moon upwards if north. Make the angle CMG equal to that which the moon's way makes with a circle of latitude; acute to the right hand, if ſhe tend to the node; or obtuſe, if ſhe be paſt it; and drawing FMG, it will be the way of the center of the moon's ſhadow upon the earth. From C let fall CH perp. to FG. Then at H will be the middle of the earth's eclipſe. With the center H, and radius HO, equal to the ſum of the ſemidiameters of the ſun and moon, deſcribe the circle QCR, which will be the moon's penumbra, by Cor. 4. Prob. II. Alſo deſcribe a ſmall circle round the center H, whoſe radius is the difference of the ſun and moon's ſemidiameters, that little circle will be the dark ſhadow of the moon, by Cor. 3. ib.

3. Then all the countries of the earth contained in the ſegment VAW will be ſucceſſively eclipſed, by the penumbra, as the ſhadow moves along the tract FG. Whilſt the other ſegment VEW ſuffers no eclipſe at all. Likewiſe all places in the line *st*, will be totally eclipſed, as the dark ſhadow, or the ſmall circle at H paſſes ſucceſſively over them. But this circle, or dark ſhadow being very ſmall, a total eclipſe at any place, continues but a ſmall time. Sometimes the ſun's ſemidiameter exceeds the moon's; and then there will be no dark circle, or total eclipſe, but a lucid ring will appear about the moon in theſe places; and this is called an *annular eclipſe*. The difference between the ſemidiameters of the ſun and moon is ſo little, that no total eclipſe laſts above 4 minutes.

4. Draw CF, CG = ſum of the ſemidiameters of the ſun and moon, and the moon's parallax;

then the moon's ſhadow will touch the earth at L and K where the eclipſe begins and ends. Fig. 121. In the triangle CFH, there is given CF, CH; to find FH = HG, which converted into time, gives half the duration, or half the time that the moon's ſhadow is upon the earth. Alſo NO meaſured, ſhews how far the eclipſe reaches. Or CO meaſured does the ſame. It may be ſufficient to meaſure all theſe by the ſcale without calculation.

5. To find the pole. Draw the arch AP, making the angle KAP equal to the ſun's longitude, and AP the diſtance of the poles of the equator and ecliptic, 23°½; then P is the pole. For AP is a part of the ſolſtitial colure, and paſſes thro' Cancer and Capricorn. And CAP is what the ſun wants of Cancer, therefore PAK is what it is paſt Aries. Thro' P draw CPT. And here we may ſuppoſe that the pole P is fixed, during the time of an eclipſe. Then in the right angled ſpherical triangle APT, there is given AP and angle A; to find AT or angle ACP. In this triangle PT is the ſun's declination, and APT or CPK his right aſcenſion from Cancer. Here note, that any place in the line CT is in the ſun's meridian. And C is the place where the ſun is vertical at the time of the eclipſe.

6. To find the ſituation of any given place, at a given hour. Make the angle CPX (with the ſun's meridian), equal to the time from noon; on the left hand, if it is before noon. And make PZ the complement of the latitude; then Z is the place required. And if it falls in the penumbra, it is eclipſed; or any where in the ſegment VAW; if its motion in the parallel circle do not carry it out, before the penumbra reaches it.

7. To find the place which is firſt or laſt touched by the penumbra, as K. Draw the arch PK. In the triangle CGH, there are given CG and CH, to find the angle GCH, from which ſubtract HCP

Fig. 121. which is known, gives the angle PCK or TK. Then in the right angled ſpherical triangle PTK, there is given TK, and PT the ſun's declination; to find PK the complement of the latitude of K, and TPK or CPK the difference of longitude of K, and the ſun. Therefore its longitude and latitude is had. And after the ſame manner you may find that of L. And by the ſame method the lat. and long. of the places *s* and *t* may be found, where the dark ſhadow firſt enters the earth's diſk, or quite leaves it.

Alſo there may be found the place which is in the line FH at any point of time; or if the place be given, what the time will be; and that by help of the horary motion, with ſeveral other particulars of like nature, which for brevity's ſake I omit.

8. The part of the ſun's diameter eclipſed by the moon, is known by the ſituation of the place within the penumbra, or its diſtance from the center of the penumbra.

And the phaſis of the eclipſe, as ſeen from any place Z, upon the diſk, will be found thus, for any time. Find the center of the ſhadow for that time, as ſuppoſe at H. Deſcribe about H, a circle whoſe radius is the moon's radius; and about Z, a circle with the ſun's radius. Then the part cut off the ſun's circle, will be the part obſcured. I omit drawing the figures, as they would perplex the ſcheme.

Fig. 121.

Example.

Let us put the former example. To find the time and phaſes of the eclipſe, June 4, 1769.

The elements are theſe following.

1. Their places —	$2^s\ 13^\circ\ 51'\ 25$
time of conjunction	$8^h\ 29\ 50$ morn.
moon's latitude —	$55'\ 32$
horizontal parallax —	60 58
diameter — —	33 12
horary motion —	38 10
ſun's declination —	22 30
his diameter — —	31 41
and horary motion —	2 23
angle of ☽ way —	84 27
her motion from ☉ —	35 47

2. The diſk of the earth being drawn, and all the particulars ſet off as required, we have CA = $60'\ 58''$. CM = $55'\ 32''$. CH = $55'\ 16''$, angle MCH = $5'\ 33''$, MH = $5'\ 22''$, and HO = $32'\ 36''$.

3. Since CO = CH — HO = 22 40. And $\frac{22\ 40}{60\ 58} = \frac{22.66}{60.96} = .3717$ the nat. ſine of CO = 21° 49′. And NO = 68° 11′. Therefore the penumbra paſſes over 68° 11′ of the northern ſurface of the earth. Likewiſe CH = 65° 3′. Therefore the tract of the dark ſhadow cuts off a ſegment at N = 24° 57′, and covers a ſpace = 1° 25′.

4. In the triangle FCH, FH = $75'\ 30''$, FCH = 53° 48′. The horary motion of the moon from the ſun is $35'\ 47''$; therefore FH or HG is deſcribed in $2^h\ 6^m\ 36^s$. And the whole duration of the penumbra upon the earth is $4^h\ 13^m\ 12^s$. And the time of the dark ſhadow, paſſing thro' *st*, is about 3 quarters of an hour.

5. In

Fig. 121. 5. In the triangle ATP, AT or angle ACP = 6° 54′.

6. In the triangle CPZ, there is given CP, PZ, and angle CPZ; to find CZ = 49° 18′, and angle PCZ = 40° 27′; whence angle ACZ = 33° 33′, and HCZ = 39° 6′. So that when the place is at Z, the dark ſhadow is at M; being the time of the conjunction. But if the time be taken 46 minutes ſooner, the place *z* would be nearer the line *s*M, and the center of the penumbra near *s*. For whilſt the place Z moves about P, towards O, by the earth's rotation, the center of the ſhadow moves from *s* towards H. And their neareſt diſtance will be about $\frac{1}{10}$ of HO.

7. The angle GCN being 53° 48′, PCG = 52° 27′ = TK; whence TPK = 73° 37′, and CPK = 106 23, the dif. longitude from the ſun; and PK = 55 44 the comp. lat. of the place K, where the eclipſe quite leaves the earth.

8. In the triangle MCZ, having MC, CZ, and angle MCZ, the place in Z may be exactly found; and the ſhadow being at M, if a circle of the penumbra be deſcribed about M, the point Z will fall a little within it; ſo that the eclipſe is not quite over at the time of the conjunction.

Cor. 1. *Hence may be known, all the places and countries thro' which the dark ſhadow paſſes.*

For the time being known when the ſhadow comes to S, it will be known what place is at S; and in like manner what place is at *r*, when the ſhadow comes there; the times being known from the horary motion. Likewiſe the time is known, when the ſhadow is at M, and at H. Then a line drawn thro' all theſe places on a globe or map; ſhows the tract of the ſhadow thereon.

Cor. 2. *Hence, the place where the ſun is vertical, in the middle of the eclipſe, is eaſily found.*

For there is given the azimuth CF (fig. 117.); and diſtance, equal to ZS, the complement of the ſun's altitude. Fig. 121.

Cor. 3. *The ſun's eclipſe is not at the ſame inſtant in all places, as it is in a lunar eclipſe.*

Cor. 4. *The middle of the eclipſe is ſeldom at or near the time of conjunction; but ſometimes ſooner and ſometimes later, according as the parallax increaſes or decreaſes the moon's longitude. Except in the place where the ſun is vertical, where there is no parallax.*

Cor. 5. *No general eclipſe of the earth can laſt above* $5\frac{1}{2}$ *hours.*

For AC + AO, or the whole ſemidiameter of the earth's diſk added to the ſemidiameter of the moon's penumbra, is $93' \, 24'' = 93.4$; and the horary motion is $35' \, 47'' = 35.78$. And $\frac{93.4}{35.78} = 2.61 = 2^h \, 36^m$, the ſemiduration.

SCHOLIUM.

Here all the circles concerned in the calculation, are ſuppoſed to be orthographically projected upon that diſk of the earth, which directly faces the ſun; and that a ſpectator is placed at the diſtance of the moon, in a line perpendicular to the diſk. Then all the parts of the ſcheme will be ſeen under their proper angles, as they are laid down therein. For the diſk itſelf will appear under an angle equal to twice the moon's horizontal parallax; and the penumbra under an angle equal to the ſum of the diameters of the ſun and moon; and being ſmall arches, are truly repreſented by right lines proportional thereto: and therefore thoſe that underſtand the nature of the orthographic projection of the ſphere, can eaſily tell how to meaſure any of theſe

Fig. 121. angles or ſides, or lay them down as required. Now if the operations be performed on a large figure, the parts may be meaſured near enough. And moreover if the ellipſis repreſenting the parallel circle of any place Z, be drawn, and divided into hours (or 15 degrees), beginning at Z, when the time of the conjunction is ſuppoſed to be known, and theſe diviſions be all marked with their proper number of hours; theſe will ſhew the places where Z is come to, at theſe ſeveral hours, and the contrary. And likewiſe if the tract FG be divided into hours, by help of the horary motion, begining at the conjunction M, whoſe time is known; and if theſe be alſo marked with their proper hours; they will ſhew the places, where the ſhadow is at theſe ſeveral hours. And from hence may be known by inſpection, what ſituation the place and the ſhadow have to one another, at any particular times; and therefore when they are neareſt together, or when the middle of the eclipſe is; thus may be known the *phaſis* of the eclipſe, that is the appearance of the ſun and moon eclipſed, at any time. But to do this, as I ſaid before, requires a very large ſcheme to work upon.

PROB. IX.

To find the period of eclipſes; or the diſtance of time, in which the eclipſes will all return in the ſame order.

Since the eclipſes depend upon the ſun and moon being near the node; it is evident that when they are in the ſyziges, and have the ſame ſituation, in reſpect to the node, that the eclipſes will return again, the ſame as before. Therefore we are to find the diſtance of time when this will happen.

The

	d	h	m	s	d
The time of the moon's ſynodic revolution is	29	12	44	3	=29.5306
the periodic revolution	27	7	43	12	=27.3216
revolves to the node in	27	5	5	35	=27.2122
to the apogee in	27	13	8	34	=27.5476

But as $27^d.3216 : 360^o :: 1^d : 13^o.1764$, the moon's motion in a day.

Let a = number of lunations, or of the moon's revolutions to the ſun, e = number of revolutions to the node, in the ſame time. Then that the ſun, moon, and node, may come together again, we muſt find a and e, ſo that $29.5306a = 27.2122e$, and $\frac{a}{e} = \frac{27.2122}{27.5306}$. But (by Prob. IX. Chap. IV. B. II. Arithmetic), we ſhall find $\frac{a}{e} = \frac{1}{1}, \frac{11}{12}, \frac{12}{13}, \frac{35}{38}, \frac{47}{51}, \frac{223}{242}, \frac{939}{1019}, \frac{16186}{17565}$ &c.

Alſo if y = number of revolutions of the moon to the apſis in the ſame time. Then $27.21e = 27.547 = 29.53a$.

To examine theſe numbers; omitting the ſmall fractions which are not exact enough; take $\frac{47}{51} = \frac{a}{e}$. Then $29.5306 \times 47 = 27.2122 \times 51$ nearly; or $1387.938 = 1387.222$, near. And $1387.938 - 1387.222 = .116^d = 1.52^o = 1\frac{1}{2}$ degrees. So that the moon, after 47 lunations, is but $1\frac{1}{2}$ degree beyond the node. And this period would do pretty well, but that the apſis is not near the node. For $\frac{27.21e}{27.54}$ or $\frac{27.21 \times 51}{27.54} = y = 50.39^d$ = ſome number of revolutions to the apſis. So that the apſis is .39 of a revolution from its former ſituation, which amounts to 10.7 days, or 140 degrees. This period would be 4 years wanting 73 days and 2 hours. But the different poſition of the apogee

Fig. diſturbs it ſo much, as to make it vary ſometimes 9 or 10 hours.

Then for the numbers $\frac{223}{242}$ or 223 lunations; here $29.5306 \times 223 - 27.2122 \times 242 = -.029^{d} = -.38^{\circ} = -23'$. So that the moon is but 23′ ſhort of the node, or of her former place. Alſo $\frac{29.53 \times 223}{27.54} = y = 239$ rev. + 3.2 days remaining. And 3.2 days = 42 degrees, and ſo far the moon is beyond the apſis; or the apſis differs 42° in ſituation. This period is 18 Julian years, 10 days, 19 hours, 46 minutes. This is called the *Plinian Period*, or *Chaldean Saros*.

Again, 939 lunations = 27729.2334, and 1019 revolutions to the node = $27729^{d}.2318$, the difference is = ,0016^{d} = .021 degrees, which the moon is paſſed the node. Alſo $\frac{27729.23}{27.54} =$ 1006.59 or 1006 revolutions to the apſis + 16 days, or 208°; ſo that the apſis is in a wrong poſition. This period is 76 Julian years, abating 29 days, 18 hours, 29 minutes. And the like for other periods.

If you vary the numbers of revolution, but a very little, you will find other periods, not contained in the former ſeries. And therefore different people uſing different numbers, will find different periods.

Cor. *Any of theſe periods being often repeated, will vary more and more; and at laſt will wear out.*

For the period of 47 lunations, runs a degree and a half beyond the node, and being 10 times repeated, it will be 15 degrees diſtant, and without the ecliptic limits.

The period of 223 lunations, or 18 years, is but 23′ ſhort of the node; but being 42 times repeated, will fall ſhort 16 degrees, and ſo will end; and Fig. that is in 757 years.

SCHOLIUM.

Here the year is ſuppoſed to conſiſt of 365 days and 6 hours; whilſt the common year conſiſts only of 365 days. Therefore for 1, 2, 3 odd years above a multiple of 4, allowance muſt be made for the 6 hours, and alſo for leap year, if it happens among the odd years.

PROB. X.

To obſerve the times and phaſes of ſolar and lunar eclipſes.

The obſervation of eclipſes is of great uſe for correcting the tables of the ſun and moon, and alſo for diſcovering the different longitudes of places.

In eclipſes of the ſun, the beginning and end are eaſily and plainly diſtinguiſhed; for the light part is ſeparated from the dark, by a line, which is the ſide of the moon's body, which makes the light part be diſtinctly terminated; for which reaſon it is viſible at the firſt inſtant, and at laſt vaniſhes in a moment. But then one muſt look at it, thro' a coloured piece of glaſs; for there is no enduring the ſun's rays.

But to ſee more exactly the beginning and end, or any other phaſis; the beſt way is to look thro' a teleſcope with a coloured glaſs to defend the eye. Then having a good clock or watch to know the times of the beginning and end; the middle of the eclipſe is eaſily known. Likewiſe the digits eclipſed muſt be meaſured.

The figure of an eclipſe of the ſun may be projected upon a paper, after this manner. Place a teleſcope in a window ſhut, and let it be directed

Fig. to the ſun that the rays may come thro' it. Then place a paper ſo far from the eye glaſs, that the ſun's diſk may appear diſtinct and of ſufficient breadth upon it. Then if a circle be drawn of the bigneſs of the diſk, and divided into degrees, and the diameter into digits; then the poſition of the horns, may be found, and the quantity of the enlightened part, or dark part, may be meaſured. If your paper be fixt at a certain place, you may make the image diſtinct upon it, by moving the eye glaſs in or out.

In lunar eclipſes, neither the beginning nor end can be diſtinctly perceived, by reaſon of the earth's penumbra; for as the real ſhadow and penumbra are not viſibly ſeparated from one another, or diſtinctly terminated; no one can exactly tell where one ends and the other begins, nor conſequently when the eclipſe begins or ends. For we do not think the moon eclipſed at all, till the darkneſs be ſo great as ſenſibly to affect the eye. Tho' we are ſure it really begins to be eclipſed, as ſoon as the penumbra touches it. Therefore the beſt way for making an obſervation is, to note the time when any remarkable place, upon the moon's ſurface, enters the ſhadow; which ſhadow we muſt judge of, as well as we can. And therefore we muſt uſe a teleſcope, as we can judge better by that, than by the bare eye, where the ſhadow begins. The place obſerved is eaſily known by a map of the moon. And it will be as well to obſerve two or three of theſe ſpots or noted places. This done the time muſt alſo be obſerved, when the ſhadow leaves the place or places; and from theſe you will have the time of the middle of the eclipſe. Thus if ſeveral people in different places obſerve any noted place agreed on upon the moon; then they will have the middle of the eclipſe in both places, nearly. This way is more exact than obſerving the beginning or end, as none can diſtinguiſh ſo exactly between the true ſhadow and the penumbra, till they both appear upon the ſun's body together.

PROB. XI.

To calculate the time of an eclipſe of Jupiter's firſt Satellite.

Having in Prob. 14. part. I. of my book of Navigation laid down the rules for doing it. I ſhall not here repeat them, but refer the reader to that book; only taking notice, that as Dr. Halley's tables being in time found to differ from obſervations; I attempted to make ſome alterations therein according to the obſervations I had made. But theſe obſervations not being ſo many as I could have wiſhed; I dare not venture to ſay, that theſe tables, will in all caſes give the time correctly. And therefore I ſhould be glad if proper perſons, who have the benefit of an Obſervatory, would take this matter into conſideration; and by a ſufficient number of obſervations, made in all poſitions of the earth to Jupiter and the Sun, conſtruct from thence a ſet of correct tables. Which would be a thing of great uſe, as theſe eclipſes happen ſo often, and are ſo eaſily obſerved. Till then we muſt be forced to uſe ſuch tables as we have. Here I ſhall make uſe of Pound's tables (which we have among Halley's aſtronomical tables), and thoſe of Wargentine, which are calculated for the meridian of Paris.

1. *By Halley's or Pound's Tables.*

The Precepts.

1. From the firſt table, take out the epocha for the year, with its correſpondent numbers A and B. And from the table of months, days,

&c. take the day, hour, &c. next leſs than the time ſought for, with the numbers A and B. Add the ſeveral numbers together; and you have the mean time of the middle of the eclipſe.

2. From the table of *the firſt equations*, with number A, take out the firſt equation of the conjunctions; and the equation of B, to be added to B.

3. From the table of the *ſecond equations*, with number B thus equated, take out the ſecond equation of the conjunctions. And from the laſt table, with number A, take out the third equation. Add all theſe equations to the mean time of the middle of the eclipſe; and you have the true equated time of the middle of the eclipſe.

4. Alſo from the laſt table, with number A, take out the ſemiduration. And if number B equated, be leſs than 500, ſubtract the ſemiduration, and you have the time of the immerſion; if it be more than 500, add it, and it will give the time of emerſion.

5. This being mean time, reduce it to the apparent time. And in leap year after February, make it a day leſs. When the numbers A or B exceed 1000, reject 1000.

Example.

To find the time of emerſion of Jupiter's firſt Satellite, July 12th, 1768.

1. *By Pound's Tables.*

Theſe tables being calculated for the old ſtile, we muſt take out for July 1,

	d	h	m	s	A	B
1768	1	2	7	29	2	262
July	1	7	5	44	42	455
mean time	2	9	13	13	44	717
1 equa.			29	3		19
2 equa.			7	5		736
3 equa.			3	27		
equat. time	2	9	52	48		
ſemidurat.		+1	4	19		
	2	10	57	7	emerſion.	
leap year	1	10	57	7		
new ſtile	12	10	57	7		
eq. time		—	5	5		
ap. time	12	10	52	2	of emerſion.	

2. *By Wargentine's Tables.*

The Precepts.

1. From Tab. 1 and 2, take the time for the given year, and alſo for the month, day, &c. next leſs than the day propoſed; with the numbers A, B, C; which add into ſeparate columns, and you have the mean time of the middle of the eclipſe.

2. From tab. 3, with number A, take out the firſt equation; and the corrections of B and C to be added to them.

3. From tab. 4, with number B, take out the ſecond equation. And from tab. 5, with number A, the third equation. And from tab. 6, with number C, the fourth equation. Add all theſe to the mean time, and you have the equated time of the conjunction.

4. From tab. 8, with number A, take out the ſemiduration. And if number B equated, be leſs than 500, ſubtract; if greater, add the ſemiduration; and you will have the time of immerſion, or emerſion, accordingly.

5. From tab. 7, take the equation of time, which

add to the mean time of immerſion or emerſion, gives the apparent time thereof at *Paris*. And ſubtracting 9′ 20″, gives the time at *London*.

N. B. In leap year, after February, take out for a day leſs. And when the numbers B and C exceed 1000, reject 1000.

Example, the ſame as before.

	d	h	m	s	A	B	C
1768	1	11	3	19	2	236	78
July	1	7	5	43	152	457	416
day	10	14	51	36	9	28	24
mean time	12	9	0	38	163	721	518
1 equa.			4	1		18	12
2 equa.			8	20		739	530
3 equa.			28	45			
4 equa.			6	56			
equat. time	12	9	48	40			
ſemidura.		+1	4	22			
end.	12	10	53	2			
eq. time			+9	39			
time at Paris	12	11	2	41			
			−9	20			
time at London	12	10	53	21	Emerſion.		

PROB. XII.

To find the longitude of a place.

To find the difference of longitude of two places, or the difference of meridians, is an eſſential point both of geography and navigation. To perform which ſeveral methods have been propoſed, ſome more practicable than others; but all of them depend upon aſtronomical principles, and therefore I give it a place here. The principal methods of doing it, I ſhall here lay down.

1 *Way*,

1 *Way, by a Time Piece.*

If a watch or time piece can be made to keep time exactly, ſuch a machine being ſet to the true time of the day in any one place, and carried to another place, it will ſtill ſhew the time at the firſt place. And therefore if the time can be known at this ſecond place; the difference of time, and therefore the difference of longitude, will be known, between one place and the other.

Now to find the hour of the day at this ſecond place, two equal altitudes of the ſun muſt be taken the ſame day, and the exact times noted by the time keeper. Then the middle between theſe two times of obſervation, ſhows what hour it is at the firſt place, when it is 12 o'clock at the ſecond place. Conſequently the difference of time between the two places is known, which gives the difference of longitude allowing 15 deg. to an hour. If the time keeper be ſhort of 12 o'clock, at the middle time, the ſecond place lies eaſtward from the firſt. But if the middle time be paſt 12, the ſecond place lies weſtward. A common watch being ſet to 12, when the time keeper is at the middle time; or, which is the ſame thing, if it be ſet ſo much paſt 12, as the time keeper is paſt the middle time; it will continue to ſhew the time in that place. This method is adapted to the ſea.

If ſuch a machine be exactly made, like Mr. Harriſon's time keeper; it may do well enough for ſhort diſtances, but will be uſeleſs for long ones, for the following reaſons. 1. Heat and cold will have an effect upon the ſprings, and conſequently will retard or accelerate the motion. 2. The toſſing of the ſhip at ſea will have an effect upon any moving machine, which will hinder it from going equally at all times. 3. Whatever may be the error generated in a ſmall diſtance, or ſmall time;

that error will be multiplied in a large diſtance and time, being proportional to the time; ſo that in long voyages it will be uſeleſs. 4. The immenſe price of ſuch a machine, when made to the utmoſt accuracy, will prevent its being generally uſeful.

2 Way, by the Moon's diſtance from a Star.

1. You muſt have an ephemeris ſhewing the moon's right aſcenſion, declination, and horary motion in right aſcenſion, every day at noon, made for a certain place, as London.

2. At any other place, when an obſervation can be had, the moon's diſtance from ſome known ſtar, muſt be exactly taken with a good inſtrument, and at the ſame time, their altitudes or zenith diſtances.

3. This apparent diſtance of the moon and ſtar, muſt be reduced to the true diſtance, by Prop. 17. Sect. VI.

4. Then the difference of right aſcenſions of the moon and ſtar muſt be found (by Cor. 2. Prob. 35. Sect. II.).

5. Then having the right aſcenſion of the ſtar from the aſtronomical tables, the right aſcenſion of the moon will be known; and conſequently (by the ephemeris) how much right aſcenſion ſhe has gained ſince noon.

6. Then ſay, as the horary motion, to 1 hour:: ſo the difference of right aſcenſions, to the time from noon.

7. This time reduced to degrees, gives the difference of longitude, between the place of obſervation and London.

Note, In finding the difference of right aſcenſions (Art. 4.) the moon's declination at noon is made uſe of. But it may be corrected by Art. 6; and then the work repeated, for more exactneſs. And the like for the horary motion.

This method may equally be applied to the ſun, as well as to a ſtar. The former way is uſed in the night; the latter in the day.

The occultation of a ſtar by the moon, or the appulſe of a ſtar to the moon, is but a particular caſe of this method. For at their firſt touching, the diſtance is equal to the moon's apparent ſemidiameter, and is therefore known without meaſuring.

There are ſeveral difficulties which attend this method, 1. The method is embarraſſed with ſo many difficult computations, that it is hardly to be performed at the time when wanted, beſides the danger of error. 2. The taking of an angle between the moon and a ſtar cannot be performed ſo exactly as the Prob. requires; eſpecially at ſea, where the motion of the ſhip fruſtrates any ſuch attempt. It has been objected, that the time of an eclipſe of a Satellite cannot be obſerved at ſea, by reaſon of the ſhip's motion; much leſs then, can an angle be taken, which requires looking at two things at once. 3. No lunar tables extant can give the place of the moon true to 2′, and there is no hopes of getting a perfect theory of the moon. So in the main, we never can be ſure we come within 4′ or 5′, of the truth. 4. Since in a revolution of 360 deg. the moon gains but 13° in right aſcenſion; therefore in 1 deg. ſhe gains but about 2′, a very ſmall matter, therefore upon this account, we cannot think to come nearer than 4′ or 5′. 5. In taking the altitudes, the horizon is ſo badly defined in the night, that they cannot be truly taken.

3 *Way, by having a Meridian.*

Here one muſt have an ephemeris, ſhewing the time of the moon's ſouthing, every night, and their differences. And at the place propoſed a meridian muſt be found, by two altitudes of the ſun or otherwiſe. Set a good watch to 12, when the ſun comes to the meridian; and note the time when the moon ſouths or comes to the meridian. Then ſay, as the difference of times (by the ephemeris), to the time by the watch : : ſo 360°, to the difference of longitude. This method is proper for the land.

The difficulties in this method, are. 1. The difficulty of drawing a meridian truly. 2. It ſuppoſes the watch to go truly all the while, which ſeldom happens. But this error may be rectified, by obſerving the watch, when the ſun is upon the meridian next day, to ſee what it gains or loſes, and allow proportionally, for the time of the moon's ſouthing. 3. It ſuppoſes the moon moves equally every hour, which ſhe does not. And this may be partly remedied, by comparing the differences between one day and another, and augmenting or diminiſhing the time by the watch, accordingly.

4 *Way, by the Moon's eclipſe.*

Let the middle time of a lunar eclipſe, be obſerved in two different places, according to the directions in Prob. X. Then the difference of the times, reduced to degrees, at 15° to an hour, will give the difference of longitude of the two places; and the place where the eclipſe was lateſt ſeen, is to the eaſtward of the other.

The difficulties here are, 1. The exact time of the ſhadow, touching any noted place upon the moon, cannot be truly known, by reaſon of the penumbra; being as dark as the ſhadow, where it borders upon it. 2. It is hard to have a clock adjuſted to true time, in every place, where the longitude is required.

The longitude cannot ſo eaſily be found by a ſolar eclipſe, but requires a deal of calculation.

5 *Way, by the Eclipſes of Jupiter's firſt Satellite.*

Here a table muſt be made, (by Prob. XI.) of the eclipſes of Jupiter's firſt Satellite, and the time when they happen at any noted place, as London. Then if the time of the eclipſe be obſerved at any other place, by a teleſcope; and this time of immerſion or emerſion be compared with the table; you will have the difference of the times; which turned into degrees, at 15° to an hour, gives the difference of longitude; which is eaſt if it happen ſooner, or weſt if later. This method is very proper for the ſea.

The greateſt defect of this method, is the errors of the tables, which do not give the times truly, ſometimes to 2 or 3 minutes of time.

SCHOLIUM

The length and difficulty of the computations made uſe of in regard to the moon, render theſe methods very incommodious for finding the longitude; and the uncertainty of ſome elements concerned therein, make the operations doubtful. It is true, there is occaſion for nothing but mere obſervation in lunar eclipſes for finding the longitude, but the uncertainty of the time of beginning, &c. will introduce an error. Of all methods, the eclipſes of the Satellites of Jupiter are the moſt proper means for finding the longitude; for that way requires no trouble, no intricate calculations. The whole buſineſs depends on this, to have a true theory of theſe Satellites, at leaſt of the firſt of them; but which is ſurprizing, no body turns their thoughts that way. It is true, it requires a great deal of time, and a great number of obſervations to ſettle the theory; but when that is done it will ſufficiciently recompence the trouble. For if we had but correct tables of this ſingle Satellite, we ſhould be enabled to calculate the exact time of the eclipſes; and if this could be truly done, it would abſolutely be the beſt method for finding the longitude, either at land or ſea, that ever was thought of. If this method cannot be put in practice at ſea, much leſs can any other methods, ſince they all require more attention and more ſteadineſs, than this does, where a man has nothing to do but look thro' a tube, at a certain object, till the obſervation be over, which need not be very long.

I have ſhewn in Cor. 3. Prop. 20. Sect. VI. That there are ſeveral irregularities that affect the moon, that quite vaniſh in this Satellite; from whence one would think it would be no hard matter to eſtabliſh the elements of its motion. Yet there is ſome difficulty in this, by reaſon of its great diſtance. As ſuppoſe we want to find the excentricity of its orbit, and the poſition of the apſides, the greateſt elongation of the Satellite from Jupiter is 112″; and if its orbit was ſimilar to the moon's, then the excentricity would be $\frac{1}{18}$ of its mean diſtance, which would only amount to 5 or 6 ſeconds; but how difficult is this to obſerve exactly? And yet the greateſt part of its irregular motion depends upon its excentricity. It would not be ſo difficult to find the place of the node, and eaſier ſtill to find the inclination of its orbit. I am perſuaded, that all theſe difficulties may be ſurmounted, by diligent obſervations made at all proper opportunities, and in all poſitions of the ſun, the earth, and Jupiter; and then reaſoning duly from the phænomena, by comparing them with the moon's motions. By this means we ſhould at laſt gain all the elements of its motion; and be able to conſtruct tables not only for the time of the eclipſes, but of its appearance upon Jupiter's diſk, or of any other phænomena. This would require ſome labour, but without pains and patience nothing is to be done.

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E R R A T A.

b ſignifies reckon from the bottom.

pag	line	read
5	6	⊕ the earth
9	8b	AFOHG,
32	1	Fig. 10.
41	1b	the earth, ⊕
42	14b	♂, ⊕, ♀,
	7b	♀ 24, ⊕ 21,
56	2b	Let CB be the
66	11b	ſtar's is leſs.
86	13b	Prob. XL.
93	4	$19''.71 =$
	11	$19''.71$ ....
	13	40.28 ...
103	17	planet, the
104	6	XXVI. Sect. III. of
107	13b	convex ſurface
136	13	2 altitudes
137	15b	or elſe that
	8b	the times of
155	18	known. Or the
168	6	1. *To find it by calculation.*
169	4	Therefore $sx =$
183	17	$\sqrt{d} : \sqrt{r} :: s :$
186	15b	B*bd*, S*db*,
	14b	SH*b* the comet's
187	11b	Fig. 62.
	4b	$s\sqrt{\frac{r}{d}}$, and
188	4	$s\sqrt{\frac{r}{d}} = s\sqrt{\frac{4p}{d}}$,
	1	Fig. 62.
192	3b	the ecliptic. The 3d
193	8b	to, Dec. 30 20 26′ add 1742.
194	4,5b	(amongſt whom is the earth),
199	9	3.1416) $= \varphi$
201	9b	NH $= sy$ ... CB (c) : CA
204	1b	y (SN) :
209	1	Fig. 70.
	10b	$y = a + nx$,
210	1b	$\sqrt{\frac{1}{2} - \frac{n + ax}{2y}} =$
216	11b	3. Prop. XI.
217	10	a' coming to
219	7	quadrants
220	12	Fig. 75.
226	10	$\sqrt{\overline{FA}^2} : \sqrt{FA \times FP}$
227	7	write out the
230	17	elongation, PES ::
233	15	⊕ place 1 21
	11b	take the ∠ ⊕ 2 2
234	8b	⊕ long. 11 17
	6b	triangle ♀ S ⊕
	5b	angle at ⊕
	2b	angle at ⊕
241	9b	$= 59^\circ.75$; therefore
243	19	as p and P,
247	5b	⊕ place is
248	5	⊕ place is
249	22	Log: Diſt. of ⊕
255	4	$\frac{2p}{y} - 1$,
258	20b	0.085068
275	4	= 3ce the ſine
	8	$s =$ ſine
278	7	$\frac{rzz}{2}$. But
296	13b	found before. When
300	16	dele 1.
302	12b	reduction. By
318	17	diſtance $ms =$
331	7b	from the node;
333	1	pa. 331
339	11	35.55. And
352	9	place Z would
355	13	$\frac{a}{e} = \frac{27.2122}{29.5306}$
	18	$27.54y = 29.53a$.
361	1	pa. 361.

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